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Semester - VI (Mechanical)

APPLIED THERMODYNAMICS

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PREFACE

The importance of **Applied Thermodynamics** is well known in various engineering fields. Overwhelming response to our books on various subjects inspired us to write this book. The book is structured to cover the key aspects of the subject **Applied Thermodynamics**.

The book uses plain, lucid language to explain fundamentals of this subject. The book provides logical method of explaining various complicated concepts and stepwise methods to explain the important topics. Each chapter is well supported with necessary illustrations, practical examples and solved problems. All the chapters in the book are arranged in a proper sequence that permits each topic to build upon earlier studies. All care has been taken to make students comfortable in understanding the basic concepts of the subject.

Representative questions have been added at the beginning of each section to help the students in picking important points from that section.

The book not only covers the entire scope of the subject but explains the philosophy of the subject. This makes the understanding of this subject more clear and makes it more interesting. The book will be very useful not only to the students but also to the subject teachers. The students have to omit nothing and possibly have to cover nothing more.

We wish to express our profound thanks to all those who helped in making this book a reality. Much needed moral support and encouragement is provided on numerous occasions by our whole family. We wish to thank the **Publisher** and the entire team of **Technical Publications** who have taken immense pain to get this book in time with quality printing.

Any suggestion for the improvement of the book will be acknowledged and well appreciated.

Authors
Anup Goel
Siddu Patil

Dedicated to God

SYLLABUS

Applied Thermodynamics - (3161910)

Credits	Examination Marks				Total Marks
C	Theory Marks		Practical Marks		
	ESE (E)	PA(M)	ESE (V)	PA (I)	
5	70	30	30	20	150

1. **Properties of gases and gas mixtures** : Avogadro's law, equation of state, Vander Waal's equation, reduced properties, law of corresponding states, compressibility chart, internal energy; enthalpy and specific heat of a gas mixtures **(Chapter - 1)**

2. **Psychrometry** : Dalton's law of partial pressure, Properties of moist air, temperature and humidity measuring instruments, psychrometric chart, psychrometric processes such as sensible heating and cooling, heating and humidification cooling and dehumidification, chemical dehumidification, adiabatic saturation **(Chapter - 2)**

3. **Refrigerant and Refrigeration cycles** : Classification, nomenclature and desirable properties of refrigerant, secondary refrigerants, ODP and GWP, Compound compression with intercooler, flash gas removal and flash intercooler, Desirable characteristics of refrigerant absorbent pair for vapor absorption cycle, Simple H_2O -NH_3 cycle, $LiBr_2$ – H_2O cycle and its working **(Chapter - 3)**

4. **Fuel Air and Actual Cycles** : Assumptions for fuel–air cycles, reasons for variation of specific heats of gases, change of internal energy and enthalpy during a process with variable specific heats, isentropic expansion with variable specific heats, effect of variable specific heats on Otto, Diesel and Dual cycle, dissociation, comparison of air standard and fuel air cycles, effect of operating variables, comparison of air standard and actual cycles, effect of time loss, heat loss and exhaust loss in petrol and diesel engines. **(Chapter - 4)**

5. **IC engine performance and Emissions** : Measurement of indicated power, brake power, friction power, fuel consumption and emission, calculation of brake thermal efficiency, brake power and brake specific fuel consumption, variable compression ratio engines, heat balance sheet, principal engine emissions, source of engine emissions, emission measurement instruments like five gas analyzer and smoke meter, Euro and Bharat standards of emissions of I.C. Engines, Emission control methods like Air injection, Exhaust gas recirculation, Catalytic converter, Evaporative emissions control **(Chapter - 5)**

6. **Fundamentals of compressible flow** : Ideal gas relationship, Adiabatic energy equation, Mach number and its significance, Mach waves, Mach cone and Mach angle, static and stagnation states, relationship between stagnation temperature, pressure, density and enthalpy in terms of Mach number, stagnation velocity of sound, reference speeds, various regions of flow, Effect of Mach number on compressibility, Area velocity relationship. **(Chapter - 6)**

7. **Reciprocating Compressors** : Construction and working, Multistage conditions for minimum work, Intercooling, Efficiency and control of air compressors

   Centrifugal Compressors : Essential parts, Static and total head properties, Velocity diagram, Degree of reaction, surging and choking, Losses in centrifugal compressor

   Axial Flow Compressors : Construction of an axial flow compressor, Aerofoil blading, Lift and drag, Performance characteristics **(Chapters - 7, 8, 9)**

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CHAPTER - 1

PROPERTIES OF GASES AND MIXTURES

Syllabus : Avogadro's law, equation of state, Vander Waal's equation, reduced properties, law of corresponding states, compressibility chart, internal energy; enthalpy and specific heat of a gas mixtures.

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1.1 Introduction to Ideal Gas

- An ideal gas is defined as a gas having no forces of intermolecular attraction.
- The gases which follows the gas laws at all ranges of temperature and pressure are considered as **ideal gases** or **perfect gases.**
- But, the real gases follow these laws at high temperatures and low pressures or at both. For example, air, oxygen, hydrogen etc.
- It is because of the forces of attraction between the molecules which tend to be very small at low pressures and high temperatures.
- An ideal gas follows the ideal gas equation (pV = mRT) but the specific heat of ideal gas varies with the temperature.
- Similarly, a perfect gas also follows the ideal gas equation (pV = mRT) but the specific heat of perfect gas is assumed to be constant with variation in temperature.
- A perfect gas is most suitable for analysis of practical machinery which uses real gases as a working substance.
- In nature, there is no ideal or perfect gas.

1.1.1 Properties of Gases

☞ **[GTU : Winter-14,16, Summer-17]**

- Gas does not have surface of its own.
- A gas can diffuse into another gas.
- After cooling gas can be converted into liquid.
- The gases can flow in any direction.
- The gas does not have definite shape or volume of its own.

1.2 Ideal Gas Laws

☞ **[GTU : Winter-15,17,18, Summer-18]**

- The behaviour of ideal gases according to the experimental results are governed by the following laws called as **ideal gas laws.**

1.	Boyle's law	2.	Charle's law
3.	Gay-Lussac's law	4.	Avogadro's law

1.2.1 Boyle's Law

- This law was invented by Robert Boyle. It states that *if unit mass of gas is maintained at constant temperature, then the volume of gas is inversely proportional to the absolute pressure of gas.*

Mathematically, $V \propto \frac{1}{p}$ ... if temperature T = Constant

$$\therefore \quad p \cdot V = C \qquad \ldots (1.1)$$

$$p_1 \cdot V_1 = p_2 \cdot V_2 \text{ or } = \frac{p_1}{p_2} = \frac{V_1}{V_2} \qquad \ldots (1.2)$$

- Consider process 1-2 as shown in Fig. 1.1 In this process, a gas is at point 1 with p_1, V_1 and at point 2 with p_2, V_2 but temperature is kept constant (T = C).
- As temperature is constant, this process is also called isothermal process.

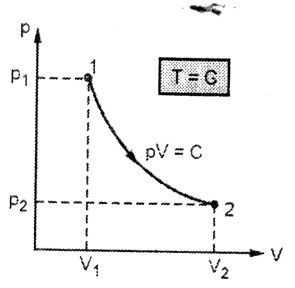


Fig. 1.1 : Boyle's Law

1.2.2 Charle's Law

- This law was inverted by Jacques Charles. It states that, *if unit mass of gas is maintained at constant pressure, then the volume of gas is directly proportional to the absolute temperature of the gas.*

Mathematically, $V \propto T$ ... if temperature p = Constant

$$\therefore \quad \frac{V}{T} = C \qquad \ldots (1.3)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \ldots (1.4)$$

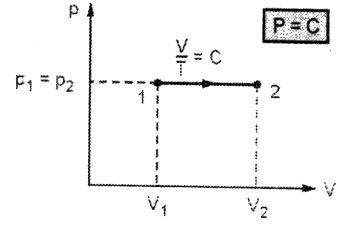


Fig. 1.2 : Charle's law

- Consider process 1-2 as shown in Fig. 1.2. In this process a gas is at point 1 with p_1, V_1, T_1, and at point 2 with p_2, V_2, T_2, but $p_1 = p_2 = C$.
- As pressure is constant, this process is also called as **isobaric process.**

1.2.3 Gay-Lussac's Law

- This law was invented by Gay-Lussac. It states that, the absolute pressure of unit mass of gas varies directly with its absolute temperature, when the volume of gas is kept constant.

Mathematically, $p \propto T$ ... if volume V = Constant

$$\frac{p}{T} = C \quad \ldots (1.5)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \ldots (1.6)$$

- Consider process 1-2 as shown in Fig. 1.3. In this process, a gas at point 1 with p_1, T_1 and at point 2 with p_2, T_2 but volume is kept constant (V = C).
- As volume is kept constant, this process is also called as **isochoric process.**

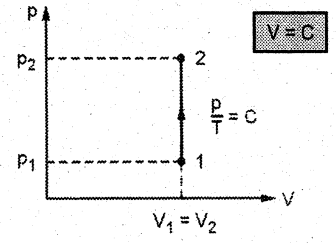


Fig. 1.3 : Gay-Lussac's law

1.2.4 Avogadro's Law

- This law was invented by Avogadro. It states that, the equal volume of different ideal gases at the same pressure and temperature contain the same number of molecules.

Mathematically,

$$\text{Number of moles (n)} = \frac{\text{Total mass (m)}}{\text{Molecular weight (M)}} \quad \ldots (1.7)$$

or $$m = n \cdot M \quad \ldots (1.8)$$

1.3 Dalton's Law of Partial Pressure

☞ [GTU : Summer-19]

- It states 'The pressure of the mixture of air and steam is equal to sum of the pressures, which each constituents would exert, if it is occupied the same space by itself.'
- Mathematical equation

$$p_T = p_a + p_s$$

p_a = Partial pressure of air

p_s = Partial pressure of steam

1.3.1 Gibb's - Dalton Law

- Dalton's law was re-constructed by Gibb's.
- Gibb's includes a second statement on properties of mixture.

- Gibb's Dalton Law – The internal energy, enthalpy and entropy of gaseous mixture are respectively equal to the sum of internal energy, enthalpies and entropies of constituents.
- Each constituents has internal energy, entropy and enthalpy, which it could have if it occupied alone that volume which is occupied by the mixture at temperature of mixture.

Let, u = Internal energy

h = enthalpy

s = entropy

$$\therefore \quad mu = m_1u_1 + m_2u_2 + m_3u_3 + \ldots + mu = \sum m_iu_i$$

$$mh = m_1h_1 + m_2h_2 + m_3h_3 + \ldots + mh = \sum m_ih_i$$

$$ms = m_1s_1 + m_2s_2 + m_3s_3 + \ldots + ms = \sum m_is_i$$

1.4 Amaget's Law

- Consider gaseous mixture with constituents A, B and C.

Using equation, pV = mRT

$$m_A = \frac{p_A \times V}{R_A T} = \frac{pV_A}{R_A T}$$

Equating above two equations

$$\frac{p_A V}{R_A T} = \frac{pV_A}{R_A T}$$

$$V_A = \frac{p_A}{P} \times V$$

For constituent B and C,

$$V_B = \frac{p_B}{P} \times V$$

$$V_C = \frac{p_C}{P} \times V$$

∴ We can write,

$$\sum V_i = \sum \frac{p_i V}{P} = \frac{V}{P} \sum p_i = V$$

Amagat's law : Volume of mixture of gases is equal to sum of volumes of individual constituent when each exists alone at a pressure and temperature of mixture.

1.5 Universal Gas Constant or Molar Constant

- Universal gas constant or molar constant (R_u) is the product of gas constant and molecular mass.

$$R_u = MR$$

- The value of R_u is some for all gases.
- S.I. unit of R_u is kJ/kg mol K or J/kg mol K
- The value of R_u is taken as 8314 J/kg mol K
- The characteristic gas equation (pV = RT) can be written in terms of molecular mass

$$pV = mRT$$

1.6 Properties of Ideal Gases

- Ideal gas consists of large number of identical molecules.
- Volume occupied by molecules themselves is negligible.
- Melecules obay Newtons law of motion.
- During the collision only molecules experiences force.
- The collisions are completely elastic.

1.7 Air and Water Mixture and Related Properties

- The designer must consider the air temperature and amount of water vapour in the air to establish proper comfort codition.
- The evaporation of water into the air can serve to lower air temperature.
- When air and water exists at low pressure, this case can be closely approximated as ideal gas.

$\therefore$ $p_a V = m_a R_a T$ ... (for the air)

and $p_v V = m_v R_v T$ ... (for water vapour)

As per Dalton's law, we can write, total pressure

$$p = p_a + p_v$$

- Total enthalpy is the sum of enthalpy of air and water vapour

$$H = H_a + H_v$$

1.7.1 Mole and Mass Fraction

- Mass fraction : Its defined as the ratio of mass of a constituent of a mixture or compound to the total mass of mixture or compound.
- **Mole fraction :** Its defined as the ratio of volume of the gas V_1 to the total volume of mixture of gases is called as the mole fraction.

Note : Mole fraction = Number of moles of individual gas in mixture/total number of moles of mixture of gases

1.8 Ideal Gas Equation or Equation of State

- An ideal gas equation or the equation of states represents the relationship between pressure, volume and temperature.
- Consider a gas on p-V diagram whose original state is at point 1 with p_1, V_1, T_1 and final state is at point 2 with p_2, V_2, T_2. Refer Fig. 1.4.

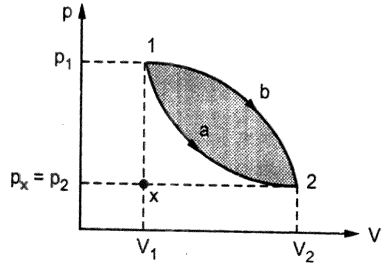


Fig. 1.4 : p-V diagram

- Let, the process from state 1-2 follow path (1-x-2) instead of following the path through a or b.
- From point 1 to x volume is constant hence it follows Gay-Lussac law whereas, from point x to 2 pressure is constant it follows Charle's law.
- Applying Gay-Lussac law for path 1 to x,

$$\frac{p_1}{T_1} = \frac{p_x}{T_x} = \frac{p_2}{T_2} \quad \dots (\because p_x = p_2)$$

... [From equation (1.6)]

$$T_x = \frac{p_2}{p_1} \cdot T_1 \quad \text{... (a)}$$

- Now applying Charle's law for path x to 2,

$$\frac{V_x}{T_x} = \frac{V_2}{T_2} = \frac{V_1}{T_x} \quad \text{... } (\because V_x = V_1)$$

... [From equation (1.4)]

$$T_x = \frac{V_1}{T_2} \cdot T_2 \quad \text{... (b)}$$

Equating equations (a) and (b),

$$\frac{p_2}{p_1} \cdot T_1 = \frac{V_1}{T_2} \cdot T_2$$

$$\frac{p_2 \cdot V_2}{T_2} = \frac{p_1 \cdot V_1}{T_1} \quad \text{... (1.9)}$$

or $$\frac{p \cdot V}{T} = \text{Constant} \quad \text{... (c)}$$

- If v = Specific volume of gas (volume per unit mass of gas)

then, $$v = \frac{V}{m} \text{ or } V = m \cdot v$$

so, the equation (c) can be written as,

$$\frac{p \cdot V}{T} = R \text{ (constant).} \quad \text{... (d)}$$

where, **R is gas constant.**

- Now, substituting the value of $v = \frac{V}{m}$ in equation (d) we get,

$$\frac{p}{T} \cdot \frac{V}{m} = R$$

$$\mathbf{p \cdot V = mRT} \quad \text{... (1.10)}$$

- The above equation (1.10) is called as **ideal gas equation.**

where, p = Pressure in N/m^2 or Pa,

V = Volume in m^3,

m = Mass of gas in kg,

T = Absolute temperature in K and

R = Gas constant in $\frac{N-m}{kgK}$ or $\frac{J}{kgK}$

For air, $R = 0.287 \frac{kJ}{kgK}$

- In the above equation, $p \cdot V = mRT$ if we put $m = n \cdot M$ (from Avogadro's law) then,

$$p \cdot V = nMRT$$

or $$p \cdot V = nR^\circ T$$

where, R° = Universal gas constant = MR

- The value of R° as found by experimental results is given by,

$$R^\circ = 8.3143 \frac{kJ}{kg_{mole}K}$$

1.9 Enthalpy

- Enthalpy is defined as the energy which is the algebraic sum of internal energy and flow work.

Mathematically,

$$\boxed{h = U + p\,V} \quad \text{... (1.11)}$$

where, h = Enthalpy in J,

U = Internal energy in J,

$p \cdot V$ = Flow work in J.

- Enthalpy is also called as **total heat** or **heat content.**
- The above equation (1.11) is applicable for unit mass gas hence **enthalpy (h) is an extensive property.**

Enthalpy as a Function of Temperature

Internal energy

$$U = C_V \cdot T$$

And from ideal gas equation of unit mass,

$$pv = mRT$$

values in equation (4.11)

$$h = U + p \cdot V = C_V \cdot T + R \cdot T$$

$$h = (C_V + R) \cdot T$$

$$h = C_p \cdot T \quad \ldots (\because C_p - C_V = R) \ldots (1.12)$$

- In the above equation (4.12), C_p is constant hence enthalpy h is the function of temperature only.
- The relation $C_p - C_V = R$ will be derived later.

1.10 Specific Heat of a Substance

☞ [GTU : Winter-16, Summer-18]

- Specific heat of any substance is defined as the amount of heat required to raise the temperature of 1 kg of substance by 1 degree of temperature.
- Transfer of heat from any system is given by,

$$\boxed{Q = m \cdot C \cdot \Delta T} \quad \ldots (1.13)$$

where, Q = Heat transfer in J,

m = Mass of substance in kg,

C = Specific heat in J/kg K,

ΔT = Change in temperature of substance in K

1.10.1 Specific Heat for Ideal Gases

- Specific heat of a gas varies according to the process.
- Following are the two important specific heats for the ideal gases :

1. Specific heat for a gas of constant volume (C_V)
2. Specific heat for a gas of constant pressure (C_p)

1. Specific heat of a gas at constant volume (C_V)

- Specific heat of a gas at constant volume is defined as the amount of heat required to raise the temperature of 1 kg of mass by one degree temperature at constant volume.

- It is denoted by C_V and its unit is J/kg K.

2. Specific heat of a gas at constant pressure (C_p)

- Specific heat of a gas at constant pressure is defined as the amount of heat required to raise the temperature of 1 kg of mass by one degree temperature at constant pressure.
- It is denoted by C_p and its unit is J/kg K.

- In case of isochoric (constant volume) process, the workdone is zero (area under the curve) hence heat supplied is used to raise the internal energy of gas.
- But in case of isobaric (constant pressure) process, heat is required for expansion of work and increase the internal energy of gas.
- Hence, heat required in constant pressure process is more than the heat required at constant volume process i.e. C_p is always greater than C_V.
- Therefore we can say that,

$$C_V = \left(\frac{dU}{dT}\right)_V \text{ or } C_p = \left(\frac{dh}{dT}\right)_p \quad \ldots (1.14)$$

where, U = Internal energy of gas in J.

h = Enthalpy (heat and work) of gas in J.

- The ratio of specific heat at constant pressure to the specific heat at constant volume for a gas is called as **adiabatic index.** It is denoted by γ.

$$\therefore \text{Adiabatic index } \boxed{\gamma = \frac{C_p}{C_V}} \quad \ldots (1.15)$$

- As $C_p > C_V$, γ is always greater than 1. Generally γ is taken as 1.4.
- Specific heat of real gases changes with rise in temperature at high pressures as well as low temperatures. It is given by.

$$C_p \text{ or } C_v = a + b^T + CT^2 + dT^3 \quad \ldots (1.16)$$

Where, a, b, c and d = constants.

1.11 Internal Energy (U)

The energy process by the system is called internal energy.

$$U = C_v \cdot T$$

$$du = C_v DT$$

Example 1.1 : *An insulated vessel of 0.5 m^3 capacity is divided by a rigid conducting diaphragm into two chambers A and B each having capacity of 0.25 m^3. Chamber A contains air at 1.4 bar pressure and 290 K temperature and the corresponding parameters for air in chamber B are 4.2 bar and 440 K. Calculate :*
i) Final equilibrium temperature.
ii) Final pressure on each side of diaphragm.
iii) Entropy change of system
For air take, C_v = 0.715 kJ/kg K and R = 0.287 kJ/kg K.

Solution : Given data : Constant volume process.

$V = V_A + V_B = 0.5\ m^3$, $V_A = V_B = 0.25\ m^3$,

$p_A = 1.4\ bar = 1.4 \times 10^5\ N/m^2 = 1.4 \times 10^2\ kN/m^2$,

$T_A = 290\ K$,

$p_B = 4.2\ bar = 4.2 \times 10^5\ N/m^2 = 4.2 \times 10^2\ kN/m^2$, $T_B = 440\ K$,

$R = 0.287\ kJ/kgK$, $C_v = 0.715\ kJ/kg\ K$.

To find : i) T ii) p'_A and p'_B iii) ΔS

Step - 1 : Calculate the equilibrium temperature (T)

Mass of air in chamber A,

$$m_A = \frac{p_A V_A}{RT_A} = \frac{1.4 \times 10^2 \times 0.25}{0.287 \times 290}$$

$$= 0.4205\ kg$$

Mass of air in chamber B,

$$m_B = \frac{p_B V_B}{RT_B} = \frac{4.2 \times 10^2 \times 0.25}{0.287 \times 440}$$

$$= 0.8314\ kg$$

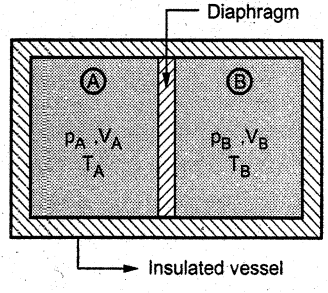


Fig. 1.5

At equilibrium condition, total internal energy of the insulated system is equal.

$$\therefore \quad U = U_A + U_B$$

$$\therefore \quad (m_A + m_B) \cdot C_v \cdot T = m_A \cdot C_v \cdot T_A + m_B C_v \cdot T_B$$

$$T = \frac{(0.4205 \times 290) + (0.8314 \times 440)}{(0.4205 + 0.8314)}$$

$$\mathbf{T = 389.6165\ K.} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate final pressure in each chamber (p'_A and p'_B)

By using ideal gas equation,

$$p'_A = \frac{m_A \cdot R \cdot T}{V_A} = \frac{0.4205 \times 0.287 \times 389.6165}{0.25}$$

$$\mathbf{p'_A = 188.0811\ kN/m^2 = 1.8808\ bar} \qquad \textbf{... Ans.}$$

$$p'_B = \frac{m_B \cdot R \cdot T}{V_B} = \frac{0.8314 \times 0.287 \times 389.6165}{0.25}$$

$$\mathbf{p'_B = 371.8683\ kN/m^2 = 3.7186\ bar} \qquad \textbf{... Ans.}$$

Step - 3 : Calculate the change in entropy of the system.

$$\Delta S = \Delta S_A + \Delta S_B$$

$$= m_A C_v ln\left(\frac{T}{T_A}\right) + m_B C_v ln\left(\frac{T}{T_B}\right)$$

$$= \left[0.4205 \times 0.715 \times \ln\left(\frac{389.6165}{290}\right)\right] + \left[0.8314 \times 0.715 \times \ln\left(\frac{389.6}{440}\right)\right]$$

$$\therefore \quad \Delta S = 0.0887 + (-0.0723) = \mathbf{0.0163\ kJ/K} \qquad \textbf{... Ans.}$$

Example 1.2 : *40 litres of nitrogen contained in cylinder behind a piston is initially at $1.05 \times 10^5\ N/m^2$ and 288 K. The gas is compressed isothermally and reversibly until the pressure is $3.8 \times 10^5\ N/m^2$. Find : i) Change of entropy*
ii) Heat flow iii) Workdone. Draw the process on p-V and T-S diagram. Molecular weight of nitrogen = 28 kg/kg mol. **[GTU : Winter-17, Summer-18]**

Solution : Given data : $V_1 = 40\ litres = 0.04\ m^3$, $p_1 = 1.05 \times 10^5\ N/m^2$, $T_1 = 288\ K$,

$p_2 = 3.8 \times 10^5\ N/m^2$,

Process is isothermal (pV = Constant) $T_2 = T_1 = 288\ K$, $M = 28\ kg/kg\ mol$.

To find : i) ΔS ii) Q iii) W

Step - 1 : Properties of the system

We have, $R = C_p - C_V$

where, $R = \frac{\text{Universal gas constant R}}{M}$

$$R = \frac{8.3143}{28} = 0.2969 \text{ kJ/kg K.}$$

and $$C_V = \frac{R}{(\gamma - 1)} = \frac{0.2969}{(1.4 - 1)} = 0.7423 \text{ kJ/kg K}$$

Now, $$C_p = R + C_V$$

$$C_p = 0.2969 + 0.7423$$

$$C_p = 1.0392 \text{ kJ/kg K.}$$

We know that,

$$p_1 V_1 = p_2 V_2$$

$$1.05 \times 0.04 = 3.8 \times V_2$$

$$V_2 = 11.0526 \times 10^{-3} \text{ m}^3$$

The gas equation is given by,

$$p_1 V_1 = mRT_1$$

$$1.05 \times 10^5 \times 0.04 = m \times 0.2969 \times 10^3 \times 288$$

$$m = \frac{(1.05 \times 10^5) \times 0.04}{(296.9 \times 288)}$$

$$\mathbf{m = 0.0491 \text{ kg}}$$

Step - 2 : Calculate change in entropy, (ΔS)

We have, $$\Delta S = S_2 - S_1 = mR\log_e\left(\frac{V_2}{V_1}\right) = mC_V\log_e\left(\frac{T_2}{T_1}\right)$$ At constant temperature process ($T_1 = T_2$),

$$\Delta S = S_2 - S_1 = mR\log_e\left(\frac{V_2}{V_1}\right)$$

$\therefore$ $$\Delta S = S_2 - S_1$$

$$= 0.0491 \times 0.2969 \times \log_e\left(\frac{11.0526 \times 10^{-3}}{0.04}\right)$$

$\therefore$ $$\mathbf{\Delta S = -0.01875 \text{ kJ/K}}$$ **... Ans.**

Step - 3 : Calculate workdone (W)

We have, $$W = p_1 V_1 \log_e\left(\frac{V_2}{V_1}\right)$$

$\therefore$ $$W = \left(\frac{1.05 \times 10^5}{10^3}\right) \times 0.04 \times \log_e\left(\frac{11.0526 \times 10^{-3}}{0.04}\right)$$

$\therefore$ $$\mathbf{W = -5.4021 \text{ kJ}}$$ **... Ans.**

Step - 4 : Calculate heat flow (Q)

For isothermal and reversible process,

$$\Delta U = 0$$

$\therefore$ $$Q = W + \Delta U$$

$\therefore$ $$Q = -5.4021 + 0$$

$\therefore$ $$\mathbf{Q = -5.4021 \text{ kJ}}$$ **... Ans.**

Example 1.3 : *A gas in a cylinder is at a pressure of 7 bar. It is expanded at constant pressure from 0.25 m^3 to 1.65 m^3 volume. Find the workdone during the process.*

☞ **[GTU : Winter-17]**

Solution : Given data : $p = 7 \text{ bar} = 7 \times 10^5 \text{ Pa}$,

$V_1 = 0.25 \text{ m}^3$, $V_2 = 1.65 \text{ m}^3$

To find : Workdone W.

Step - 1 : Calculate the workdone during the process

Refer Fig. 4.16.

Workdone from state 1 to 2 for constant pressure process is given by,

$$W_{1-2} = \int_1^2 p \cdot dV = p \cdot [V_2 - V_1]$$

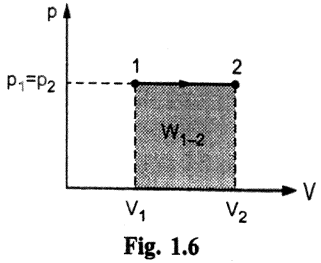


Fig. 1.6

$\therefore \quad W_{1-2} = 7 \times 10^5 \times [1.65 - 0.25]$

$\therefore \quad \mathbf{W_{1-2} = 980 \times 10^3\ J = 980\ kJ}$ **... Ans.**

Example 1.4 : *A spherical balloon contains 5 kg of air at 200 kPa and 500 °K. The balloon material is such that the temperature inside is always proportional to the square of the diameter. Determine the workdone when the volume of balloon doubles as a result of heat transfer. Make your comments on workdone.* ☞ **[GTU : Summer-16]**

Solution : Given data : m = 5 kg, $p_1 = 200$ kPa $= 200 \times 10^3$ Pa, $T_1 = 500$ K,

$T \propto d^2$ (d is diameter of the balloon). $V_2 = 2\,V_1$

To find : Workdone W.

Step - 1 : Calculate the initial and final volume of the piston

We know that, ideal gas equation is,

$$p_1 \cdot V_1 = m \cdot R \cdot T_1$$

$\therefore \quad 200 \times 10^3 \times V_1 = 5 \times 287 \times 500$ ... ($\because$ R = 287 J/kgK)

$\therefore \quad V_1 = 3.5875\ m^3$

But, $V_2 = 2\,V_1$

$= 2 \times 3.5875$

$\therefore \quad V_2 = 7.175\ m^3$

Step - 2 : Calculate the value of constant of proportionality

From ideal gas equation,

$$T = \frac{p \cdot V}{m \cdot R}$$

and volume of balloon (sphere) is,

$$V = \frac{\pi}{6} \cdot d^3 \quad \text{or} \quad d = \left(\frac{6V}{\pi}\right)^{1/3}$$

It is given that, $T \propto d^2$

$\therefore \quad \dfrac{p \cdot V}{m \cdot R} \propto \left[\left(\dfrac{6V}{\pi}\right)^{1/3}\right]^2$

$\therefore \quad p \propto \dfrac{m \cdot R}{V}\left(\dfrac{6V}{\pi}\right)^{2/3}$

$\therefore \quad p = C \cdot (V)^{-1/3}$ ... (where, C is constant)

The value of C can be found by using initial conditions i.e.

$$C = \frac{p_1}{(V_1)^{-1/3}} = \frac{200 \times 10^3}{(3.5875)^{-1/3}}$$

$\therefore \quad C = 306.155 \times 10^3$

Step - 3 : Calculate the workdone when the volume of balloon doubles

We know that, workdone is given by,

$$W_{1-2} = \int_1^2 p \cdot dV = \int_1^2 C \cdot (V)^{-1/3} \cdot dV \qquad \text{... } (\because\ p = C \cdot V^{-1/3})$$

$$\therefore \quad W_{1-2} = C \cdot \left[\frac{V^{(-1/3+1)}}{\left(-\frac{1}{3}+1\right)}\right]_1^2 = C \cdot \left[\frac{V^{2/3}}{2/3}\right]_1^2$$

$$\therefore \quad W_{1-2} = \frac{3}{2} \cdot C \cdot \left[(V_2)^{2/3} - (V_1)^{2/3}\right]$$

$$\therefore \quad W_{1-2} = \frac{3}{2} \times 306.155 \times 10^3 \times [(7.175)^{2/3} - (3.5875)^{2/3}]$$

$\therefore \quad \mathbf{W_{1-2} = 632.162 \times 10^3\ J = 632.162\ kJ}$ **... Ans.**

Example 1.5 : *Air is initially at 1 bar and 27°C is compressed reversibly and adiabatically in a reciprocating engine to final pressure of 25 bar. Find work done, change in enthalpy and change in entropy per kg of air. Assume C_p and C_v of air 1.005 kJ/KgK and 0.717 kJ/kgK respectively.*

Solution : Given data : $p_1 = 1$ bar $= 1 \times 10^5$ N/m^2, $T_1 = 27°$, C = 300 K,

$P_2 = 25$ bar $= 25 \times 10^5$ N/m^2, $C_{pa} = 1.005$ kJ/kgK, $C_v = 0.717$ kJ/kgK

To find : i) Work done (W)

ii) Change in enthalpy (ΔH) iii) Change in entropy (ΔS)

Step- 1 : Calculate the work done (W)

We know,

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \qquad \therefore \quad \frac{300}{T_2} = \left(\frac{1\times10^5}{25\times10^5}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \qquad T_2 = 752.54 \text{ K}$$

Work done, $$W = \frac{mR\,[T_2 - T_1]}{\gamma - 1} = \frac{1\times0.287\times[752.54-300]}{1.4-1}$$

$$\therefore \qquad \mathbf{W = 324.697\ kJ} \qquad \text{... Ans.}$$

Step 2 : Calculate change in enthalpy (ΔH)

$$\text{Change in enthalpy } (\Delta H) = m\cdot C_p\,[T_2 - T_1]$$

$$= 1\times 1.005\,[752.54 - 300]$$

$$\mathbf{\Delta H = 454.803\ kJ/kg} \qquad \text{... Ans.}$$

Step 3 : Calculate change in entropy (ΔS)

During reversible adiabatic process/isentropic process, there is no change in entropy.

... Ans.

Example 1.6 : *A gas has density of 1.875 kg/m³ at pressure of 1 bar and with a temperature of 15 °C. A mass of 0.9 kg of the gas requires a heat transfer of 175 kJ to raise the temperature to 250 °C, while pressure of gas remains constant. Determine :*
i) Characteristic gas constant. *ii) C_p and C_V of the gas.*
iii) Change in internal energy. *iv) Work transfer.*

Solution : Given data :

Constant pressure (isobaric) process, $\rho_1 = 1.875\ kg/m^3$,

$p_1 = p_2 = 1\ bar = 1\times10^5\ N/m^2 = 1\times10^2\ kN/m^2$,

$T_1 = 15\ °C = 273 + 15 = 288\ K$,

$m = 0.9\ kg$, $Q = 175\ kJ$, $T_2 = 250°\ C = 273 + 250 = 523\ K$

To find : i) R, ii) C_p and C_V iii) ΔU iv) W

Step - 1 : Calculate the characteristics gas constant for given process

From the ideal gas equation,

$$p_1V_1 = mRT_1$$

$$p_1 = \rho_1 RT_1 \qquad \ldots \left(\because \rho = \frac{m}{V}\right)$$

$$1\times10^2 = 1.875\times R\times 288$$

$$\mathbf{R = 0.1851\ kJ/kg\ K} \qquad \text{... Ans.}$$

Step - 2 : Calculate the specific heat at constant pressure and volume (C_p and C_V)

Specific heat at constant pressure (C_p) is,

$$Q = mC_p(T_2 - T_1)$$

$$175 = 0.9\times C_p\times(523-288)$$

$$\mathbf{C_p = 0.8274\ kJ/kg\ K} \qquad \text{... Ans.}$$

As, $$C_p - C_V = R$$

$$C_V = C_p - R = 0.8274 - 0.1851$$

$$\mathbf{C_V = 0.6423\ kJ/kgK} \qquad \text{... Ans.}$$

Step - 3 : Calculate the change in internal energy (ΔU)

$$\Delta U = U_2 - U_1 = mC_V(T_2 - T_1)$$

$$= 0.9\times0.6423\times(523-288)$$

$$\mathbf{\Delta U = 135.8464\ kJ} \qquad \text{... Ans.}$$

Step - 4 : Calculate the work transfer in given process

Work done in isobaric process is given by,

$$W = mR(T_2 - T_1)$$

$$= 0.9\times0.1851\times(523-288)$$

$$\mathbf{W = 39.1486\ kJ} \qquad \text{... Ans.}$$

1.12 Real Gases

- The gases which does not obays the ideal gas equation i.e. pV = RT are termed as real gases.
- Practically there is no real gas because we can't maintain any thermodynamic properties constant during the process.
- For example : Nitrogen, oxygen, carbon dioxide, helium, etc.

1.13 Van-der Waal's Equation of State ☞ [GTU : Winter-12,14, Summer-16]

- The ideal gas equation of state pV = RT
- Practically there is no real gas which obays the gas law perfectly.
- Following assumptions were made,

i) There is no attraction or repulsion between the molecules.

ii) The volume occupied by molecules is very small compare to volume of gas.

- In actual practice, molecules of real gases occupy some space and attract each other.
- Therefore there is no real gas which confirms to perfect gas equation pV = RT.
- The Vander Waal is the first scientist who corrects this equation by considering two factors.

i) Correction for size of molecule

- Consider some amount of gas in vessel.
- The gas molecule can have some size and hence space available for molecule to move is less than volume of gas.

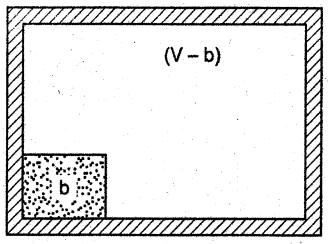


Fig. 1.7 Correction for size of molecule

V - Volume of gas

b - Volume of molecule

$\therefore$ Space available = V – b

ii) Correction for mutual attraction of molecules

- Molecule A is present in vessel which is attracted by all other molecules in all direction.

- When this molecule strike the wall of the vessel it is pulled back some other molecules.
- When pressure of gas drops momentum also drops.
- This decrease in pressure is proportional to number of attracting molecules and number of attracting molecules striking unit area of wall

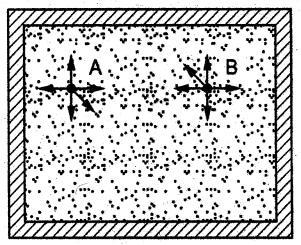


Fig. 1.8 Correction for attraction

Decrease in pressure $(p') \propto \frac{1}{v^2}$

or $p = \frac{a}{v^2}$

Where a is proportionality constant.

Corrected pressure $= p + p' = p + \frac{a}{v^2}$

$\therefore$ equation pV = RT becomes

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

This equation known as Vander Waal's equation

1.13.1 Evaluation of Constant 'a' and 'b'

We have the equation

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

$$pV - pb + \frac{a}{V^2}V - \frac{a}{V^2}b - RT = 0$$

$$pV - pb + \frac{a}{V} - \frac{ab}{V^2} - RT = 0$$

Multiplying both sides by $\frac{V^2}{p}$

we get,

$$pV \times \frac{V^2}{p} - pb \times \frac{V^2}{p} + \frac{a}{V} \times \frac{V^2}{p} - \frac{ab}{V^2} \times \frac{V^2}{p} - \frac{RT}{p} V^2 = 0$$

$$\therefore \quad V^3 - \left(b + \frac{RT}{p}\right) V^2 + \frac{av}{p} - \frac{ab}{p} = 0$$

- At critical point three roots are equal

$$\therefore \quad (V - V_c)^3 = 0 \qquad \ldots(1.17)$$

Where, suffix c denotes condition at critical point

$$V^3 - \left(b + \frac{RT_c}{p_c}\right) V^2 + \frac{av}{p_c} - \frac{ab}{p_c} = 0 \qquad \ldots(1.18)$$

Equation (1.17) and (1.18) are identical equating for coefficients.

$$3V_c = b + \frac{RT_c}{p_c}$$

$$3V_c^2 = \frac{a}{p_c}$$

$$V_c^3 = \frac{ab}{p_c}$$

By reducing above equations we have.

$$\left.\begin{aligned} V_c &= 3b \\ p_c &= \frac{a}{27b^2} \\ T_c &= \frac{8}{27} \cdot \frac{a}{bR} \end{aligned}\right\} \ldots(1.19)$$

From above relation we can find value of a and b.

$$a = 3p_c V_c^2 = \frac{9}{8} RT_c V_c$$

$$\boxed{a = \frac{27}{64} \frac{R^2 T_c^2}{P_c}}$$

$$b = \frac{V_c}{3}$$

$$\boxed{b = \frac{RT_c}{8P_c}}$$

Table 1.1 constants of Vander Waals equation are as follows :

Sr.no	Substance	a $Nm^2(kgmol)^2$	b $m^3/kg\text{-}mol$
1.	Hydrogen (H_2)	25105	0.0262
2.	Oxygen (O_2)	139250	0.0314
3.	Carbon dioxide (CO_2)	362850	0.0423
4.	Helium (He)	3417620	0.0228
5.	Air	135522	0.0362
6.	Water (H_2O)	551130	0.0300
7.	Mercury (Hg)	2031940	0.0657

Table 1.1

1.14 Redlich - Kwong Equation

- This equation is an empirical algebric equation which relates pressure, temperature and volume of gases.

$$p = \frac{RT}{V_m - b} - \frac{a}{\sqrt{T}\, V_m (V_m + b)}$$

Where,

p = Pressure of gas

R = Gas constant

T = Temperature

V_m = Molar volume (V/n)

a = Constant that corrects for attractive potential of moleculs.

b = Constant that corrects for volume

1.15 Compressibility Chart and Compressibility Factor

- The dimensionless compressibility factor, Z, for a gaseous species is defined as the ratio

$$Z = \frac{p\overline{V}}{RT} \qquad \ldots(1.20)$$

If the gas behaves ideally Z = 1.

- The extent to which Z differs from 1 is a measure of the extent to which the gas is behaving nonideally.
- The compressibility can be determined from experimental data where Z is plotted versus a dimensionless reduced pressure p_R and reduced temperature T_R defined as.

$$p_R = p/p_c \text{ and } T_R = T/T_c$$

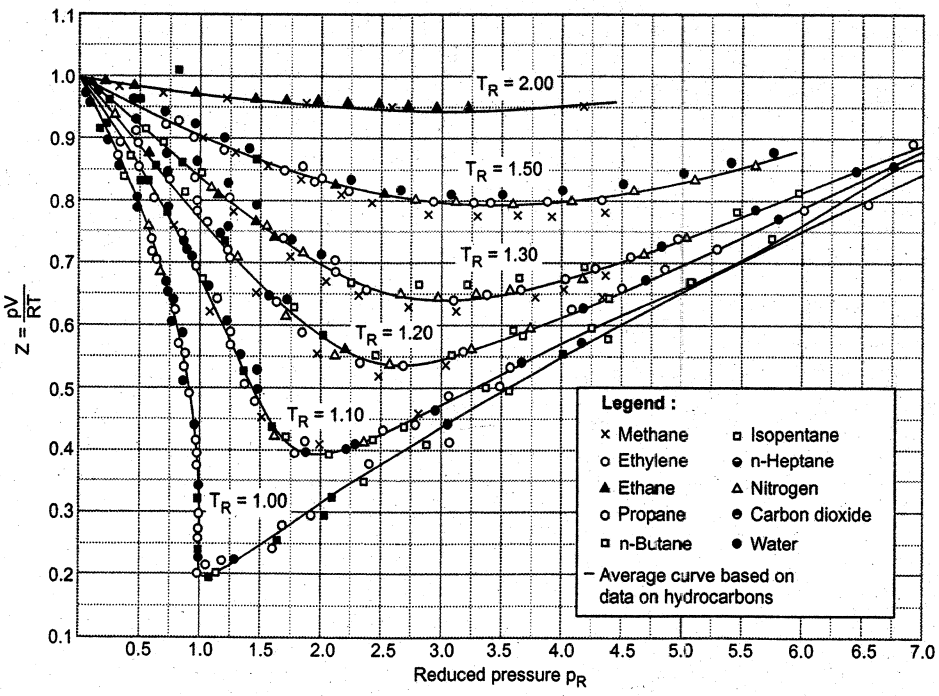


Fig. 1.9 Generalized compressibility chart for various gases

- If these expression, p_c and T_c denote the critical pressure and temperature, respectively.
- A generalized compressibility chart of the form $Z = f(p_R, T_R)$ is shown in different gases.
- The solid lines represent the best curves fitted to the data.
- It can be seen from Fig. 9.16 that the value of Z tends to unity for all temperatures as pressure zero and Z also approaches unity for all pressure at very high temperature.
- If the p,V and T data are available in table format or computer software then you should not use the generalized compressibility chart to evaluate p, V, and T since using Z is just another approximation to the real data.

1.16 Law of Corresponding States

- It states, if any two gases have the same values of reduced pressure and reduced temperature, then they have same values of reduced volume.

i.e. $V_R = f(T_R, P_R)$ for all gases and function is same.

1.17 Difference between Ideal and Real Gases

Sr. No.	Ideal gas	Real gas
1.	• It obays equation of state i.e pV = RT.	• It does not obay equation of state.
2.	• These gases does not have definite volume.	• These gases have definite volume.
3.	• Ideal gas has no mass.	• Real gas has mass.
4.	• Collision of ideal gas particle is elastic.	• Collision of real gases particles is non elastic.
5.	• Pressure is high in ideal gas.	• Pressure is low in real gases.
6.	• No energy in involved during collision of particles.	• Collision of particles in real gas has attracting energy.

1.18 Solved Examples

Example 1.7 : *Determine the pressure exerted by carbon dioxide in a container of 1.5 m^3 capacity when it contains 5 kg at 27 °C by using i) Ideal gas equation ii) Vander Waal's equation.*
Take the Vander Wall's constant a = 365.6 $kN-m^4/(kgmol)^2$;
b = 0.0428 $m^3/kgmol$ Universal gas const = $\bar{R}$ = 8.3144 kJ/kgmol K.

☞ **[GTU : Summer-16,18]**

Solution : Given data : V = 1.5 m^3, Mass of CO_2 m = 5 kg ,

Temperature T = 27 °C = 27 + 273 = 300 K,

a = 365.6 kN-m^4 / (kg mole)^2 = 365600 N-m^4 / (kg mole)^2,

b = 0.0428 m^3 / kg mol, $\bar{R}$ = 8.3144 kJ/kg mole K = 8314 J/kg mole K.

- Pressure exerted by CO_2

Using ideal gas equation,

$$R = \frac{\bar{R}}{m} \quad \text{... m - Molar mass}$$

$$= \frac{8314}{44} = 188.95 \text{ Nm / kg K}$$

Now, $pV = mRT$

$$p = \frac{mRT}{V} = \frac{5 \times 188.95 \times 300}{1.5} = 188950 \text{ N/m}^2 = \mathbf{1.889 \text{ bar}}$$

- Pressure exerted by CO_2

Using Vander wall's equation

$$\left(p + \frac{a}{\bar{V}^2}\right)(\bar{V} - b) = \bar{R}T$$

$$\bar{V} = \text{Molar specific volume} = \frac{1.5 \times 44}{5} = 13.2 \text{ m}^3 / \text{kg mol}$$

$$p = \frac{8314 \times 300}{13.2 - 0.0428} - \frac{365600}{(13.2)^2} = 187470.95 \text{ N/m}^2 = \mathbf{1.87 \text{ bar}}$$

Example 1.8 : *A balloon of spherical shape 6 m in diameter is filled with hydrogen gas at a pressure of 1 bar abs and 230 °C. At a later time, the pressure of the gas is 94 % of its original pressure at the same temperature.*
i) What mass of the original gas must have escaped if the d dimensions of the balloon is not changed ?
ii) Find the amount of heat removed to cause the same drop in pressure at constant volume. Take C_v for hydrogen as 10400 J/kgK.

Solution : Given data : D = 6 m, Pressure of H_2 - gas (p_1) = 1 bar abs,

Temperature of H_2 -gas(T_1) = 20 °C = 293 K.

After some time pressure of gas (p_2) = 0.94 p_1 at 293 K.

- Mass of original gas escaped,

$$\Delta m = \text{Initial mass } (m_1) - \text{Final mass } (m_2)$$

We know, $$\frac{p_1V_1}{RT_1} = \frac{p_2V_2}{RT_2} = \frac{V_1}{RT_1}(p_1 - p_2) = \frac{V_1}{RT_1}(p_1 - 0.94p_1) = \frac{p_1V_1}{RT_1}(1 - 0.94)$$

$$\% \text{ of mass escaped} = \frac{\Delta m}{m_1} \times 100 = \frac{\frac{p_1V_1}{RT_1}(1 - 0.94)}{\frac{p_1V_1}{RT_1}} \times 100 = 6 \%$$

Amount of heat to be removed,

$$\frac{p_1V_1}{T1} = \frac{p_2V_2}{T_2}$$

$$\frac{p_1}{T_1} = \frac{0.94p_1}{T_2} \quad \text{... } (\because V_1 = V_2 \text{ and } p_2 = 0.94p_1)$$

$$T_2 = 0.94\ T_1 = 0.94 \times 293 = 275.4 \text{ K}$$

To find out the mass $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times \frac{4}{3}\pi \times 3^3}{\frac{8314}{2} \times 293} = 9.28 \text{ kg}$$

We know,

Heat removed $$Q = mC_V(T_1 - T_2) = 9.28 \times 10400(293 - 275.4) = \mathbf{1.69 \text{ MJ}}$$

Example 1.9 : *A mixture of ideal gases consists of 3 kg of nitrogen and 54 kg of carbon dioxide at a pressure of 300 kPa and a temperature of 20 °C. Find :*
i) Mole fraction of each constituent ii) The equivalent molecular weight of mixture iii) The equivalent gas constant of the mixture. iv) The partial pressure of each gas.

Solution : Given data : $m_{N_2} = 3$ kg, $m_{CO_2} = 54$ kg, $p_1 = 300$ kPa,
$T_1 = 20\ °C = 293$ K

$$\text{Number of moles of } N_2 = (n_{N_2}) = \frac{m_{N_2}}{M_{N_2}} = \frac{3}{28}$$

$$= 0.1071 \text{ kg mole.}$$

$$\text{Number of moles of } CO_2 = (n_{CO_2}) = \frac{m_{CO2}}{M_{CO2}} = \frac{54}{44}$$

$$= 1.22 \text{ kg mole.}$$

$$\text{Total number of moles (n)} = n_{N_2} + n_{CO_2} = 0.1071 + 1.22$$

$$= 1.334 \text{ mole.}$$

$$\text{Mole fraction of } N_2 = x_{N_2} = \frac{n_{N_2}}{n} = \frac{0.1071}{1.334} = 0.080$$

$$\text{Mole fraction of } CO_2 = x_{CO_2} = \frac{n_{CO_2}}{n} = \frac{1.22}{1.334} = 0.91$$

Equivalent molecular weight

$$M_{mixture} = x_{N_2} M_{N_2} + x_{CO_2} M_{CO_2} = 0.080 \times 28 + 0.91 \times 44 = 42.28 \text{ kg/kg mole.}$$

$$\text{Gas constant } R_{mix} = \frac{\bar{R}}{M_{mixture}} = \frac{8.314}{42.28} = 0.1966 \text{ kJ/kgK}$$

- Partial pressure ratio = Mole fraction,

$$\frac{p_{N_2}}{p} = x_{N_2}$$

$$p_{N_2} = P \times x_{N_2} = 300 \times 0.080 = 24 \text{ kPa}$$

Similarly, $p_{CO_2} = 300 \times 0.91 = 273$ kPa

Example 1.10 : *Determine the specific volume of hydrogen gas when its pressure is 60 bar and temperature is 100 K by using i) Compressibility chart, ii) Vander Waal's equation Take for H_2 $T_C = 239.76$ °C, $p_C = 12.92$ bar, $a = 0.25105 \times 10^5$ $Nm^2 / kgmol^4$, $b = 0.0262$ m^3 / kg mole.*

Solution : Given data : Pressure = 60 bar, T = 100 K, $T_C = 239.76$ °C, $p_C = 12.92$ bar,
$a = 0.25105 \times 10^5$ Nm^2 / kg $(mole)^4$

$b = 0.0262$ m^3/kg mole, $T_C = -239.76$ °C $= -239.76 + 273 = 33.24$ K, T = 100 K,
p = 60 bar

To find reduced temperature and pressure

$$T_R = \frac{T}{T_C} = \frac{100}{33.24} = 3.008 \text{ K}$$

$$p_R = \frac{p}{p_C} = \frac{60}{12.92} = 4.65 \text{ bar}$$

Using the values of T_R and p_r,

From compressibility chart, we can find compressibility factor (Z) and its value is,
Z = 1.035

$$p_V = Z R T = Z \times \frac{\bar{R}}{M} \times T$$

$$V = \frac{1.035 \times 8.314 \times 100}{60 \times 10^2 \times 2} = 0.0717 \text{ m}^3 / \text{kg}$$

- Now, by using Vander Wall's equation,

$$\left(p + \frac{a}{\bar{V}^2}\right)(\bar{V} - b) = \bar{R}T$$

$$\left[60 + \frac{0.25105 \times 10^5}{\bar{V}^2}\right][\bar{V} - 0.0262] = 8.314 \times 100$$

$$\bar{V} = 0.14 \text{ m}^3/\text{kg mole,}$$

$$v = \bar{V} \times \frac{m}{M} = 0.14 \times \frac{1}{2} = 0.07 \text{ m}^3/\text{kg}$$

Example 1.11 : *One kg of CO_2 has volume of 1 m^3 at 101 °C compute the pressure by i) Vander Waal's equation ii) Perfect gas equation.*

Solution : Molar specific volume (V) = 1 × 44

= 44 $m^3/kg\,mol$ (M for CO_2 = 44)

- From table 1.1 a = 362850 $Nm^4/(kg\,mol)^2$

b = 0.0423 $m^3/kg\,mol$

R = 8314 Nm/kg mol K

T = 101 °C = 374 K

- Using Vander Waals's equation

$$\left(P+\frac{a}{V^2}\right)(V-b) = RT$$

$$P = \left(\frac{RT}{V-b}-\frac{a}{V^2}\right) = \frac{8314\times374}{(44-0.0423)}-\frac{362850}{(44)^2}$$

= 70549.58 N/m^2 = **0.7054 bar**

- Using perfect gas equation

$$pV = RT$$

$$p = \frac{RT}{V} = \frac{8314\times374}{44} = 70669\ N/m^2 = \textbf{0.7066 bar}$$

Example 1.12 : *2.5 kg of N_2 at 15 bar and 50 °C is contained in rigid vessel. Adequate quantity of O_2 is added to increase the pressure to 20 bar while temperature remains constant at 50 °C.*

Solution : Given data : m_{N_2} = 2.5 kg, M_{N_2} = 28, Initial pressure p_{N_2} = 15 bar, $p_{Total} = p_{N_2} + p_{O_2}$ = 20 bar

Number of N_2 mole $\quad n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{2.5}{28} = 0.0893$

O_2 is added p_{N_2} = 15 bar $\quad p_{O_2} = p_{Total} - p_{N_2} = 20 - 5 = 5$ bar

$$p_{N_2} = \frac{n_{N_2}R_OT}{V} \text{ and } P_{O_2} = \frac{n_{O_2}R_OT}{V}$$

Equating above equation $\quad \frac{p_{O_2}}{p_{N_2}} = \frac{n_{O_2}}{n_{N_2}}$

$$n_{O_2} = \frac{p_{O_2}+n_{N_2}}{p_{N_2}} = \frac{5\times10^5\times0.0893}{15\times10^5} = \textbf{0.0297}$$

Mass of O_2 added $\quad m_{O_2} = n_{O_2}\times M_{O_2} = 0.0297 \times 32$

= **0.9504 kg**

Review Questions

1. *Explain Dalton's law.*
2. *Compare between real and ideal gases.*
3. *What is Vander Wall's equation.*
4. *Explain compressibility chart.*
5. *Explain Gib's Dalton law.*
6. *Explain Amagat's law.*
7. *Explain Dalton's law of partial pressures.*
   i) Vander Waal's equation of states. iii) Generalized compressibility chart.
8. *Explain Dalton's law of partial pressure.*
9. *Explain Amagat's law of additive volumes.*
10. *Explain Law of corresponding states.*
12. *Explain Compressibility factor*
15. *Clearly distinguish between ideal and real gases.*
16. *Write the vander equation of state. In what says, it is an improvement over the ideal gas equation.*
18. *Explain Reduced properties.*
25. *State Gibb's Dalton Law of partial pressures and hence derive an expression for the gas constant 'R' of a mixture of gases.*
26. *Explain the reasons for deviations of Vander Walls equation from ideal gas equations.*
25. *Write a note on compressibility chart.*
26. *Explain Dry bulb temperature*
27. *Explain Wet bulb temperature.*
28. *Explain Dew point temperature.*

Properties of Gases and Mixturs ends ...

Notes

CHAPTER - 2

PSYCHROMETRY

Syllabus : Dalton's law of partial pressure, Properties of moist air, temperature and humidity measuring instruments, psychrometric chart, psychrometric processes such as sensible heating and cooling, heating and humidification cooling and dehumidification, chemical dehumidification, adiabatic saturation

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2.1 Introduction

- An air conditioning is defined as the process of conditioning the air for the simultaneous control of indoor air within the acceptable limits.
- Temperature control can be performed by refrigeration systems, but the simultaneous control of temperature, relative humidity, air movement and purity of air in the space can be performed by air conditioning systems.
- The air conditioning composed of components and equipment arranged in sequence to condition the air to provide comfort conditions. Most air conditioning systems perform the following functions :
  - Condition the supply air i.e. heat or cool, humidify or dehumidify, clean and purify and reduce any objectionable noise produced by the equipment.
  - Distribute the conditioned air, containing sufficient outdoor air, to the conditioned space.
  - Control and maintain the indoor environmental parameters such as temperature, humidity, cleanliness, air movement, sound level and pressure differential between the conditioned space and surroundings within predetermined limits.
- Air conditioning systems can be classified according to their applications as follows :

a) Comfort air conditioning systems and
b) Industrial air conditioning systems.

(a) Comfort Air Conditioning Systems :

- Comfort air conditioning systems provide occupants with a comfortable and healthy indoor environment to carry out their activities. The various sectors of the economy using comfort air conditioning systems are as follows :
  - The commercial sector includes office buildings, supermarkets, department stores, shopping centers, restaurants and others. Many high-rise office buildings use complicated air conditioning systems to satisfy multiple tenant requirements.

  - The institutional sector includes schools, colleges, universities, libraries, museums, indoor stadiums, cinemas, theaters, concert halls and recreation centers.
  - The residential and lodging sector consists of hotels, motels, apartment houses and private homes.
  - The health care sector encompasses hospitals, nursing homes and convalescent care facilities. Special air filters are generally used in hospitals to remove bacteria and particulates of sub-micrometer size from areas such as operating rooms, nurseries and intensive care units.
  - The transportation sector includes aircraft, automobiles, railroad cars, buses and cruising ships.

(b) Industrial Air Conditioning Systems :

- Industrial air conditioning systems provide needed indoor environmental control for manufacturing, product storage or other research and development processes. The following areas are examples of industrial air conditioning systems :
  - In textile mills, natural fibers and manufactured fibers are hygroscopic. Proper control of humidity increases the strength of the yarn and fabric during processing.
  - Many electronic products require clean rooms for manufacturing like integrated circuits, since their quality is adversely affected by airborne particles.
  - Precision manufacturers always need precise temperature control during production of precision instruments, tools and equipment.
  - Pharmaceutical products require temperature, humidity and air cleanliness control.

2.2 Basic Psychrometry

- The **psychrometry** is the field of engineering science concerned with the study of behaviour of gas and vapour mixtures. The word psychometric is derived from the Greek words 'psuchron' means "cold" and 'metron' means "way of measurement".
- From the subject point of view, psychometry is the study of various thermodynamic properties of air and water vapour mixture. It is used extensively to express and analyze the characteristics of various air conditioning processes.

- Understanding of properties of air and water vapour mixture and the various processes involving air is fundamental of air conditioning design.
- The moist air or air is a mixture of various gases, water vapour (moisture) and a number of pollutants. The air in absence of water vapour is called as dry air.
- The ordinary atmospheric air is always contains some amount of water vapour so the pure dry air doesn't really exist.
- The dry air is composed of various gases, mainly nitrogen (78 %) and oxygen (21 %). The remaining 1 % of the gases includes carbon dioxide and very small quantities of inert gases like hydrogen, helium, neon and argon.
- Dry air is used as the basis for defining properties of air since its composition is approximately remains same under any sets of conditions.
- However, the amount of moisture in air keep changing from one location to other and depending on the atmospheric conditions at a particular location. The places near to the sea areas contain more moisture while the desert areas contain less moisture.
- Similarly, during the rainy season, the moisture content of the air is high whereas during summer and winter seasons, it is low.
- It is important to note that, the water vapour exists in the superheated condition, but when it is cooled or heated there is change in its phases, hence it absorbs or liberates sensible heat as well as the latent heat due to changes in its phases. This is what makes the whole process of air conditioning highly complicated.
- Cooling of water vapour results in its condensation, while its heating leads to superheating.
- The following Table 2.1 represents properties of air constituents :

Table 2.1 : Properties of air constituents

Gas	Ratio compared to dry air (%)		Molecular mass- M - (kg/kmol)	Chemical symbol	Boiling point (°C)
	By volume	By weight	(kg/kmol)		
Oxygen	20.95	23.20	32.00		– 196
Nitrogen	78.09	75.47	28.02		– 183
Carbon dioxide	0.03	0.046	44.01		– 78.5
Hydrogen	0.00005	~ 0	2.02		– 2 52.87
Argon	0.933	1.28	39.94	Ar	– 186
Neon	0.0018	0.0012	20.18	Ne	– 246
Helium	0.0005	0.00007	4.00	He	– 269

2.3 Psychrometric Terms

[GTU : Winter-14,16,18. Summer-15,17,19]

- There are number of psychrometric terms, some of them are as follows :

1) Dry air	2) Moist air	3) Saturated air
4) Degree of saturation	5) Specific humidity	6) Absolute humidity
7) Relative humidity	8) Dry bulb temperature	9) Wet bulb temperature
10) Wet bulb depression	11) Dew point temperature	12) Dew point depression

1) **Dry air :** It is a mixture of number of gases such as oxygen, nitrogen, carbon dioxide, hydrogen, argon, helium, etc.

2) **Moist air :** It is a mixture of dry air and water vapour. The amount of water vapour present in the air depends upon the absolute pressure and temperature of the mixture.

3) **Saturated air :** It is a mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it.

4) **Degree of saturation :** It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.

5) **Specific Humidity (w) [Moisture content] :** It is the ratio of mass of water vapour to mass of dry air in a particular volume of moist air (dry air plus the water vapour). **Humidity** or **humidity ratio** or **Specific humidity** is related to mixing ratio on mass basis. It is expressed in grams per kg of dry air (g/kg of dry air).

6) **Absolute Humidity (AH) :** Absolute Humidity is the mass of water vapour present in unit volume of moist air at a given temperature and pressure. It is measured in grams per cubic metre of air (g/m^3).

7) **Relative Humidity (ϕ) :**

- Relative Humidity (RH) is the most commonly used psychrometric unit. The "relative" in relative humidity expresses the relation between the amount of water vapour present and the maximum amount that is physically possible at that temperature.
- In other words, relative humidity (expressed in percentage) is the partial water vapour pressure in relation to it's saturation pressure.

8) **Dry Bulb Temperature (DBT) :**

- The Dry Bulb Temperature refers basically to the ambient air temperature as measured by a standard thermometer with a dry sensing bulb.
- It is called as Dry Bulb because the air temperature is indicated by a thermometer not affected by the moisture of the air. It is denoted by t_d or t_{db}.

9) **Wet Bulb Temperature (WBT) :**

- Temperature of air as measured by a thermometer using a sensing bulb covered by a wet cloth.
- The adiabatic evaporation of water from the thermometer and the cooling effect is indicated by a WBT.
- As the thermometer is moved through the air, water will evaporate from the cloth at a rate determined by the relative humidity of the surrounding air.
- The wet bulb temperature is always lower than the dry bulb temperature but at 100 % relative humidity, it will be identical as there would no moisture evaporation from the wet cloth. It is denoted by tw or twb.

10) **Wet bulb depression :** The wet-bulb depression is the difference between the dry-bulb temperature and the wet-bulb temperature. It indicates relative humidity of the air.

11) **Dew Point Temperature (DPT) :**

- The mass of water vapour that air can hold is dependent on the air temperature (and pressure). When the air is cooled, the amount of water it can hold is reduced and the relative humidity subsequently rises.
- If air is cooled to the point where it becomes 100 % saturated, then any further cooling will cause moisture to deposit out of the air in the form of condensation or dew.

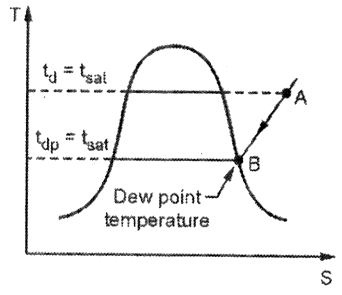


Fig. 2.1 : Dew point temperature

- The temperature at which this saturation occurs is the 'dew point' of the air. Above this temperature the moisture will stay in the air.
- Thus, the dew point temperature can be defined as the temperature at which condensation of moisture begins when the air is cooled. The dew point is always lower than (or equal to) the air temperature.
- If unsatured air containing superheated water vapour is cooled at constant pressure, the partial pressure remains constant until the water vapour reaches the saturated state as shown in Fig. 2.1 (Point B).
- At this point B dew will be formed, hence the corresponding temperature is called as dew point temperature. It is denoted by t_{dp}.

12) **Dew point depression :** It is the difference between the dry bulb temperature and dew point temperature of air.

In addition to psychrometric terms it is necessary to revise the following terms.

13) **Specific Volume (v) :** The specific volume is the volume of unit mass of dry air at a given temperature normally expressed as m^3/kg and is also shown in the psychrometric chart. Specific volume is the inverse of density.

14) **Sensible Heat (SH) :** It can be defined as heat which when applied to (or removed from) a substance, causes only a change in temperature. Sensible heat is measured in joules per kilogram (J/kg).

15) **Latent Heat (LH) :** It can be defined as heat which when applied to (or removed from) a substance, produces a change in state without any change to temperature. Latent heat is measured in joules per kilogram (J/kg).

16) **Total heat :** Total heat is the sum of sensible and latent heat.

2.3.1 Psychrometer

- To measure the different psychrometric properties psychrometer is used.
- There are different types of psychrometer but the sling psychrometer is commonly used.
- It consists of a dry bulb thermometer and wet bulb thermometer mounted side by side in a protective case. Refer Fig. 2.2.
- The protective case is attached to a handle by a swivel connection so that it can be easily rotated as per the requirement.
- The sling psychrometer is rotated in the air for almost 1 minute and the readings of both the thermometers are taken.
- The dry bulb thermometer is directly exposed to air and measures the actual temperature of air.

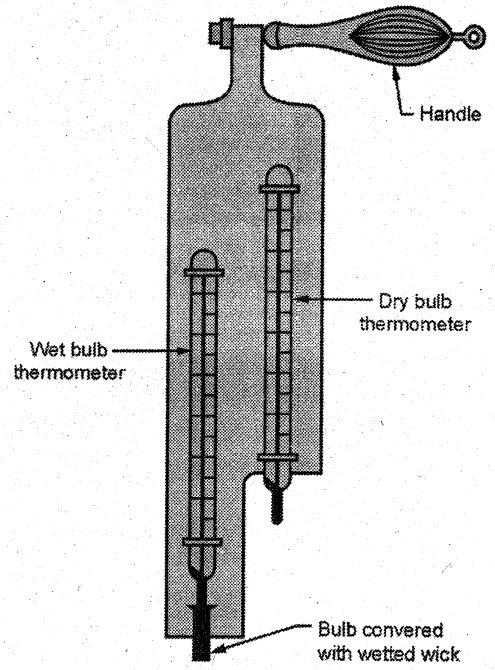


Fig. 2.2 : Sling psychrometer

- But, the bulb of wet bulb thermometer is covered by a wick (throroughly wetted by using distilled water).
- Now the temperature measured by this wick covered bulb is the temperature of liquid water in the wick which is known as wet bulb temperature.

2.4 Psychrometric Relations

- In previous section, we have studied psychrometric terms used in air conditioning. These terms have some co-relation between one another. Some of the important co-relations are discussed in this section as given below :

(1) Dalton's law of partial pressure : [GTU : Summer-17]

- It is used to evaluate the pressure of individual constituent of mixture of gases.
- This law states that, "The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent would exert, if it is occupied the same space by itself."
- In simple words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Thus, in case of moist air the barometric pressure is given by,

$$p_b = p_a + p_v$$

where, p_b = Barometric pressure exerted by moist air

p_b = Partial pressure exerted by dry air

p_v = Partial pressure exerted by water vapour

- According to Dr. Carrier, the partial pressure of water vapour can be determined as follows:

$$p_v = p_w - \frac{[p_f - (p_{vs})_{wb}]\,(t_{db} - t_{wb})}{1527.4 - 1.3\,t_{wb}}$$

where, p_w = Saturation pressure of water vapour corresponding to wet bulb temperature (from steam table)

p_b = Barometric pressure of moist air

t_{db} = Dry bulb temperature (°C)

t_{wb} = Wet bulb temperature (°C)

(2) Specific humidity or humidity ratio (w) :

- It is defined as, the mass of water vapour per unit mass of dry air in the moist air. It is denoted by 'w'.
- Mathematically specific humidity is given by,

$$\text{Specific humidity (w)} = \frac{\text{Mass of water vapour}}{\text{Mass of dry air}}$$

$$w = \frac{m_v}{m_a} \quad \ldots(2.1)$$

Air is assumed to be perfect ideal gas, therefore using ideal gas equation, we have,

$$m_a = \frac{p_a v_a}{R_a T_a} \quad \ldots(2.2)$$

and

$$m_v = \frac{p_v v_v}{R_v T_v} \quad \ldots(2.3)$$

where,

p_a = Partial pressure of dry air

p_v = Partial pressure of water vapour

v_a = Volume of dry air

v_v = Volume of water vapour

T_a = Temperature of dry air

T_v = Temperature of water vapour

R_a = Characteristic gas constant of dry air

R_v = Characteristic gas constant of water vapour

From equation (2.2) and (2.3),

$$w = \left(\frac{p_a v_a}{R_v T_a}\right) \Big/ \left(\frac{p_a v_v}{R_a T_v}\right)$$

$$= \frac{p_v v_a}{R_v T_a} \times \frac{R_a T_v}{p_a v_v}$$

$$w = \frac{R_a p_v}{R_v p_a} \quad \ldots (\because T_a = T_v = T_d \text{ and } v_a = v_v)$$

But,

$$R_a = \frac{R_o}{m_a} = \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K}$$

and

$$R_v = \frac{R_o}{m_v} = \frac{8.3143}{18} = 0.462 \text{ kJ/kg K}$$

Thus,

$$w = \frac{0.287}{0.462} \times \frac{p_v}{p_a} = 0.622 \frac{p_v}{p_a}$$

$$\mathbf{w = 0.622 \left(\frac{p_v}{p_b - p_v}\right)} \quad (\because p_b = p_a + p_v) \quad \ldots(2.4)$$

❑ (3)Relative Humidity (RH) (ϕ) :

☞ **[GTU : Summer-16, Winter-17]**

According to definition of relative humidity,

$$RH(\phi) = \frac{\text{Mass of water vapour associated with unit mass of dry hair}}{\text{Mass of water vapour associated with unit mass of dry saturated air}}$$

$$\phi = \frac{m_v}{m_{vs}} = \left(\frac{p_v v_v}{R_v T_v}\right) \left(\frac{p_{vs} \cdot v_{vs}}{R_{vs} \cdot T_{vs}}\right)$$

But, $v_v = v_{vs}$, $T_v = T_{vs}$ and $R_v = R_{vs}$

$$\phi = \frac{p_v}{p_{vs}} \qquad \text{Also, } \phi = \frac{v_{vs}}{v_v} \quad \ldots(2.5)$$

- Thus, the relative humidity can also be defined as the ratio of partial pressure of water vapour in an unsaturated air at a given temperature to the partial pressure of saturated air at the same temperature. It is generally expressed in percentage.
- When p_v is equal to p_{vs}, RH becomes unity and the air becomes fully saturated and is considered to have 100 % RH. Relative humidity indicates the closeness of air to its saturated condition. It decides the rate of evaporation.

As, $RH(\phi) = \frac{p_v}{p_{vs}}$

But, we know, $w = 0.622 \frac{p_v}{p_a}$ or $p_v = \frac{w}{0.622} p_a$

$$\phi = \frac{p_v}{p_{vs}} = \frac{w}{0.622} \frac{p_a}{p_{vs}}$$

❑ (4) Degree of saturation (μ) :

- It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.
- In other words, it is also defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature.
- Degree of saturation is the another property of air that gives relation of air to its saturation state. It is generally, denoted as and mathematically written as,

$$\mu = \frac{w}{w_s}$$

where, w and w_s are specific humidities of air and saturated air respectively.

- From the equations of specific humidity above equation can be modified as,

$$\mu = \frac{0.622 \left(\frac{p_v}{p_b - p_v}\right)}{0.622 \left(\frac{p_{vs}}{p_b - p_{vs}}\right)}$$

$$\mu = \frac{p_v}{p_{vs}}\left(\frac{p_b - p_{vs}}{p_b - p_v}\right) = \phi\left(\frac{p_b - p_{vs}}{p_b - p_v}\right)$$

$$\phi = \mu\left(\frac{p_b - p_v}{p_b - p_{vs}}\right) = \frac{\left(1 - \frac{p_v}{p_b}\right)}{1 - \left(\frac{p_{vs}}{p_b}\right)} \quad \ldots (2.6)$$

❑ (5) Enthalpy of moist air :

- The total heat (enthalpy) of moist air is mathematically equal to the enthalpy of dry air plus the enthalpy of water vapour associated with dry air.

  Mathematically, $h = h_a + w \cdot h_v$

  Enthalpy of 1 kg of dry air is,

  $h_a = C_{pa}\, t_{db}$

  where, C_{pa} = Specific heat of dry air = 1.005 kJ/kg K

  t_{db} = Dry bulb temperature

  and enthalpy of water vapour associated with 1 kg of dry air.

  $h_v = w\, h_s$

  where, w = Mass of water vapour in 1 kg of dry air

  h_s = Enthalpy of water vapour per kg of dry air at dew point temperature (t_{dp})

- If the moist air is superheated, then the enthalpy of water vapour is,

  $h_v = w\, C_{ps}\, (t_{db} - t_{dp})$

  Where, C_{ps} = Specific heat of superheated water vapour (generally taken as 1.88 kJ/kg K)

  $t_{db} - t_{dp}$ = Degree of superheat of water vapour

  Now, the total enthalpy of superheated water vapour is given by,

  $$h = (C_{pa} + w\, C_{ps})\, t_{db} + w\, [h_{fgdp} + 2.3\, t_{dp}] \quad \ldots (2.7)$$

- The term ($C_{pa} + w\, C_{ps}$) is called **humid specific heat (C_{pm}).** It is the specific heat or heat capacity of moist air i.e. (1 + w) kg/kg of dry air. The general value of humid specific heat in air conditioning range is taken as 1.022 kJ/kg K.

  Enthalpy of moist air is given by,

  $$h = 1.022\, t_{db} + w\, (h_{fgdp} + 2.3\, t_{dp}) \text{ kJ/kg}$$

where, h_{fgdp} = Latent heat of vapourisation of water corresponding to dew point temperature (from steam table)

= 2500 kJ/kg.

- Better approximation may be obtained by following equation,

  $$h = 1.005\, t_{db} + w\, (2500 + 1.88\, t_d) \text{ kJ/kg} \quad \ldots (2.8)$$

2.5 Solved Examples

Ex. 2.1 : For a sample of air having DBT of 22 °C, RH 30 % at barometric pressure of 0.76 m of Hg determine the following parameters by using psychrometric chart :
i) Vapour pressure ii) Humidity ratio iii) Vapour density iv) Enthalpy.
☞ **[GTU : Winter-16, Summer18]**

Sol. : Given data :

DBT (t_d) = 22 °C, RH ϕ = 30 % = 0.3, p_b = 0.76 m of Hg

To find :

i) p_v ii) w iii) ρ iv) h

- Mark the initial condition of air i.e. 22 °C DBT and 30 % RH on a psychrometric chart as shown in Fig. 2.3. (Say point A).

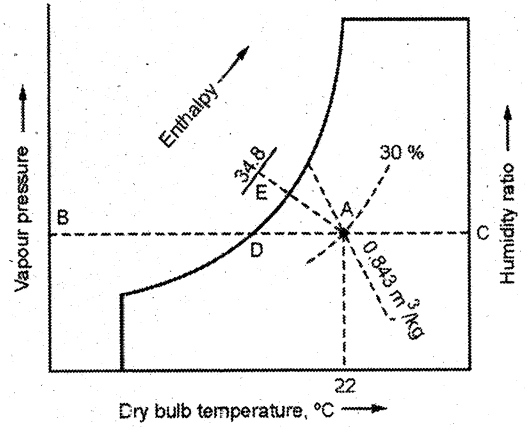


Fig. 2.3

- Now, from point A draw a horizontal line which meets the vapour pressure line at B and humidity ratio line at C.

  By measurement on psychrometric chart, we get,

(i) Vapour pressure at point B,

$$p_v = 5.95 \text{ mm of Hg} = 5.95 \times 10^{-3} \text{ m of Hg}$$

$$p_v = 5.95 \times 10^{-3} \times 13.6 \times 9810 = 793.8252 \text{ Pa} \quad \ldots (\because p = \gamma h)$$

$$\mathbf{p_v = 793.8252 \times 10^{-5} \text{ bar} = 0.007938 \text{ bar}} \quad \ldots \textbf{Ans.}$$

(ii) Humidity ratio at point C,

$$w = 5 \text{ g/kg of dry air} = 5 \times 10^{-3} \text{ kg/kg of dry air} \quad \ldots \text{Ans.}$$

(iii) Specific volume at point A,

$$v_a = 0.843 \text{ m}^3/\text{kg of dry air}$$

(iv) Vapour density,

$$\rho_v = \frac{w}{v_a} = \frac{\mathbf{5 \times 10^{-3}}}{\mathbf{0.843}}$$

$$= \mathbf{5.9311 \times 10^{-3} \text{ kg/m}^3} \quad \ldots \textbf{Ans.}$$

Also draw a line from point A to the WBT line meeting the enthalpy line at point E.

(v) Enthalpy from the chart is,

$$\mathbf{h = 34.8 \text{ kJ/kg of dry air}} \quad \ldots \textbf{Ans.}$$

Ex. 2.2 : Atmospheric air at 1.0132 bar has a DBT of 32 °C and a WBT of 26 °C. Compute i) Partial pressure of water vapour ii) Specific humidity iii) Dew point temperature iv) Relative humidity v) Degree of saturation vi) Density of air in the mixture vii) Density of vapour in the mixture and viii) Enthalpy of the mixture. ☞ **[GTU : Winter-16, Summer-19]**

Sol. : Given data :

$p_b = 1.0132$ bar, $t_{db} = 32$ °C, $t_{wb} = 26$ °f

To find : p_v, w, t_{tp}, ϕ, μ, ρ_v, h

✦ **Step 1 : Calculate partial pressure of water vapour.**

Partial pressure of water vapour is,

$$p_v = p_w - \frac{(p_b - p_w)(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}$$

From psychrometric chart,

At t_{db} = 32 °C, $p_{vs} = 0.0479$ bar

At t_{wb} = 26 °C, $p_w = 0.0338$ bar

$$p_v = 0.0338 - \frac{(1.0132 - 0.0338)(32 - 26)}{1544 - 1.44\,(26)}$$

$$\mathbf{p_v = 0.02989 \text{ bar}} \quad \ldots \textbf{Ans.}$$

✦ **Step 2 : Calculate specific humidity and dew point temperature.**

As, $$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.02989}{1.0132 - 0.02989}$$

$$\mathbf{w = 0.01890 \text{ kg/kg of dry air.}} \quad \ldots \textbf{Ans.}$$

From steam table,

At p_v = 0.02989 bar, $\mathbf{t_{dp} = 23}$ °C $\quad \ldots \textbf{Ans.}$

✦ **Step 3 : Calculate relative humidity and degree of saturation**

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.02989}{0.0479} = 0.6240$$

$$\boldsymbol{\phi = 62.40\ \%} \quad \ldots \textbf{Ans.}$$

Now, $$(W_{sat}) = \frac{0.622\, p_{vs}}{p_b - p_{vs}} = \frac{0.622 \times 0.0479}{1.0132 - 0.0479}$$

$$W_{sat} = 0.03086 \text{ kg/kg of dry air}$$

Now, degree of saturation is given by,

$$\mu = \frac{w}{w_{sat}} = \frac{0.01890}{0.03086} = \mathbf{0.6124} \quad \ldots \textbf{Ans.}$$

✦ **Step 4 : Calculate density of air and density of vapour in the mixture.**

We know that,

$$\rho_v = \frac{w\,(p_b - p_v)}{R \cdot t_{db}}$$

$$= \frac{0.01890\,(1.0132 - 0.02989)\,10^5}{287 \times (32 + 273)}$$

$$\mathbf{\rho_v = 0.0237 \text{ kg/m}^3 \text{ of dry air}} \quad \ldots \textbf{Ans.}$$

Specific volume of dry air $= \dfrac{R \cdot t_{db}}{p_b - p_v}$

$$v_a = \frac{287 \times (32 + 273)}{(1.0132 - 0.02989)\,10^5} = 0.8902 \text{ m}^3/\text{kg of dry air}$$

$$\text{Now, } \rho_a = \frac{1}{v_a} = \frac{1}{0.8902}$$

$$= 1.123 \text{ kg/m}^3 \text{ of dry air}$$

✦ **Step 5 : Calculate the enthalpy**

Enthalpy is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$h = 1.005\,(32) + 0.01890\,(2500 + 1.88\,(32))$$

$$\therefore \quad \mathbf{h = 80.547 \text{ kJ/kg}} \qquad \textbf{... Ans.}$$

Ex. 2.3 : A sling thermometer reads 40 °C DBT and 28 °C WBT. Find the following :
i) Specific humidity ii) Relative humidity
ii) Dew point temperature iv) Vapour density. ☞ **[GTU : Summer-14, Winter-17]**

▣ **Sol. :**

Given : $t_{dp} = 40$ °C, $t_{wb} = 28$ °C

To find : i) w ii) ϕ iii) t_{dp} iv) ρ_v

✦ **Step 1 : Calculate specific humidity and relative humidity**

From steam table,

$$\text{at } t_{dp} = 40 \text{ °C}, \; p_{vs} = 0.0737 \text{ bar}$$

$$\text{at } t_{wb} = 28 \text{ °C}, \; p = 0.03778 \text{ bar}$$

By Carrier's equation,

$$p_v = p_w - \left[\frac{(p_b - p_w)\,(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}\right]$$

$$\therefore \quad p_v = 0.03778 - \left[\frac{(1.0132 - 0.03778)\,(40 - 28)}{1544 - 1.44 \times 28}\right]$$

$$= 0.03 \text{ bar}$$

Specific humidity is given by,

$$w = 0.622\left[\frac{p_v}{p_b - p_v}\right]$$

$$= 0.622\left[\frac{0.03}{1.0132 - 0.03}\right]$$

$$\mathbf{w = 0.0189 \text{ kg/kg of dry air}} \qquad \textbf{... Ans.}$$

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.03}{0.0737} = 0.407 = \mathbf{40.7\ \%} \qquad \textbf{... Ans.}$$

✦ **Step 2 : Calculate dew point temperature and vapour density**

From steam table,

$$\text{at } p_v = 0.03 \text{ bar}, \; t_{dp} = \mathbf{26\ °C} \qquad \textbf{... Ans.}$$

$$\text{Specific volume of dry air } = v_a = \frac{R \cdot t_{tb}}{p_a} = \frac{R \cdot t_{db}}{p_b - p_v}$$

$$v_a = \frac{287 \times (40 + 273)}{(1.0132 - 0.03) \times 10^5} = 0.9136 \text{ m}^3/\text{kg}$$

We know that,

$$\rho_v = \frac{w}{v_a} = \frac{0.0189}{0.9136} = \mathbf{0.0206 \text{ kg/m}^3} \qquad \textbf{... Ans.}$$

Ex. 2.4 : Calculate : i) Relative humidity ; ii) Humidity ratio ; iii) Dew point temperature ; iv) Density and v) Enthalpy of atmospheric air when the DBT is 35 °C, WBT = 23 °C and the barometer reads 750 mm of Hg.

▣ **Sol. : Given :**

$t_{db} = 35$ °C, $t_{wb} = 23$ °C, $p_b = 750$ mm of Hg

To find : i) ϕ ii) w iii) t_{dp} iv) ρ_v v) h

✦ **Step 1 : Calculate dew point temperature**

From steam table, saturation pressure at $t_{wb} = 23$ °C is,

$$p_w = 0.028 \text{ bar}$$

We know that,

$$p_b = 750 \text{ mm of Hg} = 0.75 \times 13.6 \times 9810$$

$$= 100.062 \times 10^3 \text{ N/m}^2$$

$$p_b = 1.0006 \text{ bar}$$

Partial pressure of water vapour is given by,

$$p_v = p_w - \frac{(p_b - p_w)\,(t_{db} - t_{wb})}{1544 - 1.44\, t_{wb}}$$

$$p_v = 0.028 - \frac{(1.0006 - 0.028)(35 - 23)}{1544 - 1.44 \times 23}$$

$$= 0.0202 \text{ bar}$$

For partial pressure of water vapour we can find saturation temperature from steam table. This saturation temperature is dew point temperature.

$$\mathbf{t_{dp} = 17\ ^\circ C}$$ **... Ans.**

✦ **Step 2 : Calculate relative humidity and specific humidity**

We can find saturation pressure of vapour corresponding to dry bulb temperature from steam table.

$$\therefore \quad p_{vs} = 0.0562 \text{ bar}$$

Relative humidity is given by,

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.0202}{0.0562} = 0.3594$$

$$\mathbf{\phi = 35.94\ \%}$$ **... Ans.**

Specific humidity is given by,

$$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.0202}{1.0006 - 0.0202}$$

$$\mathbf{w = 0.0128\ kg/kg\ of\ dry\ air}$$ **... Ans.**

✦ **Step 3 : Calculate vapour density and enthalpy**

Vapour density is given by,

$$\rho_v = \frac{w\,(p_b - p_v)}{R \cdot t_{db}}$$

$$= \frac{0.0128\,(1.0006 - 0.0202) \times 10^5}{287 \times (273 + 35)}$$

$$\mathbf{\rho_v = 0.01419\ kg/m^3\ of\ dry\ air}$$ **... Ans.**

Enthalpy is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$h = 1.005 \times 35 + 0.0128\,(2500 + 1.88 \times 35)$$

$$\mathbf{h = 68.0172\ kJ/kg\ of\ dry\ air}$$ **... Ans.**

Ex. 2.5 : Moist air at 35 °C has a dew point of 15 °C. Calculate its relative humidity, specific humidity and enthalpy.

Sol. : Given data : $t_{db} = 35\ ^\circ C = 308\ K$, $t_{dp} = 15\ ^\circ C = 288\ K$

To find : i) ϕ ii) w iii) h

✦ **Step 1 : Calculate relative humidity and specific humidity**

From Psychrometric chart,

At $t_{db} = 35\ ^\circ C$, $p_{va} = 0.0563$ bar

At $t_{dp} = 15\ ^\circ C$, $p_v = 0.017$ bar

We know that,

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{vs}} = \frac{0.017}{0.0563} = 0.3019$$

$$\mathbf{\phi = 30.19\ \%}$$ **... Ans.**

Specific humidity is given by,

$$w = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.017}{1.01325 - 0.017}$$

$$\mathbf{w = 0.01061\ kg/kg\ of\ air}$$ **... Ans.**

✦ **Step 2 : Calculate the enthalpy of moist air**

Enthalpy of moist air is given by,

$$h = 1.005 + w\,(2500 + 1.88\,)$$

$$h = (1.005\ \ 35) + 0.01061\ (2500 + 1.88\ \ 35)$$

$$\mathbf{h = 62.39\ kJ/kg}$$ **... Ans.**

Ex. 2.6 : Moist air at 20 °C, 1.01325 bar has relative humidity of 80 %, without using psychrometric chart determine degree of saturation, dew point temperature, specific volume and density of air. **[GTU : Summer-16]**

Sol. : Given :

$t_{db} = 20\ ^\circ C$, $p_b = 1.0132$ bar, $\phi = 80\ \% = 0.8$

To find : i) μ ii) t_{dp} iii) v_a iv) ρ_a

✦ **Step 1 : Calculate dew point temperature**

From steam table,

$$\text{at } 20\ ^\circ C,\ p_{vs} = 0.02337 \text{ bar}$$

$$\text{Relative humidity } \phi = \frac{p_v}{p_{va}}$$

$$0.8 = \frac{p_v}{0.02337} \quad \therefore \quad p_v = 0.01869 \text{ bar}$$

From steam table, at $p_v = 0.01869$ bar, we get dew point temperature as,

$$t_{dp} = \mathbf{16\ °C} \quad \textbf{... Ans.}$$

✦ **Step 2 : Calculate degree of saturation and mass of air**

Degree of saturation is given by,

$$\mu = \phi\left(\frac{p_b - p_{vs}}{p_b - p_v}\right)$$

$$= 0.8\left(\frac{1.01325 - 0.02337}{1.01325 - 0.01869}\right) = \mathbf{9.6443} \quad \textbf{... Ans.}$$

$$\text{We know that, } p_b = p_a + p_v$$

$$p_a = p_b - p_v = 1.01325 - 0.01869$$

$$= 0.9945 \text{ bar}$$

Now,

$$\text{Specific volume} = v_a = \frac{RT}{p_a} = \frac{287 \times 293}{0.9945 \times 10^5}$$

$$= 0.8455 \text{ m}^3/\text{kg}$$

$$\text{Specific humidity} = w = 0.622\left(\frac{p_v}{p_b - p_v}\right)$$

$$= 0.622\left(\frac{0.01869}{10132 - 0.01869}\right)$$

$$w = 0.0118 \text{ kg/kg of dry air}$$

Now,

$$\rho_v = \frac{w}{v_a} = \frac{0.0118}{0.8455}$$

$$= \mathbf{0.0139\ kg/m^3 \text{ of dry air}} \quad \textbf{... Ans.}$$

Ex. 2.7 : Atmospheric air at 101.325 kPa has 30° DBT and 15°C DPT. Without using psychrometric chart calculate partial pressure of air and vapour, Specific humidity, Relative humidity, vapour density and enthalpy of moist air.

Sol. : Given :

$p_b = 101.325$ kPa $= 1.0132$ bar, $t_{db} = 30$ °C, $t_{dp} = 15$ °C

To find : i) p_a and p_v ii) w iii) ϕ iv) ρ_v v) h

✦ **Step 1 : Calculate partial pressure of air and water vapour**

From steam table,

$$\text{at } t_{db} = 30\ °C,\ p_{vs} = 0.04242 \text{ bar}$$

$$\text{at } t_{dp} = 15\ °C,\ p_v = \mathbf{0.01704 \text{ bar}} \quad \textbf{... Ans.}$$

$$\text{We know that, } p_b = p_a + p_v$$

$$\therefore \quad p_a = p_b - p_v = 1.0132 - 0.01704 = \mathbf{0.99616 \text{ bar}} \quad \textbf{... Ans.}$$

✦ **Step 2 : Calculate specific humidity and relative humidity**

$$\text{Specific humidity} = w = 0.622\frac{p_v}{p_a} = 0.622 \times \frac{0.01704}{0.99616}$$

$$w = \mathbf{0.01063 \text{ kg/kg of dry air}} \quad \textbf{... Ans.}$$

$$\text{Relative humidity} = \phi = \frac{p_v}{p_{vs}} = \frac{0.01704}{0.04242} = 0.4016$$

$$\phi = \mathbf{40.16\ \%} \quad \textbf{... Ans.}$$

✦ **Step 3 : Calculate enthalpy and vapour density**

Enthalpy of moist air is given by,

$$h = 1.005\, t_{db} + w\,(2500 + 1.88\, t_{db})$$

$$h = 1.005 \times 30 + 0.01063\,(2500 + 1.88 \times 30)$$

$$h = \mathbf{57.3245 \text{ kJ/kg of dry air}} \quad \textbf{... Ans.}$$

$$\text{Specific volume of air} = v_a = \frac{R \cdot t_{db}}{p_a} = \frac{287 \times 303}{0.9961 \times 10^5} = 0.873 \text{ m}^3/\text{kg}$$

$$\text{Vapour density} = \rho_v = \frac{w}{v_a} = \frac{0.01063}{0.873} = \mathbf{0.0121\ kg/m^3} \quad \textbf{... Ans.}$$

Ex. 2.8 : A room measures 5 m × 5 m × 3 m contains atmospheric air at 100 kPa, DBT = 30 °C and relative humidity = 30 %. Find the mass of dry air and the mass of associated water vapour in the room. Solve the problem without using psychrometric chart. **[GTU : Winter-17]**

Sol. : Given : Volume $= 5 \times 5 \times 3 = 75\ m^3$, $p_b = 100$ kPa $= 1$ bar,

$\phi = 30\ \% = 0.3$, $t_{db} = 30$ °C

To find : i) m_a ii) m_v

✦ **Step 1 : Calculate mass of air**

From steam table,

At DBT = 30 °C, p_{vs} = 0.04242 bar = 4.242 kPa

We know that, $\phi = \frac{p_v}{p_{vs}}$ $\therefore$ $0.3 = \frac{p_v}{4.242}$

p_v = 1.2726 kPa

For moist air barometric pressure is given by,

$p_b = p_a + p_v$ $\therefore$ $p_a = p_b - p_v$

$p_a = 100 - 1.2726 = 98.7274$ kPa

$= 98.7274 \times 10^3$ Pa

By using ideal gas equation,

$$m_a = \frac{p_a v}{R_a T} = \frac{98.7274 \times 10^3 \times 75}{287 \times 303}$$

$\therefore$ $\mathbf{m_a = 85.1479\ kg}$ **... Ans.**

✦ **Step 2 : Calculate mass of vapours**

Characteristic gas constant for ideal vapour = R_v = 461.8

By using ideal gas equation

$$m_v = \frac{p_v v}{R_v T} = \frac{1.2726 \times 10^3 \times 75}{461.8 \times 303}$$

$\mathbf{m_v = 0.6821\ kg}$

... Ans.

2.6 Psychrometric Chart

☞ **[GTU : Winter-16,17, Summer-18]**

- The psychrometric chart is a plot of various thermodynamic properties of moist air. It is very useful to find out the various properties of air required in air condition and eliminate tedious calculations.
- The psychrometric chart gives the interrelationships between dry air, moisture content and enthalpy of air.
- For a proper understanding of air conditioning, it is essential to get familiar with the psychrometric chart.

- It is suitable to design charts at a constant barometric pressure because barometric pressure does not change significantly over much of the occupied surface of the earth.
- There is a slight variation in the charts prepared by different air-conditioning manufactures but basically they all are same.
- The psychrometric chart is generally drawn at standard atmospheric pressure of 0.76 m of Hg or 1.01325 bar.
- In a psychrometric chart, DBT is taken as abscissa (°C) and moisture content i.e specific humidity as ordinate (g/kg of dry air).
- Now the saturation curve is drawn by plotting the various saturation points at corresponding DBT.
- The saturation curve shows 100 % relative humidity at various DBT. It also represents the wet bulb temperature dew point temperature.
- The psychrometric chart has a number of details, some of the important lines are as follows:

1) Dry bulb temperature lines
2) Specific humidity lines
3) Dew point temperature lines
4) Wet bulb temperature lines
5) Enthalpy or total heat lines
6) Specific volume lines
7) Relative humidity lines.

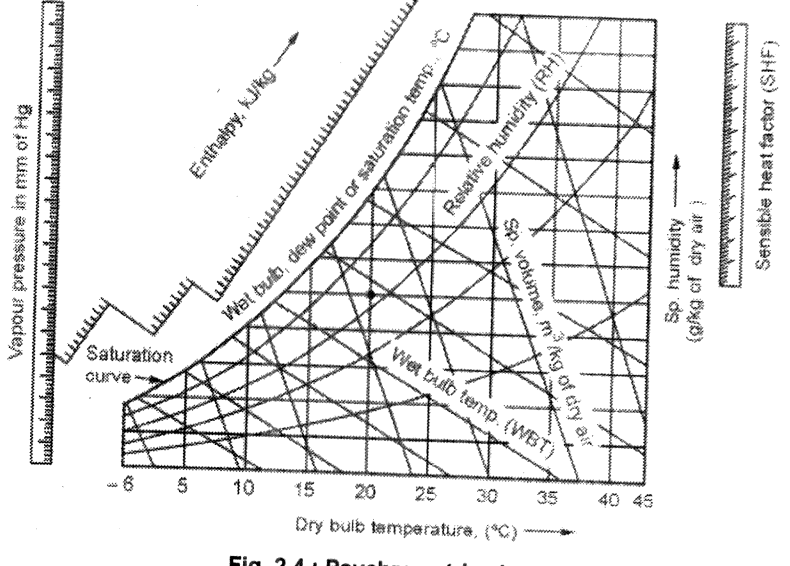


Fig. 2.4 : Psychrometric chart

❑ (1) Dry bulb temperature lines :

- These lines are vertical i.e. parallel to the ordinate and uniformly spaced as shown in Fig. 2.5.
- The temperature range of these lines on psychrometric chart is from – 6 °C to 45 °C.
- These lines are drawn with difference of every 5 °C and up to the saturation curve.

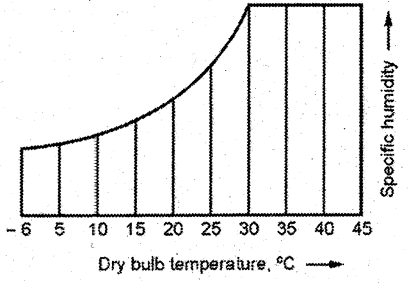


Fig. 2.5 : Dry bulb temperature lines

❑ (2) Specific humidity or moisture content lines :

- These lines are horizontal i.e. parallel to the abscissa and are also uniformly spaced as shown in Fig. 2.6.

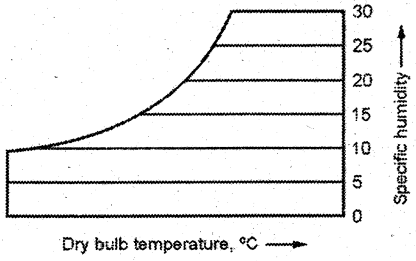


Fig. 2.6 : Specific humidity lines

- The moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air).
- The moisture content lines are drawn with a difference of every 1 g (0.001 kg) and up to the saturation curve.

❑ (3) Dew point temperature lines :

- These lines are horizontal i.e. parallel to the abscissa and non-uniformly spaced as shown in Fig. 2.7.

- At any point on the saturation curve, the dry bulb temperature and dew point temperature are same.
- The values of DPT are generally given along the saturation curve of the chart.

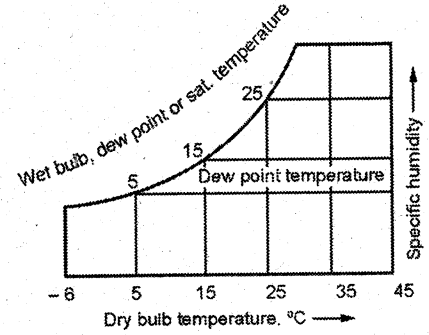


Fig. 2.7 : Dew point temperature lines

❑ (4) Wet bulb temperature lines :

- These lines are inclined straight lines and are not uniformly spaced as shown in Fig. 2.8.

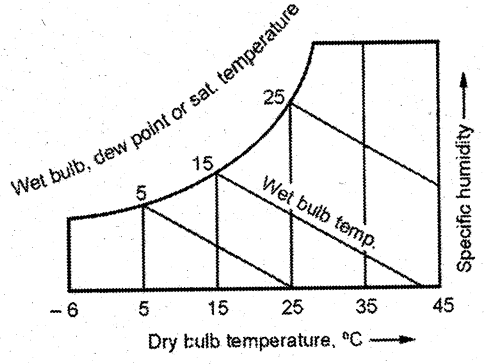


Fig. 2.8 : Wet bulb temperature lines

- At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.
- The values of WBT are generally given along the saturation curve of the chart.

❑ (5) Specific enthalpy lines :

- These lines are inclined straight lines and uniformly spaced as shown in Fig. 2.9.
- These lines are parallel to the wet bulb temperature lines and are drawn up to the saturation curve. Some lines coincide with the WBT lines.

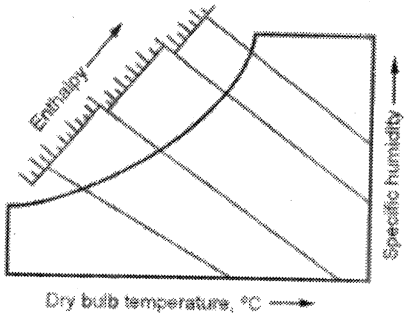


Fig. 2.9 : Enthalpy lines

- The values of total enthalpy are given on a scale above the saturation curve.
- The specific enthalpy will increase with DBT and moisture content.

❑ (6) Specific volume lines :

- These lines are obliquely inclined straight lines and uniformly spaced as shown in Fig. 2.10.
- These lines are drawn up to the saturation curve. The values of volume lines are generally given at the base of the chart.

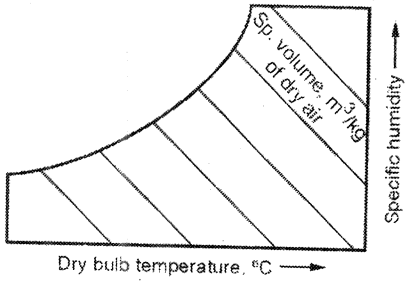


Fig. 2.10 : Specific column lines

❑ (7) Relative humidity lines :

- These lines are curved lines and follow the saturation curve. Generally, these lines are drawn with values of relative humidity 10 %, 20 %, 30 % etc. and up to 100 %.
- The saturation curve represents 100 % relative humidity line. The values of relative humidity lines are generally given along the lines themselves.

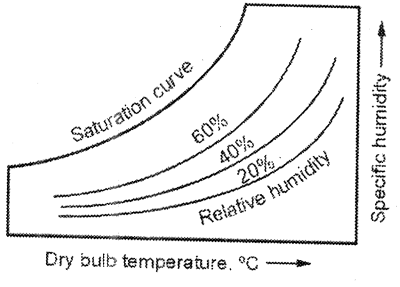


Fig. 2.11 : Relative humidity lines

2.7 Psychrometric Processes

- After developing the relationships between all the different properties of moist air, the next step in the design and analysis of air conditioning system is to study the various processes being performed on air.
- All these processes can be plotted easily on a psychrometric chart for quick visualization of changes in important properties such as temperature, moisture content, relative humidity, enthalpy, etc. In this section the various air conditioning processes are treated individually at first and later combined to form complete cycles.
- The important process that air undergoes in a typical air conditioning system are :

(1) Sensible cooling	(2) Sensible heating
(3) Humidification and dehumidification	(4) Cooling and dehumidification
(5) Cooling with adiabatic humidification	(6) Heating and humidification
(7) Heating and dehumidification	(8) Adiabatic mixing of two streams

2.7.1 Sensible Cooling

- The cooling of air, without change in its moisture content (i.e. specific humidity) is known as sensible cooling.
- When air at temperature t_{d1} passes over a cooling coil whose temperature (t_{d3}) is below the DBT of entering air and above the DPT of entering air (t_{dp1}) then moisture content is unaffected. So this process is called as sensible cooling as only sensible heat is removed from the air.
- In this process, the dew point temperature and latent heat content of air remains constant. Therefore, the process is indicated as a horizontal line from right to left, based on the end conditions of air, as shown in Fig. 2.12.
- Fig. 2.12 (a) shows the cooling process on psychrometric chart as horizontal line 1-2. The point 3 represents the surface temperature of the cooling coil.
- If the cooling coil is perfect (i.e. 100 % effective) then the leaving temperature of air (t_{d2}) will be equal to the surface temperature of coil (t_{d3}).
- But in actual practice, the leaving air temperature will be higher than the cooling coil temperature as it requires contact of air with coil surface for enough period of time.
- During the sensible cooling process, the specific humidity of air remains constant (i.e. $w_1 = w_2$). The DBT of air reduces from t_{d1} to t_{d2} and RH of air increases from ϕ_1 to ϕ_2 as shown in Fig. 2.12 (b).

- The heat rejected by air is obtained from the psychrometric chart by enthalpy difference $(h_1 - h_2)$ as shown in Fig. 2.12 (b).

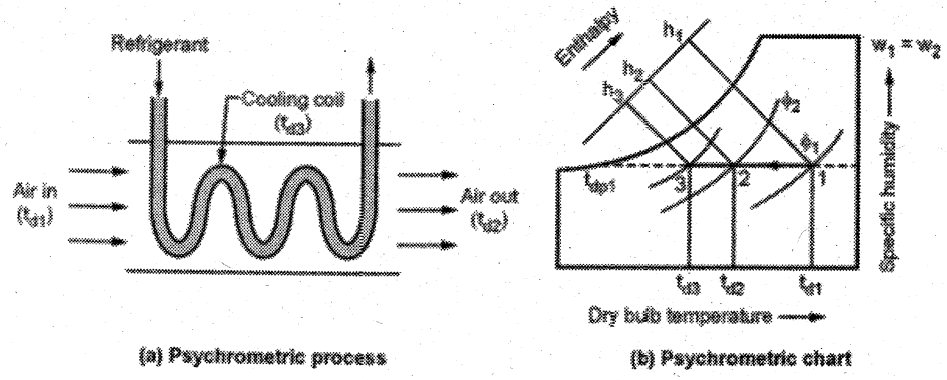


Fig. 2.12 : Sensible cooling

The total heat rejected during sensible cooling is equal to the change in sensible heat and is given by,

Heat rejected, $q = h_1 - h_2$

$= C_{pa}(t_{d1} - t_{d2}) + w\,C_{pv}\,(t_{d1} - t_{d2})$ ... ($\because h = C_p \cdot \Delta T$)

$q = (C_{pa} + wC_{pv})(t_{d1} - t_{d2})$

$= C_{pm}(t_{d1} - t_{d2})$

- The term $(C_{pa} + C_{pv})$ is called **humid specific heat (C_{pm})** and its value is taken as 1.022 kJ/kg K.

$\therefore$ Heat rejected, $q = 1.022\,(t_{d1} - t_{d2})$ kJ/kg

- The cooling capacity of coil(sensible heat per minute) is,

$$Q = m_a\,C_{pm}(t_{d1} - t_{d2})\ \text{kJ/min}$$

where, $\dot{m}_a$ = Mass flow rate of air in kg/min.

- For air conditioning applications generally flow rate is mentioned in cmm i.e (m^3/min.). Therefore, the sensible heat transfer per min is given as,

$$SH = V \cdot r \cdot C_{pm}\,(\Delta t),\ \text{kJ/min}$$

where, V = Flow rate of air in cmm i.e. m^3/min

ρ = Density of moist air = 1.2 kg/m^3

C_{pm} = Humid specific heat = 1.022 kJ/kg K and

Δt = Temperature difference between entering and leaving air condition

Putting the values of ρ and C_{pm} in above equation we get,

$SH = v \times 1.2 \times 1.022 \times (\Delta t)$

$= 1.2264\,V \times (\Delta t)$ kJ/min $= \dfrac{1.2264\,v \times \Delta t}{60}$

$$\mathbf{SH = 0.0204\,V \times t,\ kW}$$

Note : The sensible cooling can be possible only upto the dew point temperature (t_{dp}) as shown Fig. 2.12 (b). The further cooling will result in condensation of moisture.

❑ Solved examples on Sensible Cooling

Ex. 2.9 : (Sensible cooling) : In a summer air conditioner, an atmospheric air enters at 30 °C DBT and 18 °C WBT. The air leaves the air conditioner at 20 °C DBT without changing its moisture content. Find the following :

(i) Initial enthalpy and specific humidity of air.

(ii) Final relative humidity of air and its WBT.

(iii) Sensible heat removed per kg of air.

▣ **Sol. :** Refer Fig. 2.13.

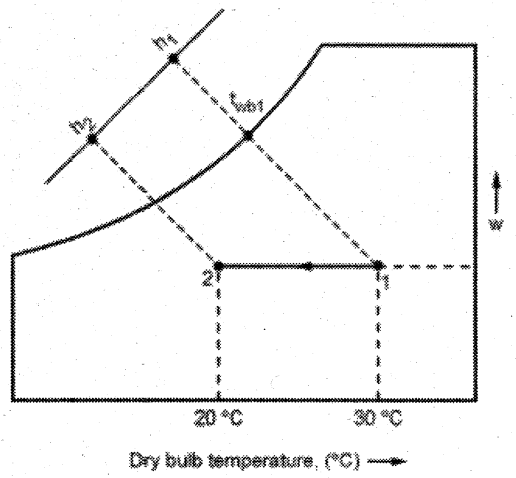


Fig. 2.13

Given data : $t_{db1} = 30$ °C, $t_{wb1} = 18$ °C, $t_{db2} = 20$ °C

(i) Initial enthalpy (h_f) and specific humidity :

- Locate point 1 at the intersection of 30 °C DBT and 18 °C WBT lines.

From psychrometric chart at point 1,

h_1 = **50 kJ/kg of air**

and w_1 = **0.008 kg/kg of dry air** **... Ans.**

(ii) Final relative humidity ϕ_2 and t_{wb2} :

- As the air is cooled without changing its moisture content ($w_1 = w_2$) upto 20 °C WBT, draw a horizontal line from point 1 that cuts the 20 °C vertical DBT line at point 2.
- From psychrometric chart,

We get, ϕ_2 = 55 % and t_{wb2} = 14.5 °C.

Also, h_2 = **42 kJ/kg** **... Ans.**

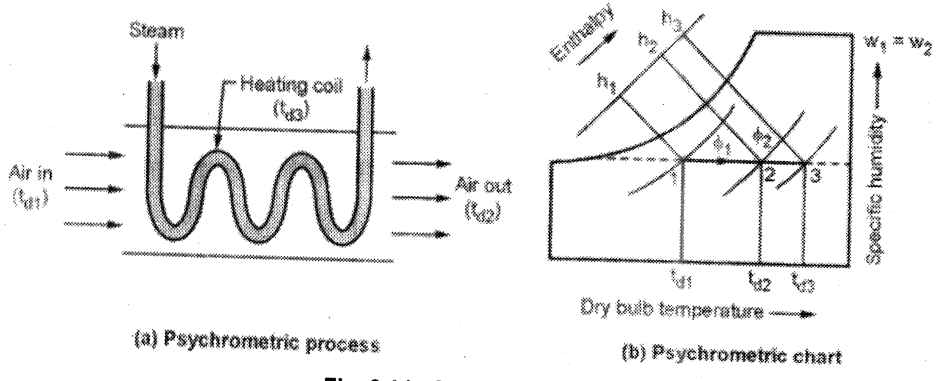
(iii) Sensible heat removed per kg of air (Q) :

$$Q = (h_1 - h_2) = (50 - 42)$$

Q = 8 kJ/kg of air **... Ans.**

2.7.2 Sensible Heating

- The heating of air, without any change in its moisture content (i.e. specific humidity), is known as sensible heating.
- When air at temperature (t_{d1}) passes over a heating coil, whose surface temperature (t_{d3}) is above DBT of entering air (t_{d1}), heating of air takes place keeping moisture constant.
- Since no moisture is added or removed from the air, the process is shown by horizontal line 1-2 from left to right as shown in Fig. 2.14 an psychrometric chart. The point 3 represents the surface temperature of heating coil.



(a) Psychrometric process (b) Psychrometric chart

Fig. 2.14 : Sensible heating

- During sensible heating, the specific humidity remains constant ($w_1 - w_2$). The DBT of entering air increases form t_{d1} to t_{d2} and RH reduces from ϕ_1 to ϕ_2 as shown in Fig. 2.14 (b).
- Generally, sensible heating is carried over the air by passing the steam or hot water through the heating coil.
- The heat absorbed by the air during sensible heating is obtained from the psychrometric chart by the enthalpy difference ($h_2 - h_1$) as shown in Fig. 2.14 (b).
- The total heat transfer during this process is equal to the change in sensible heat and is given as,

Heat added, $q = h_2 - h_1 = C_{pa}(t_{d2} - t_{d1}) + w\, C_{pv}(t_{d2} + t_{d1})$

$$q = (C_{pa} + w\, C_{pv})(t_{d2} - t_{d1}) = C_{pm}(t_{d2} - t_{d1})$$

The term (C_{pa} + w C_{pv}) is called humid specific heat (C_{pm}) and its value is taken as 1.22 kJ/kg K.

Heat addedSH = $1.022\,(t_{d2} - t_{d1})$, kJ/kg

and the heating capacity of coil is given by,

$$Q = \dot{m}_a\, C_{pm}\,(t_{d2} - t_{d1})\ \text{kJ / min}$$

where, $\dot{m}_a$ = Mass flow rate of air in kg/min

For air conditioning purpose it given as,

Heating capacity of coil, $Q = 0.0204\ V \times t$, kW

where v = Volume flow rate of air in cmm (m^3/min)

❑ Solved example on Sensibile Heating

Ex. 2.10 : (Sensible heating) : Moist air of mass flow rate 200 m^3/min at 15 °C DBT and 75 % RH is heated until its temperature reaches to 25 °C. Find the following :
(i) RH of heated air (ii) Wet bulb temperature of heated air (iii) Heat added to air.
[GTU : Summer-16, Winter-19]

Sol. : Refer Fig. 2.15.

Given data : V = 200 m^3/min t_{db1} = 15 °C,

ϕ_1 = 75 %, t_{db2} = 25 °C

- Locate point 1 on psychrometric chart at the intersection of 15 °C DBT and 75 % RH lines. Through point 1 draw a constant specific humidity line ($w_1 - w_2$) to cut 25 °C DBT line and get point 2.

- Read the following values from the psychrometric chart :

h_1 = 354.4 kJ/kg and h_2 = 45.2 kJ/kg,

V_1 = 0.825 m^3/kg of dry air

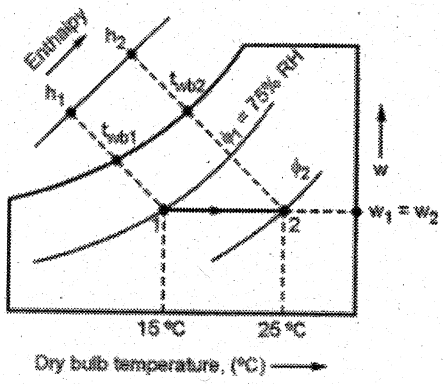


Fig. 2.15

(i) RH of heated air (from chart at point 2) :

ϕ_2 = 41 %

(ii) WBT of heated air (from chart at point 2) :

t_{wb} = 16.1 °C

(iii) Heat added to air :

$$\dot{m} = \frac{V}{V_1} = \frac{200}{0.825} = 242.42 \text{ kg/min}$$

$$Q = \dot{m}\,(h_2 - h_1) = \frac{242.42}{60}\,(45.2 - 35.4)$$

Q = 39.59 kJ/kg of air **... Ans.**

2.7.3 Bypass Factor of Heating and Cooling Coil

☞ **[GTU : Winter-17]**

- In sensible heating and cooling, we have discussed that, temperature of air leaving the apparatus (t_{d2}) is less than surface temperature of coil (t_{d3}) in a heating case and more than t_{d3} in cooling case.
- This is due to the fact that, not all air comes in contact with surface of coil and contacted air will not remain in touch with surface for sufficient time. To understand this phenomenon the concept of by pass factor is very important in air conditioning design.

- Let us consider the air at temperature t_{d1} passes over coil having surface temperature of t_{d3} as shown in Fig. 2.16.

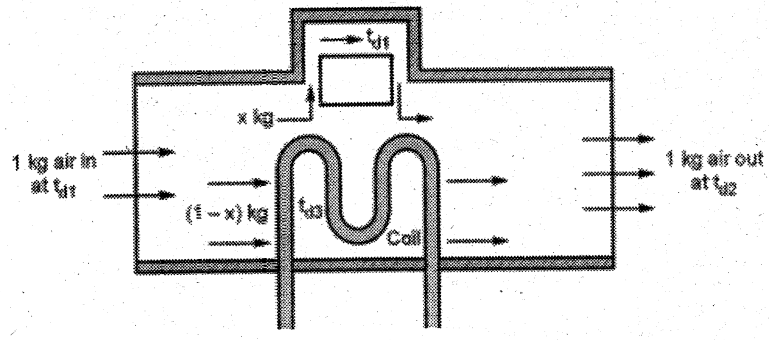


Fig. 2.16 : By pass factor

- If the mass of air is 1 kg, then some quantity of this air say 'x' kg just by-passes the coil untouched and remaining (1–x) kg comes in contact with the coil, this phenomenon of by-passing the air is measured in terms of a by-pass factor.
- The by-pass factor depends upon the following factors :
  - i) Coil surface area
  - ii) The number of tubes
  - iii) The number of fins provided in a unit length
  - iv) The number of rows in a coil
  - v) Pitch of the fins
  - vi) Velocity of air
- Applying the energy balance, we can write,

$$x\,C_{pm}\,t_{d1} + (1 - x)\,C_{pm}\,t_{d3} = 1 \times C_{pm}\,t_{d2}$$

where, C_{pm} = Humid specific heat (kJ/kg K)

$\therefore\ x\,t_{d1} + (1 - x)\,t_{d3} = t_{d2}$ or $x\,t_{d1} + t_{d3} - x\,t_{d3} = t_{d2}$

$\therefore\ x\,(t_{d3} - t_{d1}) = t_{d1} - t_{d2}$

$$x = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

where, x is called the **by-pass factor of the coil** and is generally written as BPF.

For heating coil, the by-pass factor is given by,

$$BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

Similarly, for cooling coil $BPF = \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$

- The term (1 - x) i.e. (1 - BPF) is called as **contact factor or efficiency of coil.**

∴ Efficiency of heating coil is,

$$\eta_H = 1 - BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}} = \frac{t_{d2} - t_{d1}}{t_{d3} - t_{d1}}$$

and efficiency of cooling coil is,

$$\eta_c = 1 - BPF = 1 - \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}} = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}}$$

❑ Solved example on BPF

Ex. 2.11 : (Sensible cooling, BPF) : Moist air enters the cooling coil at 40 °C DBT and 20 °C WBT. Finally, the air is sensibly cooled to 26 °C DBT. Plot the process on psychrometric chart and determine
(i) Final WBT of air.
(ii) The total heat transferred in kW, if air is flowing at the rate of 100 m^3/min.
If the cooling coil surface temperature is 22 °C, find the bypass factor of coil.

☞ **[GTU : Summer-18]**

Sol. : Refer Fig. 2.17.

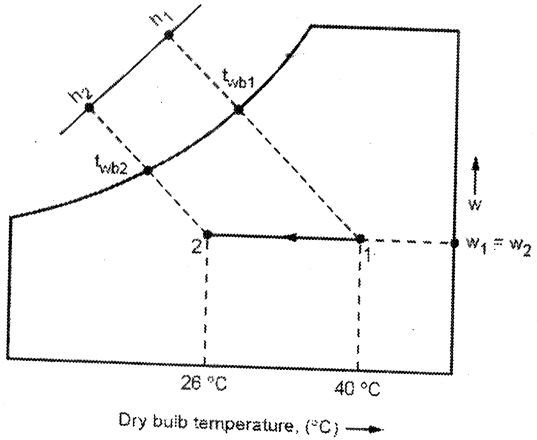


Fig. 2.17

Given data : $t_{db1} = 40$ °C, $t_{wb1} = 20$ °C, $t_{db2} = 26$ °C

$v_a = 100$ m^3/min, ADP = 22 °C

- Locate point 1 at the intersection of 40 °C DBT and 20 °C WBT lines.
- Through point 1 draw a horizontal line that cuts the saturation curve at point 2.
- Line 1-2 represents sensible cooling process as shown in Fig. 2.17.
- Locate point 3 at the intersection of 26 °C DBT line and line 1-2.

i) WBT of air : From psychrometric chart, at point '3' we get

$t_{wb3} = 15.5$ °C, $h_3 = 41.5$ kJ/kg

ii) Total heat transferred (Q) :

Mass flow rate of air is,

$$m_a = \frac{v}{v_1}$$

From psychrometric chart, at point 7

$v_1 = 0.896$ m^3/kg dry air, $h_1 = 57.5$ kJ/kg

$$\therefore \quad m_a = \frac{100}{0.896 \times 60} = 1.8601 \text{ kg/s}$$

Thus $Q = m_a (h_1 - h_3) = 1.8601 (57.5 - 41.5)$

$$\therefore \quad Q = \mathbf{29.7616 \text{ kW}}$$

iii) By pass factor of coil is

$$BPF = \frac{T_3 - T_2}{T_1 - T_2} = \frac{26 - 22}{40 - 22} = \mathbf{0.22}$$ **... Ans.**

Ex. 2.12 (Sensible heating) : In a heating application, moist air enters a steam heating coil at 15 °C DBT and 10 °C WBT at the flow rate of 5 kg/sec. The temperature of heating coil is 40 °C, while its bypass factor is 0.4. Determine DBT, WBT, RH of the air leaving the coil. Also determine the capacity of heating coil. ☞ **[GTU : Winter-17]**

Sol. : Refer Fig. 2.18.

Given data : $t_{db1} = 15$ °C ; $t_{wb1} = 10$ °C, $t_{db3} = 40$ °C, BPF = 0.4

- Locate point 1 at the intersection of 15 °C DBT and 10 °C WBT lines as shown an psychrometric chart in Fig. 2.18.
- Through point 1 draw a horizontal lines to intersect the vertical line at the DBT of heating coil at 40 °C to get point 2. Then, line 1-2 represents sensible heating process.

- Point 3 lies on the line 1-2. Let tdb3 be the temperature of air leaving the heating coil.

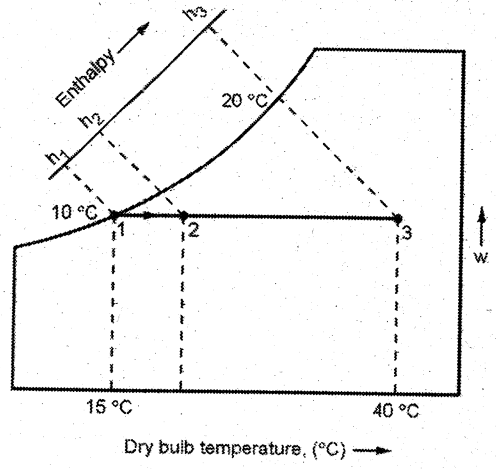


Fig. 2.18

(i) DBT of air leaving the coil :

$$\text{We know that, BPF} = \frac{t_{db3} - t_{db2}}{t_{db3} - t_{db1}}$$

$$0.4 = \frac{40 - t_{d2}}{40 - 15}$$

$$\therefore \quad t_{db2} = 30\ °C \qquad \text{... Ans.}$$

(ii) WBT of air leaving the coil (t_{wb2}) and relative humidity (ϕ_2) :

- Locate point 3 an vertical line of DBT = 30 °C, that cuts the line 1-2 at point 3.
- From the psychrometric chart, the outlet state at point 3 is :

$$t_{wb2} = 16\ °C \text{ and } \phi_2 = 21\ \%$$

(iii) Capacity of heating coil (Q) :

From psychrometric chart,

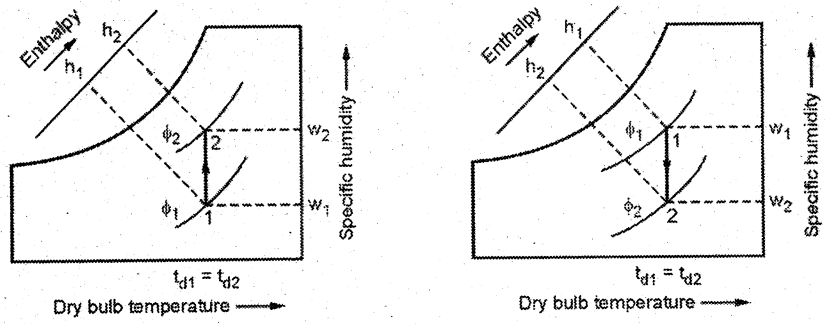
$$h_1 = 29.5 \text{ kJ/kg} \qquad h_3 = 45 \text{ kJ/kg}$$

$$\text{Capacity of heating coil (Q)} = \dot{m}\,(h_3 - h_1) = 50\,(45 - 29.5)$$

$$= 775 \text{ kW} \qquad \text{... Ans.}$$

2.7.4 Humidification and Dehumidification ☞ [GTU : Summer-16, Winter-18]

- **Humidification** is the process of addition of water vapour in the air at constant dry bulb temperature. Similarly, removal of water vapour from the air, without change in its dry bulb temperature is known as **dehumidification.**
- During perfect humidification process, the dry bulb temperature of air is remain constant, therefore it is represented on psychrometric chart as a vertical line from bottom to top as shown in Fig. 2.19 (a). It is to be noted that in humidification, the RH of air increases from ϕ_1 to ϕ_2 and specific humidity also increases from w_1 to w_2.
- Similarly, ideal dehumidification process is represented on psychrometric chart as a vertical line from top to bottom as shown in Fig. 2.19 (b). During dehumidification process, the RH and specific humidity decreases from ϕ_1 to ϕ_2 and w_1 to w_2 respectively.
- Since the DBT of air remains constant during humidification and dehumidification process, sensible heat also remains constant.
- Therefore, the total heat transfer during these processes is the latent heat which causes the change in enthalpy per kg of dry air due to increased moisture content equal to $(w_2 - w_1)$ kg per kg of dry air.



(a) Humidification (b) Dehumidification

Fig. 2.19 : Humidification and dehumidification

- Thus, the latent heat transfer during humidification and dehumidification is given by,

$$\text{LH} = h_2 - h_1 \qquad \text{... for humidification}$$

$$= h_{fg}\,(w_2 - w_1)$$

where, h_{fg} = Latent heat of vapourisation at DBT (t_{d1})

- Similarly, for dehumidification the above equation may be written as,

$$LH = (h_1 - h_2) = h_{fg}(w_1 - w_2)$$

In air conditioning practice, the latent heat load per minute is given as :

$$LH = \dot{m}_a (\Delta h)\ \dot{m}_a \cdot h_{fg} (\Delta w) \qquad (\because \dot{m}_a = V \cdot \rho)$$

$$LH = V \cdot \rho\, h_{fg} (\Delta w)$$

where, ρ = Density of moist air = 1.2 kg/m^3

h_{fg} = Latent heat of vapourisation of water = 2500 kJ/kg

(Δw) = Specific humidity difference between inlet and exit air condition

Putting these values in above equation,

$$LH = V \times 1.2 \times 2500 \times (\Delta w) = 3000\ V \times (\Delta w)\ \text{kJ/min}$$

$$LH = \frac{3000\ V \times \Delta w}{60} = 50\ V \times \Delta w\ \text{kJ/s or kW}$$

2.7.5 Sensible Heat Factor (SHF)

- The heat added during a psychrometric process may be divided into sensible heat (SH) and latent heat (LH).
- Sensible Heat Factor (SHF) or Sensible Heat Ratio (SHR) is the ratio of sensible heat to the total heat. It is given as,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH}$$

- SHF is shown on the right side of psychrometric chart.

2.7.6 Cooling and Dehumidification

- In summer air conditioning system, the outside air is cooled and dehumidified to obtain desired comfort conditions.
- During cooling of coil, the moisture is removed from the air only when air is cooled to a temperature below its dew point temperature.
- Therefore, for dehumidification the effective surface temperature of cooling coil i.e. apparatus dew point (ADP) (t_{d4}) must be less than the dew point temperature of entering air (t_{dp1}).
- The cooling and dehumidification process is represented by process 1-2 on psychrometric chart as shown in Fig. 2.20 (a).

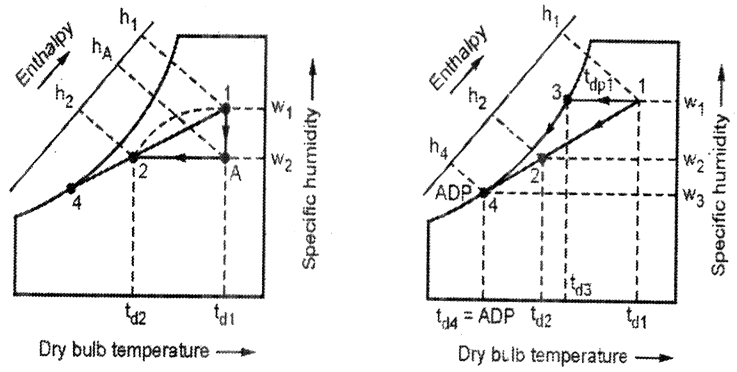


Fig. 2.20 : Cooling and dehumidification

- Let, t_{d1} = DBT of air entering the coil

t_{dp1} = DPT of air entering the coil = t_{d3}

t_{d2} = DBT of leaving air

t_{d4} = Effective surface temperature of coil

- In ideal conditions, the DBT of air leaving the coil (t_{d4}) should be same as that of surface temperature of coil (ADP), but in actual practice it is not possible due to inefficiency of cooling coil.
- Therefore, actual condition of air leaving the cooling coil is indicated by the point '2' an straight line joining the points 1 and ADP.
- In this case BPF is given by,

$$BPF = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{w_2 - w_4}{w_1 - w_4} = \frac{h_2 - h_4}{h_1 - h_4}$$

- For calculation of psychrometric properties, only end points are considered.
- Thus, the cooling and dehumidification process as shown by a line 1-2 is divided as 1-A (dehumidification) and A-2 (cooling).

The total heat removed from the air during cooling and dehumidification process is,

$$q = h_1 - h_2 = (h_1 + h_A) + (h_A - h_2) = LH + SH$$

where, LH = Latent heat removed due to condensation of moisture

SH = Sensible heat removed

- The total heat absorbed by the coil is sum of SH and LH and is given by enthalpy difference $(h_1 - h_2)$ as shown in Fig. 2.20 (a). The proportion of sensible heat to the total heat is expressed in sensible heat factor.
- The SHF of process gives the slope of line representing the process an psychrometric chart as shown in Fig. 2.20 (a) by line 1-ADP.

Thus, from Fig. 2.20 (a),

$$SHF = \frac{SH}{SH + LH} = \frac{h_A - h_2}{(h_A - h_2) + (h_1 - h_A)}$$

$$= \frac{h_A - h_2}{h_1 - h_2}$$

❑ Examples on Cooling and Dehumidification

Ex. 2.13 (Cooling and dehumidification) : In a cooling application, air at 32 °C DBT and 20 °C WBT is passed through a cooling coil maintained at 5 °C. The heat removed by the cooling coil from air is 14 kW and air flow rate is 42.5 m^3 /min
Determine : (i) DBT and WBT of the air leaving the coil. (ii) Coil by-pass factor.

☞ **[GTU : Winter-17,19]**

▣ **Sol. :** Refer Fig. 2.21.

Given data : $t_{db1} = 32$ °C, $t_{wb1} = 20$ °C, $t_{db2} = 5$ °C,

$V = 42.5\ m^3/min$

Heat removed by the cooling = 14 kW

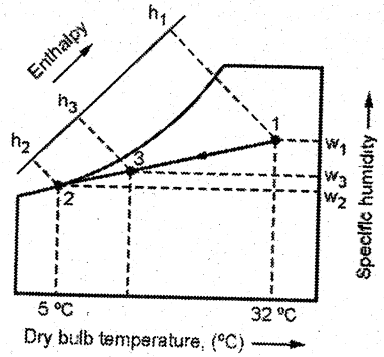


Fig. 2.21

- Locate the point 1 at the intersection of 32 °C DBT and 20 °C WBT lines.
- Locate the point 2 at the intersection of 5 °C DBT and the saturation curve.

Join point 1 and 2.

Line 1-2 represents cooling and dehumidification process.

From psychrometric chart, corresponding to points 1 and 3

We have, $w_1 = 0.0098$ kg/kg of dry air and

$h_1 = 57.6$ kJ/kg

$v_1 = 0.875$ /kg,

$w_2 = 0.0054$ kg/kg of dry air

$h_2 = 18.7$ kJ/kg

Mass of air passed through the coil is given by,

$$\dot{m}_a = \frac{v}{v_1} = \frac{42.5}{0.875} = 48.57 \text{ kg/min}$$

Heat removed per kg of air by the cooling coil is,

$$Q = \dot{m}_a \cdot (h_1 - h_3)$$

$$\therefore \quad 14 \times 60 = 48.57 \times (57.6 - h_3)$$

$$\therefore \quad h_3 = 40.31 \text{ kJ/kg}$$

(i) DBT and WBT of the air leaving the coil

The equation for the condition line can be written as :

$$\frac{h_1 - h_3}{h_1 - h_2} = \frac{w_1 - w_3}{w_1 - w_2}$$

$$\therefore \quad \frac{57.6 - 40.31}{57.6 - 18.7} = \frac{0.0098 - w_3}{0.0098 - 0.0054}$$

$$w_3 = 0.00785 \text{ kg}$$

- Corresponding to $h_3 = 40.31$ kJ/kg and $w_3 = 0.00785$, locate the point 3 on the psychrometric chart and final :

$$\mathbf{t_{db3} = 20.3\ °C}$$

$$\mathbf{t_{wb3} = 14.4\ °C}$$ **... Ans.**

(ii) Coil by-pass factor

$$BPF = \frac{t_{db3} - t_{db2}}{t_{db1} - t_{db2}} = \frac{20.3 - 5}{32 - 5}$$

$$\mathbf{BPF = 0.56}$$ **... Ans.**

Ex. 2.14 (Cooling and dehumidification) : The moist air at 30 °C DBT and 75 % RH enters a refrigeration coil at the rate of 120 m^3/min. The coil dew point temp. is 14 °C and the by-pass factor of the coil is 0.1. Determine :
(i) The temp. of air leaving the cooling coil
(ii) The capacity of the cooling coil in TR and in kW
(iii) The amount of water vapour removed per min.
(iv) The sensible heat factor for the process.

Sol. : Refer Fig. 2.22.

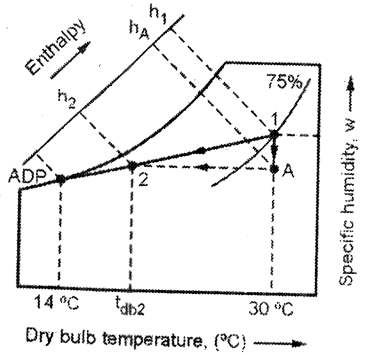


Fig. 2.22

Given data : t_{db1} = 30 ° C, ϕ_1 = 75 %, V = m^3/min , ADP = 14 °C, BPF = 0.1

- Locate the initial condition of air (point 1) at the intersection of 30 °C DBT and 75 % RH lines on psychrometric chart as shown in Fig. 2.22.
- From psychrometric chart, we have

$$T_{dp1} = 25.2\ °C$$

Since the dew point temperature (~ADP) is less than dew point temperature of entering air, therefore it is a process of cooling and dehumidification.

(i) Temperature of air leaving the cooling coil, (t_{db2})

We know that, $$BPF = \frac{t_{db2} - ADP}{t_{d1} - ADP}$$

$$\therefore \quad 0.1 = \frac{t_{db2} - 14}{30 - 14}$$

$$\therefore \quad t_{db2} = \mathbf{15.6\ °C} \qquad \textbf{... Ans.}$$

(ii) Capacity of cooling coil,

The air coming out of the coil is indicated by point 2 on the line joining 1 and ADP. as shown in Fig. 2.22. The line 1-2 represents the cooling and dehumidification process, which is assumed to have followed the path 1-A (i.e. dehumidification) and A-2 (i.e. cooling). Now from psychrometric chart, we get

Specific humidity of entering air at point 1,

$$w_1 = 0.0202\ kg/kg\ of\ dry\ air$$

specific humidity of leaving air at point 2,

$$w_2 = 0.011\ kg/kg\ of\ dry\ air$$

Specific volume of entering air at point 1,

$$v_1 = 0.884\ m^3/kg\ of\ dry\ air$$

Enthalpy at point 1, h_1 = 82 kJ/kg

Enthalpy at point A, h_A = 58 kJ/kg

Enthalpy at point 2, h_2 = 43.5 kJ/kg

We know that, mass flowing through the cooling coil,

$$\dot{m}_1 = \frac{v}{v_1} = \frac{120}{0.884} = 135.75\ kg/min$$

Capacity of the cooling coil in kW $= \dot{m}_1 (h_1 - h_2)$

$$= \frac{135.75}{60} (82 - 43.5)$$

$$= \mathbf{87.106\ kW} \qquad \textbf{... Ans.}$$

Capacity of the cooling coil in TR $= \mathbf{87.106/3.516}$

$$= \mathbf{24.77\ TR} \qquad \textbf{... Ans.}$$

(iii) Amount of water vapour removed

$$= \dot{m}_1 (w_1 - w_2) = 135.75 \times (0.0202 - 0.011)$$

$$= \mathbf{1.25\ kg/min} \qquad \textbf{... Ans.}$$

(iv) Sensible heat factor of process is,

$$SHF = \frac{h_A - h_2}{h_1 - h_2} = \frac{58 - 43.5}{82 - 43.5} = \mathbf{0.377}$$

Ex. 2.15 (Cooling and dehumidification, SHF line) : In air conditioning application, the atmospheric air enters a dehumidifying coil at initial condition of 30 °C DBT and 20° WBT respectively. It is desired that, the air leaving the coil have 17 °C DBT and 14 °C WBT. Determine :

(i) The effective surface temperature of coil

(ii) The by-pass factor of coil

(iii) The sensible heat factor of coil.

(iv) The sensible heat removed per kg of air

(v) The latent heat removed per kg of air

(vi) The mass of water vapour condensed per kg of air. **[GTU : Summer-16,18, Winter-17]**

Sol. :

Refer Fig. 2.23.

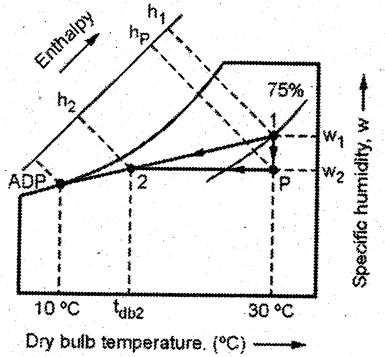


Fig. 2.23

Given data : $t_{db1} = 30$ °C, $t_{wb1} = 20$ °C, $t_{db2} = 17$ °C, $t_{wb2} = 14$ °C

- Locate the entering condition of air i.e. point 1 act the intersection of 30 °C DBT and 20 °C WBT lines.
- Similarly, locate the leaving condition of air i.e. point 2 at the intersection of 17 °C DBT and 14 °C WBT lines.
- Join the points 1 and 2. The line 1-2 represent sensible heat factor line to provide simultaneous cooling and dehumidification of air as shown in Fig. 2.23 an psychrometic chart.

(i) The effective surface temp. of coil (ADP) :

- Extend the process line 1-2 till it cuts the saturation curve at point 3 as shown in Fig. 2.23. The intersection point '3' gives the required surface temperature of coil. From psychrometric chart,

ADP = $t_{db3} = 10$ °C

(ii) The by-pass factor of coil (BPF) :

$$\text{We know that, BPF} = \frac{t_{db2} - t_{db3}}{t_{db1} - t_{db3}} = \frac{17 - 10}{30 - 10}$$

$$\therefore \quad \textbf{BPF = 0.35} \quad \textbf{... Ans.}$$

(iii) The Sensible Heat Factor of Coil (SHF) :

- Locate a reference point an psychrometric chart as shown in Fig. 2.23 (at 25° C DBT and 50 % RH.)
- Through a reference point 'O', draw a line parallel to process line 1-2 that will cut the SHF scale at point 'A' as shown in Fig. 2.23. The intersection point of line OA and SHF scale gives the required sensible heat factor of coil.
- From SHF scale of psychrometric chart, we get

SHF = 0.73 **... Ans.**

(iv) Sensible heat removed per kg of air (SH) :

- The cooling and dehumidification process as shown in Fig. 2.23 is assumed to have followed the path 1-P (i.e. dehumidification) and P-2 (i.e. cooling).

From psychrometric chart,

$h_1 = 57.60$ kJ/kg, $h_2 = 39.5$ kJ/kg and $h_p = 52.50$ kJ/kg

We know that, sensible heat removed is,

$$Q_s = SH = \dot{m}_1 (h_p - h_2)$$

$$= 1 (52.50 - 39.5)$$

$$= \textbf{13 kJ/kg of dry air} \quad \textbf{... Ans.}$$

(v) Latent heat removed per kg of air (LH) :

$$\text{It is given by, LH} = Q_l = \dot{m}_a (h_1 - h_p)$$

$$= 1 (57.60 - 52.50) = \textbf{5.1 kJ/kg of dry air} \quad \textbf{... Ans.}$$

$$\text{Thus total heat removed} = SH + LH = 13 + 5.1 = \textbf{18.1 kJ/kg} \quad \textbf{... Ans.}$$

(vi) The mass of water vapour condensed per kg of air :

From psychrometric chart,

$w_1 = 0.01063$ kg/kg of dry air

and $w_2 = 0.0088$ kg/kg of dry air

The mass of locater vapour condensed is,

$$= \dot{m}_a\ (w_2 - w_1) = 1\ (0.01063 - 0.0088)$$

$$= \mathbf{0.00183\ kg/kg\ of\ dry\ air} \quad \textbf{... Ans.}$$

Ex. 2.16 (Cooling and dehumidification) : 40 m^3/min of air is at 31 °C DBT and 19 °C WBT is passed over the cooling coil whose surface temperature is 5 °C. The cooling capacity is 4 TR under the given conditions of air. Determine DBT and WBT of the air leaving the cooling coil. Also calculate by-pass factor of cooling coil. **[GTU : Summer-18]**

Sol. :

Refer Fig. 2.24.

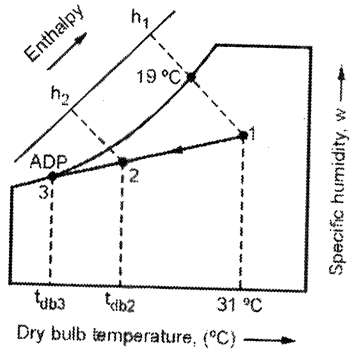


Fig. 2.24

Given data : $v_a = 40\ m^3$/min, $t_{db1} = 31$ °C, $t_{wb1} = 19$ °C, ADP = $t_{d3} = 5$ °C, $Q_C = 4$ TR

- Locate the entering air condition i.e. point 1 at the intersection of 31 °C DBT and 19 °C WBT lines.
- Locate the surface temperature of cooling coil i.e. point '3' on saturation surve at 5 °C DBT as shown in Fig. 2.24 on psychrometric chart.
- Join the points 1 and 3. The line 1-3 represents cooling and dehumidification process.
- Let, point '2' be the condition of air leaving the cooling coil.

(i) DBT and WBT of leaving air (t_{db2}, t_{wb2}) :

From psychrometric chart, we get,

$$v_1 = 0.874\ m^3/kg,\ h_1 = 54\ kJ/kg\ of\ dry\ air$$

We know that, Mass flow rate of air is given by,

$$\dot{m}_a = \frac{v_a}{v_1} = \frac{40}{0.874 \times 60} = 0.7628\ kg/sec.$$

Thus, capacity of cooling coil is

$$Q = \dot{m}_a\ (h_1 - h_2)$$

$$\therefore \quad 4 \times 3.516 = 0.7628\ (54 - h_2)$$

$$\therefore \quad h_2 = 35.56\ kJ/kg\ of\ dry\ air$$

- Now, plot point '2' on line 1-3 such that, the enthalpy

$$h_2 = 35.56\ kJ/kg.$$

From psychrometric chart,

DBT of leaving air is, t_{db2} = **17.5 °C.** **... Ans.**

and WBT of leaving air is, t_{wb2} = **12.6 °C.** **... Ans.**

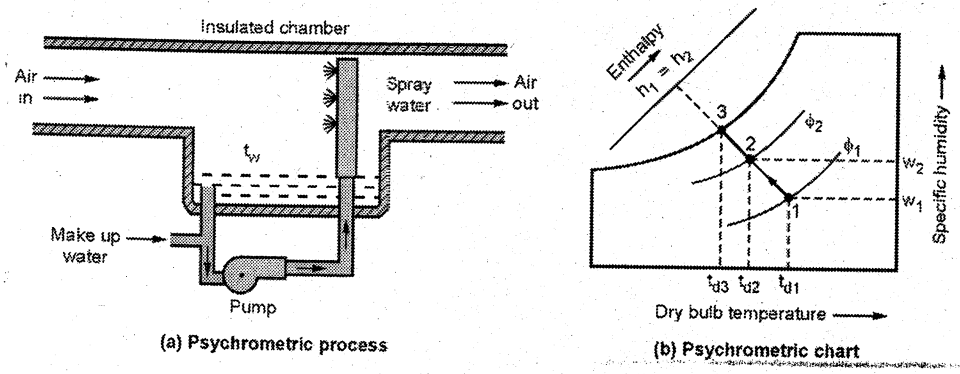
(ii) By-pass factor of cooling coil

$$\text{It is given by, BPF} = \frac{t_{db2} - t_{db3}}{t_{db1} - t_{db3}} = \frac{17.5 - 5}{31 - 5}$$

$$\therefore \quad \mathbf{BPF = 0.48} \quad \textbf{... Ans.}$$

2.7.7 Cooling with Adiabatic Humidification / Evaporative Cooling

- When the air is passed through an insulated chamber, having sprays of water maintained at a temperature (t_w) higher than the dew point temperature of entering air (t_{dp1}), but lower than its dry bulb temperature (t_{db1}) of entering air or equal to the wet bulb temperature of the entering air (t_{wb1}), then the air is said to be cooled and humidified.
- Since no heat is supplied or rejected from the spray water (water is circulated again and again) this process is called as **adiabatic cooling process.**
- This process is shown by line 1-3 or the psychrometric chart and follows the path along the constant wet bulb temperature line or constant enthalpy line. Refer Fig. 2.25 (a) and (b).
- During the process, the DBT of air decreases from t_{d1} to t_{d2} and RH increases from ϕ_1 to ϕ_2.
- When humidification is perfect (ideal condition), the final condition of air would be point 3. But practicly, perfect humidification is never achieved. Therefore, the final condition of air at the outlet is indicated by point 2 and the line 1-3 as shown in Fig. 2.25 (b).



(a) Psychrometric process (b) Psychrometric chart

Fig. 2.25 : Cooling with adiabatic humidification

- The performance of spray chamber is measured by the term effectiveness humidifying efficiency as,

$$\eta_H = \frac{\text{Actual dropin DBT}}{\text{Ideal drop in DBT}}$$

or
$$\eta_H = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}} = \frac{w_2 - w_1}{w_3 - w_1}$$

❑ Solved examples on Cooling with Humidification

Ex. 2.17 (Cooling humidification) : On a particular day, the atmospheric air conditions are recorded as 30 °C DBT and 40 % RH. Determine the dew point and wet bulb temperature of air. If this air cooled in the air washer using recirculated spray water and having a humidifying efficiency of 90 %, what is the dry bulb temperature and dew point temperature of air leaving the air washer ? **[GTU : Summer-19]**

Sol. : Refer Fig. 2.26.

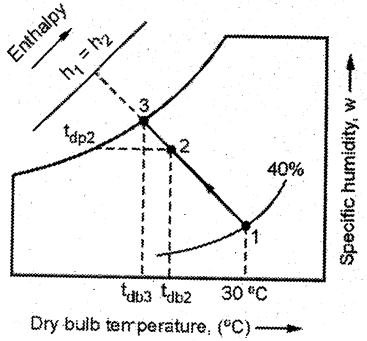


Fig. 2.26

Given data : $t_{db1} = 30$ °C, $\phi_1 = 40$ %, $\eta_H = 0.9$

(i) Dew point temperature and wet bulb temperature (t_{dp1}, t_{wb1}) :

Locate initial condition of air i.e. point 1 at the intersection of 30 °C DBT and 40 % . RH as shown in Fig. 2.26 on psychrometric chart.

From psychrometric chart, we get,

$$t_{dp1} = 15 \text{ °C}$$

and $$t_{wb1} = t_{d3} = 19.8 \text{ °C}$$

(ii) Dry bulb temperature and dew point temperature of air leaving air leaving air washer :

Let t_{dp2} be the DBT of air leaving the air washer.

In an ideal case i.e when the humidification is perfect the final condition of air will be at point '3' as shown in Fig. 2.26 (i.e. at temperature , 100 % RH and humidifying efficiency as 100 %).

However, in given case air washer is only 90 % efficient.

Let, the condition of air leaving the air washer is indicated by point 2 at temperature of as shown in figure an line 1-3.

Humidifying efficiency of air washer is,

$$\eta_H = \frac{t_{dp1} - t_{dp2}}{t_{dp1} - t_{dp3}} = \frac{30 - t_{dp2}}{30 - 19.8} = 0.9$$

$$t_{dp2} = 20.82 \text{ °C} \quad \text{... Ans.}$$

- Now, plot point '2' on the constant wet bulb temperature line 1-3 such that, $t_{dp2} = 20.82$ °C.

From psychrometric chart.

The dew point temperature at point 2 is,

$$t_{dp2} = 19.4 \text{ °C} \quad \text{... Ans.}$$

Ex. 2.18 (Cooling with adiabatic humidification) : For a hall to be air conditioned, Outdoor conditions : DBT = 40 °C,WBT = 20 °C, Required conditions : DBT = 20 °C, RH = 60 %. Seating capacity of hall = 1500, Amount of outdoor air supplied = 0.3 m^3/min person. If required conditions are achieved first by adiabatic humidification and then by cooling, estimate : (i)Capacity of cooling coil in TR
(ii) Capacity of humidifier in kg/hr.

Sol. : Refer Fig. 2.27.

Given data : Outside conditions (1) = 40 °C DBT and 20 °C WBT

Required conditions (3) = 20 °C DBT and 60 % RH

No. of persons = 1500, V_a = 0.3 /min. person.

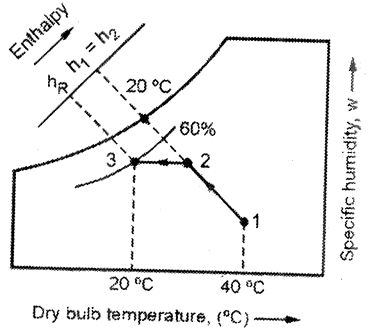


Fig. 2.27

- Locate outside conditions of air i.e. point '1' at the intersection of 40 °C DBT and 20 °C WBT lines.
- Similarly, locate required conditions of air i.e. point '3' at the intersection of 20 °C DBT and 60 % RH lines.
- According to given condition first adiabatic humidification is carried over air and then cooling process.
- Therefore, draw constant enthalpy line through point '1' and draw horizontal line through 3. The intersection of these lines gives the point '2' which represents the end of adiabatic humidification and begining of cooling process as shown in Fig. 2.27 an psychrometric chart.
- From psychrometric chart,

At point '1' :

$$\text{Enthalpy } h_1 = 57.4 \text{ kJ/kg of dry air}$$

$$\text{Specific humidity } w_1 = 0.0066 \text{ kg/kg of dry air}$$

$$\text{Specific volume } v_1 = 0.896 \text{ /kg of dry air}$$

At point '2' :

$$\text{Enthalpy } h_2 = h_1 = 57.4 \text{ kJ/kg of dry air}$$

$$\text{Specific humidity } w_2 = 0.0088 \text{ kg/kg of dry air}$$

At point '3' :

$$\text{Enthalpy } h_3 = 42.6 \text{ kJ/kg of dry air}$$

Total air supplied, V = Seating capacity × Air flow/min/ person

$$= 1500 \times 0.3 = 450 \text{ m}^3/\text{min}$$

(i) Cooling capacity of coil $= \dfrac{\text{Air supplied}}{\text{Specific volume} \times 60} \times (h_2 - h_3)$

$$= \frac{450}{0.896 \times 60}(57.4 - 42.6) = 123.88 \text{ kW}$$

$$= \frac{123.88}{3.516} = \mathbf{35.23\ TR} \quad \textbf{... Ans.}$$

(ii) Capacity of humidifier in kg/hr $= \dot{m}_a (w_2 - w_1)$

$$= \frac{V}{V_o}(w_2 - w_1)$$

$$= \frac{450}{0.896} \times (0.0088 - 0.0066)$$

$$= 1.1049 \text{ kg/min}$$

$$= \mathbf{66.29\ kg/hr} \quad \textbf{... Ans.}$$

Ex. 2.19 (Cooling with adiabatic humidification) : 200 m^3 of air per min is passed through adiabatic humidifier. The condition of air at inlet is 40 °C DBT and 15 % relative humidity. The outlet condition is 23 °C DBT and specific humidity of 0.12 kg/kg of dry air. Find DPT and amount of water vapour added.

Sol. : Refer Fig. 2.28.

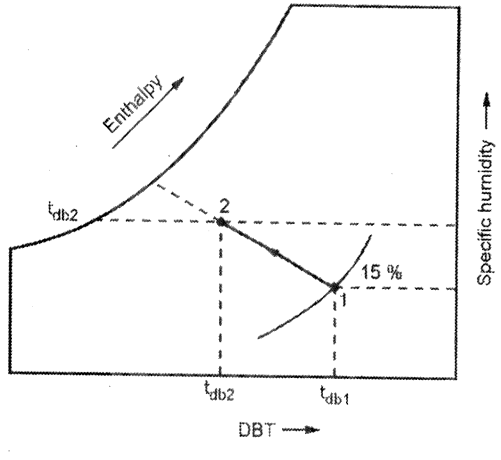


Fig. 2.28

Given data : $V = 200\ m^3/min$, $t_{db1} = 40\ °C$, $\phi = 15\ \%$, $t_{db2} = 23\ °C$, $w_2 = 0.12\ kg/kg$ of dry air

- Mark inlet condition of air at 40 °C DBT and 15 % RH lines. This is point 1 as shown in Fig. 2.28.
- Outlet condition is marked on psychrometric chart at 23 °C DBT. Specific humidity of 0.012 kg/kg of dry air. This is point 2 as shown in Fig. 2.28.

(i) Dew point temperature

Take a horizontal line from point 2 on psychrometric chart upto saturation curve from that point dew point temperature is given by,

$$t_{dp2} = 18.8\ °C \quad \text{... Ans.}$$

(ii) Amount of water vapour added to air per minute

From psychrometric chart,

At point 1 $v_1 = 0.895\ m^3/kg$ of dry air

$w_1 = 0.0066$ kg/kg of dry air

Mass of air supplied

$$\dot{m}_a = \frac{V}{V_1} = \frac{200}{0.88} = 227.27\ kg/min$$

Mass of water vapour added

$$= \dot{m}_a (w_2 - w_1)$$

$$= 227.27\ (0.012 - 0.0066) = \mathbf{1.22\ kg/min} \quad \text{... Ans.}$$

2.7.8 Heating and Humidification

- It is the reverse process of cooling and dehumidification.
- It is generally used in winter air conditioning to warm and humidify the air as the outside air is at lower temperature with lower moisture content.
- To achieve heating and humidfication process, the temperature of water to be sprayed in the air stream is kept at higher temperature than the DBT of entering air so that heat of vapourization of water will be transferred to air to make it hot.
- In this way, the unsaturated air reaches the condition of saturation and the heat of vapourizisation of water will be transferred to air to make it hot.
- The heat of vapourization of water is absorbed from the spray water itself so that the spray water gets cooled.

- The process of heating and humidification is represented by line 1-2 on psychrometric chart as shown in Fig. 2.29.
- During this process, the specific humidity, DBT, WBT, DPT and enthalpy of air increases whereas RH of air may increase or decrease. The air enters at state 1 and leaves at state 2 as shown in Fig. 2.29.

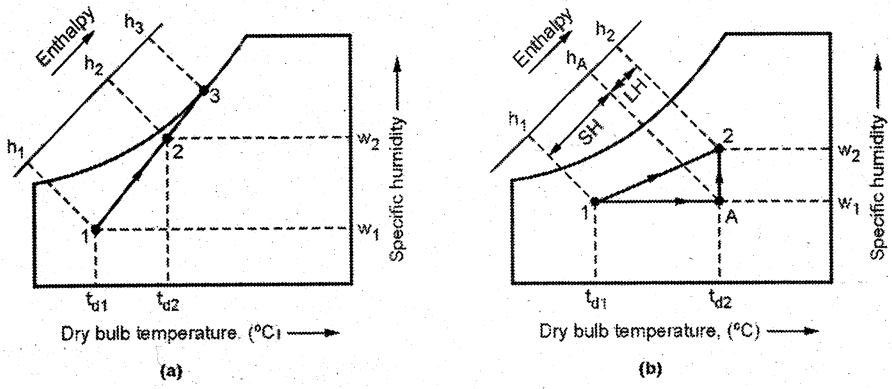


Fig. 2.29 : Heating and humidification

The mass balance for the water spray is,

$$(m_{w1} - m_{w2}) = m_a (w_2 - w_1)$$

where, m_{w1} and m_{w2} = Mass flow rates of water entering and leaving the humidifier in kg

m_a = Mass of entering dry air

w_1 and w_2 = Specific humidity of entering and leaving air respectively.

The enthalpy balance is,

$$m_{w1} - h_{fw1} - m_{w2} f_{fw2} = m_a (h_2 - h_1)$$

Where, h_{fw1} and h_{fw2} = Enthalpy of spray water entering and leaving the humidifier respectively.

The heating and humidification process shown by line 1-2 is assumed to have followed the path 1-A (i.e. heating) and A-2 (i.e. humidification) as shown in Fig. 2.29 (b). Thus, the total heat added to the air during heating and humidification is,

$$q = h_2 - h_1 = (h_2 - h_A) + (h_A - h_1)$$

$$q = SH + LH$$

where, $SH = h_A - h_1$ = Sensible heat added

$$LH = h_2 - h_A$$

= Latent heat of vapourization of increased moisture content $(w_2 - w_1)$

We know that, sensible heat factor is,

$$SHF = \frac{SH}{\text{Total heat}} = \frac{SH}{SH + LH}$$

$$SHF = \frac{h_A - h_1}{h_2 - h_1}$$

❑ Solved examples on Heating with Humidification

Ex. 2.20 (Heating and humidification) : The air of 19 °C DBT and 13 °C WBT is heated in a furnace to get the humidified air of specific humidity 0.019 kg/kg of dry air and 35 °C DBT. Find heat gain and moisture gain alongwith SHF.

▣ Sol. :

Refer Fig. 2.30.

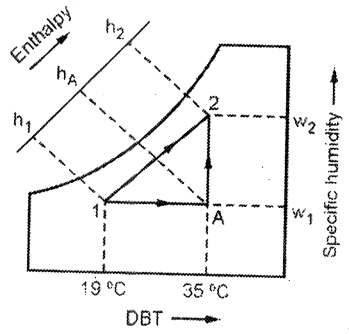


Fig. 2.30

Given data : $t_{db1} = 19$ °C, $t_{wb1} = 13$ °C, $t_{db2} = 35$ °C, $w_2 = 0.019$ kg/kg of dry air

- In psychrometric chart mark point 1 at 19 °C DBT and 13 °C WBT.
- Also mark point 2 as 35 °C DBT and 0.012 kg/kg of dry air specific humidity. Join 1 - 2.
- Draw vertical line through 2 and horizontal line from 1. Mark intersection as A.
- From psychrometric chart,

at point 2, $h_2 = 84$ kJ/kg

at point A, $h_A = 53$ kJ/kg

at point 1, $h_1 = 37$ kJ/kg

(i) heat added to the air,

$= h_2 - h_1 = (84 - 37)$

$= \mathbf{47\ kJ/kg}$ **... Ans.**

(ii) Moisture added to the air

at point 1 $w_1 = 0.007$ kg/kg of dry air

at point 2 $w_2 = 0.012$ kg/kg of dry air

Moisture added to the air,

$= w_2 - w_1 = 0.012 - 0.007$

$= \mathbf{0.005\ kg/kg\ of\ dry\ air}$ **... Ans.**

(iii) Sensible heat factor

$$SHF = \frac{h_A - h_1}{h_2 - h_1} = \frac{53 - 37}{84 - 37} = \mathbf{0.34}$$ **... Ans.**

Ex. 2.21 (Heating and humidification) : Cold air at DBT 10 °C and 60 % RH enters into a room heater at the rate of 100 /min .
It is heated at DBT of 25 °C and moisture content increased to 0.011 kg/kg of dry air. Find sensible and latent heat added, SHF and capacity of heating coil.

▣ Sol. :

Refer Fig. 2.31.

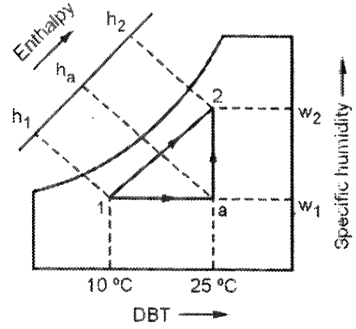


Fig. 2.31

Given data : $t_{db1} = 10$ °C, $\phi_1 = 60$ %, $V = 100$ m^3/min,

$T_{db1} = 25$ °C, $w_1 = 0.011$ kg/kg of dry air

- Mark point 1 on psychrometric chart at 10 °C DBT and 60 % RH.

- Mark point 2 on psychrometric chart at 25 °C DBT and 0.011 kg/kg of dry air join 1 - 2.
- Draw vertical line from 2 and horizontal line from 1. Mark the intersection as 'a'.
- From psychrometric chart,

at point 1, $h_1 = 21.5$ kJ/kg, $v_1 = 0.81$ m^3/kg of dry air

$w_1 = 0.0046$ kg/kg of dry air

at point 2, $h_2 = 53$ kJ/kg

at point a, $h_a = 36.5$ kJ/kg

Now, mass of air-supplied,

$$\dot{m}_a = \frac{V}{v_1} = \frac{100}{0.81} = 123.46 \text{ kg/min}$$

(i) Sensible heat added

$$SH = \dot{m}_a (h_a - h_1) = 123.46 (36.5 - 21.5)$$

SH = 1851.9 kJ/min **... Ans.**

(ii) Latent heat added

$$LH = \dot{m}_a (h_2 - h_a) = 123.46 (53 - 36.5)$$

LH = 2037.09 kJ/min **... Ans.**

(iii) Sensible heat factor

$$SHF = \frac{SH}{SH + LH} = \frac{1851.9}{1851.9 + 2037.09}$$

= 0.47 **... Ans.**

(iv) Capacity of heating coil

$$C = \dot{m}_a (h_2 - h_1) = \frac{123.46}{60} = (53 - 21.5)$$

C = 64.8165 kW = 18.4347 TR **... Ans.**

2.7.9 Heating and Dehumidification / Chemical Dehumidification

- In some applications of air conditioning, the moisture requirement is very low which is not possible to achieve by a cooling coil due to restriction of its temperature. In such cases, moisture from air is absorbed by a chemical in its path.

- In this process, the outside air is passed over chemicals which have strong affinity for water vapours.
- As air comes in contact with these chemicals, the water vapour gets condensed out of the air by giving its latent heat. This latent heat of condensation is used to heat the air and hence increases its dry bulb temperature.
- In ideal case, the latent heat released by condensation of water vapour is taken up by the air as sensible heat raising its temperature. Therefore, this process is called adiabatic dehumidification and is represented by constant enthalpy line 1-2 as shown on psychrometric chart in the Fig. 2.32.

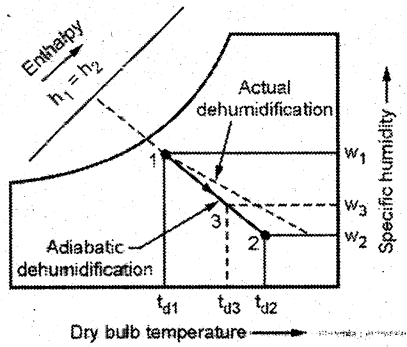


Fig. 2.32 : Heating and dehumidification

- Due to condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air. This increases its DBT.
- The efficiency of dehumidifier or effectiveness is given as,

$$\eta_H = \frac{t_{d3} - t_{d1}}{t_{d2} - t_{d1}}$$

- Heating and dehumidification process is mainly used in industrial air conditioning and can also be used for some comfort air conditioning requiring either a low RH or low dew point temperature in the room.
- The two types of chemicals are commonly used for dehumidification viz **absorbents and adsorbents.**
- The absorbents are substance which can take up moisture from air and changes chemically and physically. These includes water solutions or brines of calcium chloride, lithium chloride, lithium bromide and enthylene glycol.
- The adsorbents are the solid substance which can take up moisture from air and do not change it chemically or physically. These includes silica gel and activated alumina.

❑ Solved examples on Heating with Dehumidification

Ex.2.22 (Adiabatic chemical dehumidification) : Saturated air at 21 °C is passed through a drier so that its final relative humidity is 20 %. The drier uses silica gel absorbent. The air is then passed through a cooler until its final temperature is 21 °C without change in specific humidity. Determine :

(i) The temperature of air at the end of drying process.
(ii) The heat rejected during at the end of cooling process.
(iii) The relative humidity at the end of cooling process.
(iv) The dew point temperature at the end of drying process.
(v) The moisture removed during the drying process. ☞ **[GTU : Summer-18,19, Winter-14]**

Sol. : Refer Fig. 2.33.

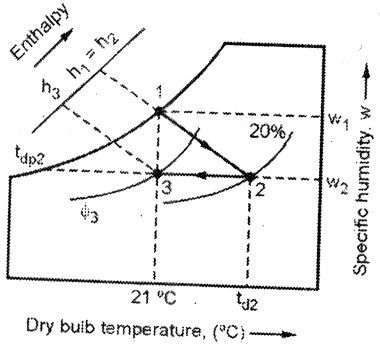


Fig. 2.33

Given data : $t_{d1} = t_{d3} = 21$ °C, $\phi_2 = 20$ %

(i) Temperature of air at the end of drying process :

- Locate the initial condition of air i.e. point '1' on the saturation curve at 21 °C DBT as shown in Fig. 2.33.
- Since the process is a chemical dehumidification process, it follows a path along constant enthalpy line as shown by the line 1-2 in Fig. 2.33. Plot the point 2 at the RH of 20 % on line 1-2.

From psychrometric chart, the temperature at the end of drying process at point '2',

$$T_{d2} = 38.5 \text{ °C}$$ **... Ans.**

(ii) Heat rejected during the cooling process :

Line 2-3 on psychrometric chart represents cooling process. From psychrometric chart,

Enthalpy of air at point '2' is $h_2 = 61$ kJ/kg of dry air

and enthalpy of air at point '3' is $h_3 = 43$ kJ/kg of dry air

Heat rejected during cooling process = $h_2 - h_3 = 61 - 43$

$\therefore$ $Q_r = 18$ kJ/kg of dry air **... Ans.**

(iii) Relative humidity at end of cooling process (ϕ_3) :

From the psychrometry chart, we get

Relative humidity end of cooling process (at point 3)

$$\phi_3 = 55 \text{ \%}$$ **... Ans.**

(iv) Dew point temperature at the end of drying process (t_{dp2}) :

From chart, $t_{dp2} = 11.6$ °C **... Ans.**

(v) Moisture removed during the drying process, :

From psychrometric chart,

Moisture in air before drying process at point 1 is,

$w_1 = 0.0157$ kg/kg of dry air.

Moisture in air after the drying process at point 2 is

$w_2 = 0.0084$ kg/kg of dry air.

Moisture removed during drying process

$= w_1 - w_2 = 0.0157 - 0.0084 = 0.0073$ kg/kg of dry air **... Ans.**

2.7.10 Adiabatic Mixing of Two Air Streams

- Consider the adiabatic mixing of different quantities of air in two different states at constant pressure.
- Let, the two air streams 1 and 2 are mixed then the final condition of the air mixture depends upon the masses involved, enthalpy and specific humidity of each of the constituent which enter the mixture.
- Let

m_1, h_1 and w_1 = Mass flow rate, enthalpy and specific humidity of entering air at '1'.

m_2, h_2 and w_2 = Corresponding values of entering air at '2'.

m_3, h_3 and w_3 = Corresponding values of mixture leaving at 3. Refer Fig. 2.34.

- Neglecting losses during the air mixing process, the mass balance for the mixing process,

$$m_1 + m_2 = m_3 \quad \text{... (i)}$$

The energy balance is given by,

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \quad \ldots (ii)$$

and mass balance for the water vapour is,

$$m_1 w_1 + m_2 w_2 = m_3 w_3 \quad \ldots (iii)$$

Substituting the value of m_3 from equation (i),

$$m_1 h_1 + m_2 h_2 = (m_1 + m_2) h_3$$

$$m_1 h_1 + m_1 h_3 = m_2 h_3 - m_2 h_2$$

$$m_1 (h_1 - h_3) = m_2 (h_3 - h_2)$$

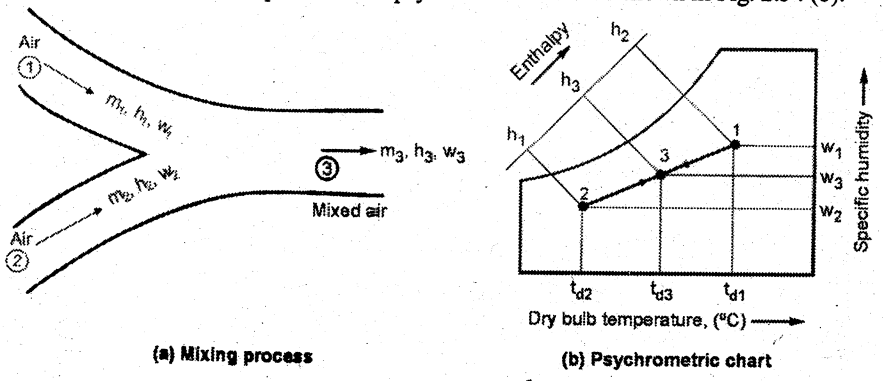
$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} \quad \ldots (iv)$$

Similarly, substituting the value of from equation (i) in equation (iii), we have,

$$\frac{m_1}{m_2} = \frac{w_3 - w_2}{w_1 - w_3}$$

$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{w_3 - w_2}{w_1 - w_3}$$

- The adiabatic process is represented on psychrometric chart as shown in Fig. 2.34 (b).



(a) Mixing process (b) Psychrometric chart

Fig. 2.34 : Adiabatic mixing of two air streams

- The final condition of mixture point 3 lies on the straight line 1-2. The point '3' divides the line 1-2 in the inverse ratio of the mixing masses. By calculating the value of w_3 from equation (iv), the point '3' can be plotted on the line 1-2.

Note : When warm and high humidity air is mixed with cold air, the resulting mixture will be a fog. In such cases, point 3 on the psychrometric chart will lie to the left or above the saturation curve. The fog can be cleared by heating the fog, mixing the fog with warmer unsaturated air or separate the water droplets from the air.

Solved examples on Mixing of Air Streams

Ex.2.23 (Mixing of two air streams) : Two air streams are mixed steadily and adiabatically. The first air stream enters at 32 °C DBT and 40 % RH while second enters at 12 °C and 90 % RH. The flow rates of the two streams are 20 m^3/min. and 25 m^3/min. respectively. Determine the specific humidity, relative humidity and flow rate after mixing.

[GTU : Winter-15, Summer-16]

Sol. : Refer Fig. 2.35.

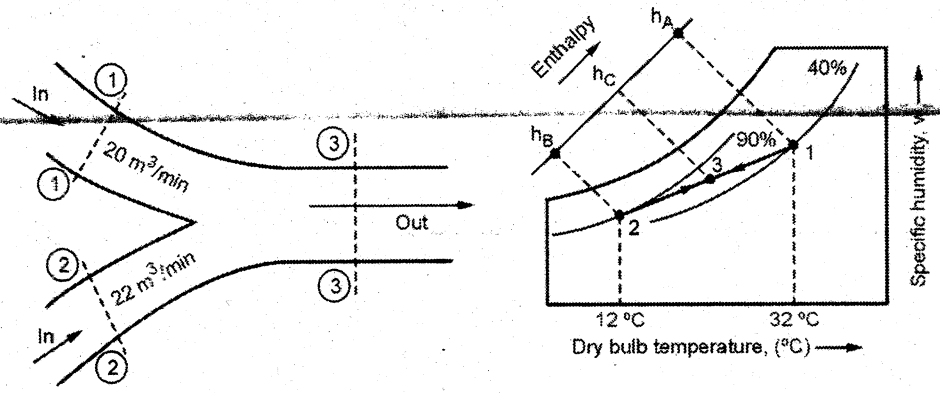


Fig. 2.35

Given data :

First air stream condition '1' = 32 °C DBT and 40 % RH .

Second air stream condition '2' = 12 °C DBT and 90 % RH

Flow rate of first air stream = $V_1 = 20\ m^3$/min.

Flow rate of second air stream = $V_2 = 25\ m^3$/min.

- Locate 1st air stream condition i.e. point '1' at the intersection of 32 °C DBT line and 40 % RH lines.
- Similarly, locate 2nd air stream condition i.e. point '2' at the intersection of 12 °C DBT line and 90 % RH lines as shown in Fig. 2.35 on psychrometric chart. Join line 1-2.
- From psychrometric chart, we get,

$$v_1 = 0.881\ m^3/kg \text{ of dry air.}$$

and $$v_2 = 0.818\ m^3/kg \text{ of dry air.}$$

Mass flow rate of 1st stream is given by,

$$\dot{m}_1 = \frac{V_1}{v_1} = \frac{20}{0.881} = 22.7 \text{ kg/min.}$$

Similarly, mass flow rate of 2nd stream is,

$$\dot{m}_2 = \frac{v_2}{v_2} = \frac{25}{0.818} = 30.56 \text{ kg/min.}$$

According to mass and energy balance, we can write,

$$(\dot{m}_1 + \dot{m}_2)\, t_{d3} = \dot{m}_1 \cdot t_{d1} + \dot{m}_2 \cdot t_{d2}$$

$$t_{d3} = \frac{22.7 \times 32 + 30.56 \times 12}{22.7 + 30.56} = 20.5 \text{ °C}$$

- Now, plot point '3' (i.e. mixing state) on line 1-2 such that $t_{d3} = 20.5$ °C.
- From psychromatric chart, $v_3 = 0.844$ m^3/kg

(i) Specific humidity of mixture at point '3' is

$$w_3 = \mathbf{0.0096 \text{ kg/kg of dry air}}$$ **... Ans.**

(ii) RH of Mixture is,

$$\phi_3 = \mathbf{64\ \%}$$ **... Ans.**

(iii) Flow rate of mixture is,

$$V_3 = m_3 \times v_3$$
$$V_3 = (22.7 + 30.56) \times 0.844$$
$$= \mathbf{44.95 \text{ m}^3\text{/min}}$$ **... Ans.**

Ex. 2.24 (Mixing of two air streams) : An air stream of 24 °C DBT and 22 °C WBT is mixed with fresh air of 32 °C DBT and 70 % RH. The volume flow rates of streams is m^3/min and 850 m^3/min respectively. Determine specific humidity, enthalpy and DPT of mixture.

☞ **[GTU : Summer-17]**

Sol. : Refer Fig. 2.36.

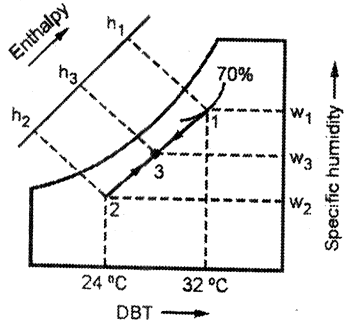


Fig. 2.36

Given data : $t_{db2} = 24$ °C, $t_{wb2} = 22$ °C, $V_2 = 850$ m^3/min, $t_{db1} = 32$ °C, $\phi_1 = 70$ %, $V_1 = 200$ m^3/min

- On psychrometric chart plot point 1 at 70 % RH and 32 °C DBT. Plot point 2 at 24 °C DBT and 22 °C WBT. Join 1-2.
- From psychrometric chart,

At point 1, $h_1 = 90$ kJ/kg,

$w_1 = 0.028$ kg/kg of dry air

$v_1 = 0.897$ kg/kg of dry air

At point 2, $h_2 = 51$ kJ/kg,

$w_2 = 0.0104$ kg/kg of dry air,

$v_2 = 0.855$ kg/kg of dry air

- Mass of fresh air at point 1,

$$\dot{m}_1 = \frac{200}{0.897} = 222.96 \text{ kg/min}$$

Mass of recirculated at point 2.

$$\dot{m}_2 = \frac{v_2}{v_2} = \frac{850}{0.855} = 994.15 \text{ kg/min}$$

Now, $\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3}$ $\therefore \frac{222.96}{994.15} = \frac{h_3 - 51}{90 - h_3}$

$$222.96\ (90 - h_3) = 994.15\ (h_3 - 51)$$

$$h_3 = \mathbf{58.14 \text{ kJ/kg}}$$ **... Ans.**

- Mark point 3 on line 1 – 2 at $h_3 = 58.14$ kJ/kg

From psychrometric chart,

$$w_3 = \mathbf{0.0126 \text{ kg/kg of dry air}}$$ **... Ans.**

$$t_{dp3} = \mathbf{17.5\ °C}$$ **... Ans.**

2.7.11 Solved Examples from University Question Papers

Ex. 2.25 : 1.5 m^3/s of moist air at a state of 28 °C dry-bulb, 21 °C wet-bulb and 101.325 kPa flows across a cooler coil and leaves the coil at 13 °C dry-bulb and specific humidity of 8.5 gm per kg of dry air. Determine the apparatus dew point, the contact factor and the cooling load.

Sol. : Given data,

Mass flow rate = 1.5 m^3/s

Entering air condition = 28 °C DBT and 21 °C WBT

Leaving air condition = 13 °C DBT and specific humidity as 8.5 $\frac{gm}{kg}$ of dry air.

Refer Fig. 2.37.

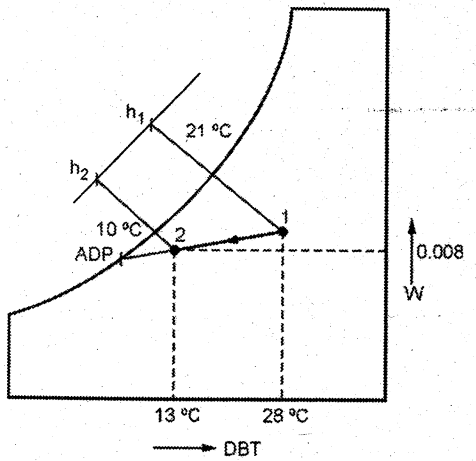


Fig. 2.37

First and all, mark the entering air condition i.e. 28 °C DBT and 21 °C WBT on psychometric chart at point 1, as shown in Fig. 2.37.

Also mark the leaving air condition i.e. 13 °C DBT and 0.0085 kg/kg of specific humidity on psychometric chart at point 2, as shown in Fig. 2.37.

Now, draw a line passing through point 1 and 2 and extend this line till saturation curve at ADP as shown in Fig. 2.37.

i) ADP of cooling coil : From psychometric chart, we find that the apparatus dew point is

ADP = 10 °C **... Ans.**

ii) Contact factor : We know that, By-pass factor is given by,

$$BPF = \frac{t_{d2} - ADP}{t_{d1} - ADP}$$

$$\therefore \quad BPF = 0.1667$$

Thus, contact factor is given by,

Contact factor = 1 – BPF = 1 – 0.1667

= **0.833** **... Ans.**

iii) Cooling load

We know that, cooling capacity of coil is given by,

$$Q = \dot{m}_a (h_1 - h_2)$$

where, $m_a = \frac{V_a}{v_a} = \frac{1.5}{0.855} = 1.754$ kg/sec

$$Q = 1.754 (61 - 34)$$

$$\mathbf{Q = 47.358\ kW}$$

Ex. 2.26 : Moist air at a state of 21 °C dry-bulb, 15 °C wet-bulb and 101.325 kPa barometric pressure enters a spray chamber. If, for each kilogram of dry air passing through the chamber, 0.002 kg of water at 100 °C is injected and totally evaporated, calculate the moisture content, enthalpy and dry-bulb temperature of the moist air leaving the chamber.

Sol. : Given data,

Entering air condition = 21 °C DBT and 15 °C WBT

Barometric pressure = 101.325 kPa

Mass of water at 100 °C = 0.002 kg

Refer Fig. 2.38.

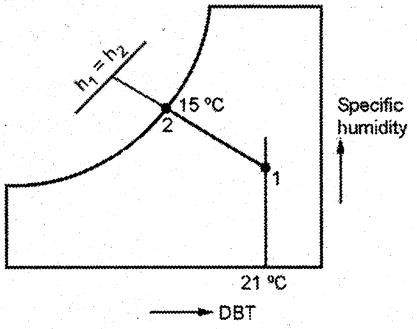


Fig. 2.38

i) Moisture content of leaving air

Now for the mass balance, we have

$$w_2 = w_1 + \frac{m_w}{m_a}$$

$$\therefore \quad w_2 = 0.0083 + \frac{0.002}{1}$$

$$\therefore \quad \mathbf{w_2 = 0.0103 \text{ kg/kg of dry air}} \quad \textbf{... Ans.}$$

ii) Enthalpy of leaving air,

From psychometric chart, we get

$$\mathbf{h_2 = 42.1 \text{ kJ/kg}} \quad \textbf{... Ans.}$$

iii) Dry bulb temperature of leaving air, from psychometric chart, we get

$$\mathbf{t_{d2} = 15 \text{ °C}} \quad \textbf{... Ans.}$$

Ex. 2.27 : Moist air at standard atmospheric pressure is passed over a cooling coil. The inlet state - DBT 30 °C, RH 50 % and exit state - DBT 15 °C, RH 90 %. Show the process on psychrometric chart. Determine the amount of heat and moisture removed per kg of dry air.

Sol. : Given data :

Inlet air condition = 30 °C DBT and 50 % RH

Exit air condition = 15 °C DBT and 90 % RH

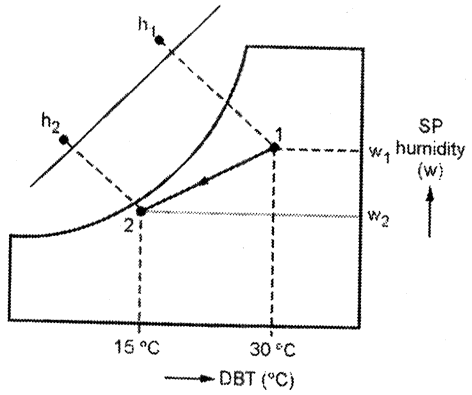


Fig. 2.39

- From psychrometric chart, we get

$$h_1 = 64.5 \text{ kJ/kg} ; h_2 = 41 \text{ kJ/kg}$$

$$w_1 = 0.0128 \text{ kg/kg d.a.} ;$$

$$w_2 = 0.0098 \text{ kg/kg d.a.}$$

i) Amount of heat removed per kg of d.a.

$$Q = h_1 - h_2$$

$$Q = 64.5 - 41 = \mathbf{23.5 \text{ kJ/kg d.a.}} \quad \textbf{... Ans.}$$

ii) Moisture removed per kg of d.a.

$$\Delta w = w_1 - w_2 = \mathbf{0.0032 \text{ kg/kg d.a.}} \quad \textbf{... Ans.}$$

Ex. 2.28 : Calculate : i) Relative humidity ii) Humidity ratio iii) Dew point temperature iv) Density v) Enthalpy of atmospheric air when the DBT is 35 °C, WBT is 23 °C and barometer reads 750 mm of Hg.

Sol. :

Given data, $t_{db} = 35$ °C, $t_{wb} = 23$ °C

barometric pressure = 750 mm of Hg

i) Relative humidity (ϕ)

$$\phi = \frac{p_v}{p_a}$$

From saturation properties of water, at a DBT of 35 °C

$$p_s = 42.5 \text{ mm Hg}$$

Also, at a WBT of 23 °C $p_{v'} = 21.05$ mm Hg

We know that,

$$p_v = p_{v'} - \frac{(p - p_{v'})(t_{db} - t_{wb})\,1.8}{2800 - 1.3\,(1.8\,t_{db} + 32)}$$

$$= 21.05 - \frac{(750 - 21.05)(35 - 23)\,1.8}{2800 - 1.3\,(3.5 \times 1.8 + 32)}$$

$$= 21.05 - 5.88$$

$$p_v = 15.189 \text{ mm Hg}$$

Thus, $$\phi = \frac{p_v}{p_s} \times 100$$

$$= \frac{15.189}{42.5} \times 100 = \mathbf{35.74\ \%} \quad \textbf{... Ans.}$$

ii) Humidity ratio (w)

$$w = 0.622 \frac{p_v}{p - p_v}$$

$$= 0.622 \frac{15.189}{750 - 15.189}$$

$$\mathbf{w = 0.01285 \text{ kg/kg d.a.}} \quad \text{... Ans.}$$

iii) Dew point temperature (t_d)

From steam table, at saturation pressure of 15.18 mm Hg

$$\mathbf{t_d = 17.7\ ^\circ C} \quad \text{... Ans.}$$

iv) Partial densities

$$\rho_a = \frac{p_a}{R_a T} = \frac{(750 - 15.189)\ 133.5}{287\ (273 + 3.5)}$$

$$\rho_a = 1.1.086 \text{ kg/m}^3$$

and

$$\rho_v = \frac{p_v}{R_v T} = \frac{15.189 \times 133.5}{287\ (273 + 35)}$$

$$\rho_v = 0.0143 \text{ kg/m}^3$$

Density of moist air is,

$$\rho = \rho_a + \rho_v = 1.1086 + 0.0143$$

$$= \mathbf{1.1229 \text{ kg/m}^3} \quad \text{... Ans.}$$

v) Enthalpy of air (h)

We know that,

$$h = 1.005\ t_{db} + w\ (2500 + 188\ t_{db})$$

$$= 1.005\ (35) + 1.0128\ (2500 + 1.88 \times 35)$$

$$= \mathbf{2633.81 \text{ kJ/kg d.a.}} \quad \text{... Ans.}$$

Ex. 2.29 : Without using psychrometric chart, calculate humidity ratio, relative humidity and enthalpy of moist air sample having DBT = 38 °C and DPT = 15 °C. Take total pressure of the sample = 1 bar.

Sol. :

Given data : DBT = 38 °C, DPT (t_{dp}) = 15 °C,

Total pressure p_t = 1 bar

(i) Humidity ratio (w) : From steam table, the saturation pressure corresponding to 15 °C temperature is,

$$p_v = 0.01704 \text{ bar}$$

$$w = 0.622 \frac{p_v}{p_t - p_v}$$

$$= 0.622 \frac{0.01704}{1 - 0.01704}$$

$$= \mathbf{0.0108 \text{ kg / kg of dry air}}$$

(ii) Relative humidity (ϕ) :

From steam table, the saturated pressure at DBT of 38 °C is,

$$p_{vs} = 0.06674 \text{ bar}$$

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.01704}{0.06674}$$

$$= \mathbf{0.2553 \text{ or } 25.53\ \%}$$

(iii) Enthalpy of air (h) :

$$h = 1.005\ t_{db} + w\ [2500 + 1.88\ (t_{dp})]$$

$$= 1.005 \times 38 + 0.0108\ [2500 + 1.88 \times 15]$$

$$= 38.19 + 27.305$$

$$= \mathbf{65.49 \text{ kJ/kg of dry air}}$$

Ex. 2.30 : 5 gms of water vapour per kg of atmospheric air is removed and the temperature of air becomes 22 °C DBT. Find the RH and Dew point temperature. Assume initial condition of air as 33 °C DBT, 65 % RH and 1.013 bar pressure. Don't use psychrometric chart.

Sol. : Given data :

Mass of water vapour removed ($w_1 - w_2$) = 5 gm/kg of air = 0.005 kg/kg of air

Initial condition :

$$t_{db2} = 33\ ^\circ C, \qquad \phi_1 = 65\ \% = 0.65$$

$$p_t = 1.013 \text{ bar}$$

Final condition :

$$t_{db2} = 22\ ^\circ C$$

(i) Relative humidity (ϕ_2) :

From steam table, the saturated vapour pressure corresponding to t_{db} = 33 °C is,

$$p_{val} = 0.0507 \text{ bar}$$

$$\therefore \quad \phi_1 = \frac{p_{val}}{p_{vsl}}$$

$$\therefore \quad p_{v1} = 0.65 \times 0.0507 = 0.03296 \text{ bar}$$

We know, $$w_1 = 0.622 \frac{p_{v1}}{p_t - p_{v1}} = 0.622 \frac{0.3296}{1.013 - 0.3296}$$

$$w_1 = 0.0209 \text{ kg/kg of dry air}$$

Given, $$w_1 - w_2 = 0.005$$

$$w_2 = 0.0209 - 0.005$$

$$w_2 = 0.0159 \text{ kJ/kg of dry air}$$

Also, $$w_2 = 0.622 \frac{p_{v2}}{p_t - p_{v2}}$$

$$0.0159 = 0.622 \frac{p_{v2}}{1.013 - p_{v2}} = 0.02549 \text{ bar}$$

From steam table, the rated vapour pressure corresponding to temperature of 22 °C is,

$$p_{vs2} = 0.02\,669 \text{ bar}$$

Now, $$\phi_2 = \frac{p_{v2}}{p_{vs2}} = \frac{0.2549}{0.2669} = \mathbf{0.955 \text{ or } 95.5\ \%}$$

(ii) Dew point temperature (t_{dp}) :

Dew point temperature is saturation temperature corresponding to partial pressure of water vapour at p_{v2} = 0.02549 bar. From steam table, temperature at p_{v2} = 0.02549 bar is, **t_{dp2} = 21.28 °C**

Ex. 2.31 : For a hall to be air conditioned, Outdoor conditions : DBT = 40 °C, WBT = 20 °C Required conditions : DBT = 20 °C, RH = 60 %. Seating capacity of hall = 1500, Amount of outdoor air supplied = 0.3 m^3/min person. If required conditions are achieved first by adiabatic humidification and then by cooling, estimate : (i) Capacity of cooling coil in TR (ii) Capacity of humidifier in kg/hr.

Sol. :

Refer Fig. 2.40

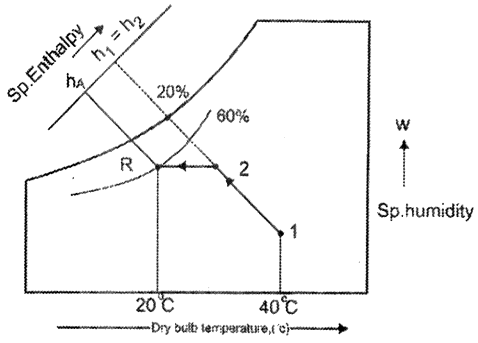


Fig. 2.40

Given data :

Outside conditions (1) = 40 °C DBT and 20 °C WBT

Required conditions (R) = 20 °C DBT and 60 % RH

No. of persons = 1500

V_a = 0.3 m^3/min. person.

At point '1' :

Enthalpy h_1 = 57.4 kJ/kg of dry air

Specific humidity w_1 = 0.0066 kg/kg of dry air

Specific volume v_1 = 0.896 m^3/kg of dry air

At point '2' :

Enthalpy h_a = h_1 = 57.4 kJ/kg of dry air

Specific humidity w_2 = 0.0088 kg/kg of dry air

At point 'R' :

Enthalpy h_f = 42.5 kJ/kg of dry air

Total air supplied = Seating capacity × Air flow/min/ person

1500×0.3 = 450 m^3/min.

i) Cooling capacity of coil

$$= \frac{\text{Air supplied}}{\text{Specific volume} \times 60} \times (h_2 \times h_R) = \frac{450}{0.896 \times 60} (57.4 - 42.6)$$

$$= 123.88 \text{ kW} = \frac{123.88}{3.516} = \mathbf{35.22 \text{ TR}}$$

ii) Capacity of humidifier in kg/hr = $\dot{m}_a$ $(w_2 - w_1)$

$$= \frac{540}{0.896} \times (0.0088 - 0.0066) = \mathbf{66.29\ kg/hr}$$

Ex. 2.32 : Air at 25 °C and 60 % RH is supplied to the drug formulation unit. It is conditioned to this state, first by cooling and dehumidifying and then by reheating it. Cooling coil surface temperature is 13 °C and ambient conditions are 32 °C DBT and 65 % RH. If the air supply rate is 15,000 m^3/hr determine : (i) Cooling coil capacity in TR (ii) Bypass factor of cooling coil (iii) Heating capacity in kW (iv) Heating coil surface temperature if bypass factor is 0.3. (v) Mass of water vapour removed per hour.

Sol. :

Refer Fig. 2.41

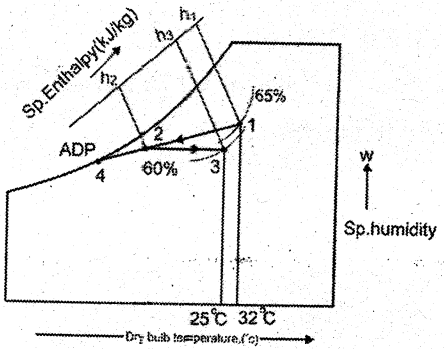


Fig. 2.41

Given data : $t_{d1} = 32$ °C, $\phi_1 = 65$ % RH, $t_{d3} = 25$ °C, $\phi_3 = 60$ % RH, ADP = 13 °C, $V_a = 15{,}000\ m^3$/hr.

$BPF_{HC} = 0.3$

- From psychrometric chart,

v_3 = 0.86 m^3 of dry air

Enthalpy at point '1' is = 82.5 kJ/kg

Enthalpy at point '2' is = 47.5 kJ/kg

(i) Cooling coil capacity in TR :

Mass flow rate of supply air is

$$m_a = \frac{V_a}{v_3} = \frac{15000}{0.86 \times 3600} = 4.845\ kg/s$$

Cooling capacity = $\dot{m}_a\ (h_1 - h_2)$

$$= 4.845\ (82.5 - 47.5) = 169.51\ kW = \frac{169.51}{3.516} = \mathbf{48.08\ TR}$$

- From psychrometric chart, $t_{d2} = 17.7$ °C.

(ii) Bypass factor of cooling coil $BPF_{c.c}$:

We know that,

$$BPF = \frac{t_{d2} - t_4}{t_{d1} - t_4} = \frac{t_{d2} - ADP}{t_{d1} - ADP} = \frac{17.7 - 13}{32 - 13} = 0.247$$

(iii) Heating capacity in kW :

From psychrometric chart, Enthalpy at '3' h_3 = 55.2 kJ/kg

$$\therefore \quad \text{Heating capacity} = \dot{m}_a\ (h_3 - h_2)$$

$$= 4.845\ (55.2 - 47.5) = \mathbf{37.3\ kW}$$

(iv) Heating coil surface temperature if bypass factor is 0.3 :

Let t_{d5} = Heating coil surface temperature

We know that,

$$BPF = \frac{t_{d5} - t_{d3}}{t_{d5} - t_{d2}}$$

$$0.3 = \frac{t_{d5} - 25}{t_{d5} - 17.7}$$

$$t_{d5} = 28.1\ °C$$

(v) Mass of water vapour removed per hour :

water vapour is removed during 1-2 process.

Mass of water vapour removed = $m_a\ (w_1 - w_2)$

From psychrometric chart,

w_1 = 0.0196 kg/kg of dry air

w_2 = 0.0112 kg/kg of dry air

Mass of water vapour removed per hour

$$= \dot{m}_a\ (w_1 - w_2) \times 3600$$

$$= 4.845\ (0.0196 - 0.0112) \times 3600 = \mathbf{146.5\ kg/hr}$$

Ex. 2.33 : Two air streams are mixed steadily and adiabatically. The first air stream enters at 32 °C DBT and 40 % RH while second enters at 12 °C and 90 % RH. The flow rates of the two streams are 20 m^3/min. and 25 /min. respectively. Determine the specific humidity, R/t and mass flow rates after mixing.

Sol : Refer Fig. 2.42.

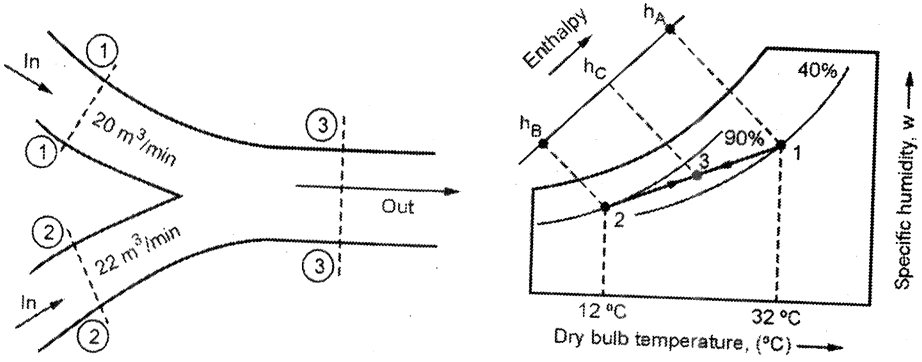


Fig. 2.42

Given data : First air stream condition '1' = 32 °C DBT and 40 % RH

Second air stream condition '2' = 12 °C DBT and 90 % RH

Flow rate of first air stream = $v_1 = 20\ m^3$ / min

Flow rate of second air stream = $v_2 = 25\ m^3$/min

- From psychrometric chart, we get,

$$v_1 = 0.881\ m^3/kg \text{ of dry air.}$$

$$v_2 = 0.818\ m^3/kg \text{ of dry air.}$$

Mass flow rate of 1st stream is given by,

$$\dot{m}_1 = \frac{v_1}{v_1} = \frac{20}{0.881} = 22.7 \text{ kg/min.}$$

Similarly, mass flow rate of 2nd stream is,

$$\dot{m}_2 = \frac{v_2}{v_2} = \frac{25}{0.818} = 30.56 \text{ kg/min.}$$

According to mass and energy balance, we can write,

$$(\dot{m}_1 + \dot{m}_2)\, t_{d3} = m_1 \cdot t_{d1} + m_2 \cdot t_{d2}$$

$$T_{d3} = \frac{22.7 + 32 + 30.56 \times 12}{22.7 + 30.56} = 20.5\ °C$$

- Now, plot point '3' (i.e. mixing state) on line 1-2 such that $t_{d3} = 20.5$ °C.
- From psychromatric chart, $v_3 = 0.844\ m^3/kg$

(i) Specific humidity of mixture at point '3' is $w_3 = 0.0096$ kg/kg of dry air

(ii) RH of mixture is,

$$\phi_3 = 64\ \%$$

(iii) Flow rate of mixture is ,

$$v_3 = m_3 \times v_3$$

$$= (22.7 + 30.56) \times 0.844 = 44.95\ m^3/min$$

2.8 Room Sensible Heat Factor (RSHF)

[GTU : Summer-18]

- Room sensible heat factor is the ratio of room sensible heat to room total heat.

$$RSHF = \frac{RSH}{RTH} = \frac{RSH}{RSH + RLH}$$

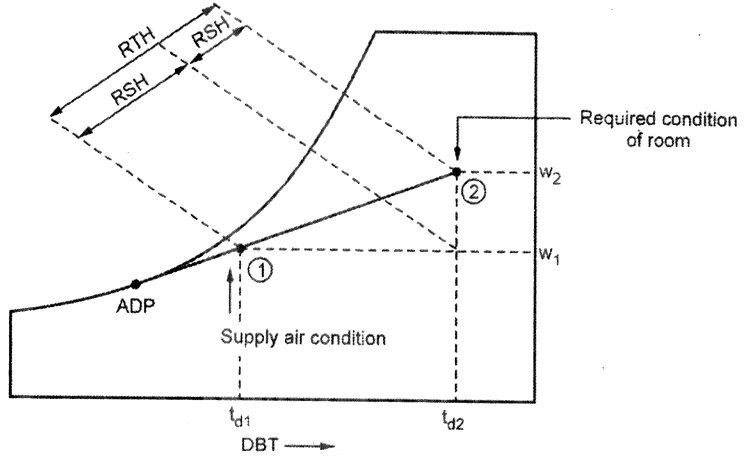


Fig. 2.43

- From the above figure,

  Point 1 - represents supply air condition

  Point 2 - represents required room design condition

- The line (1-2) is called as room sensible heat factor line (RSHF-line).

- The slope on this line gives the ratio of room sensible heat (RSH) AND Room Latent Heat (RLH).
- The supply air in to the room must have the capacity to absorb room sensible heat load and room latent heat load.
- RSHF line generally used to find out supply air condition.

2.8.1 Steps to Find Out Supply Air Condition and to Draw RSHF Line

- Calculate the RSHF

$$RSHF = \frac{RSH}{RSH + RLH}$$

- Mark the point 'X' on Sensible Heat Factor (SHF) scale which is at right side of psychrometric chart.
- Locate the guide point 'Y' (Considering 26 °C DBT and 50 % RH)
- Join the line XY. This XY line is known as guide line.
- Mark the point 'Z' (Which represents the design condition of room).
- From the point Z draw the line which is parallel to guide line (XY).
- This line represents the RSHF line.

2.9 Grand Sensible Heat Factor (GSHF)

GSHF is defined as the ratio of the total sensible heat to grand total heat which the cooling coil handle.

$$GSHF = \frac{TSH}{GTH}$$

$$GSHF = \frac{TSH}{TSH + TLH}$$

But,

TSH - Total sensible heat = RSH + Outside air sensible heat
= RSH + OASH

TLH - Total latent heat = RLH + Outside air latent heat
= RLH + OALH

$$GSHF = \frac{RSH + OASH}{(RSH + OASH) + (RLH + OALH)}$$

GTH - Grand total heat

$$GTH = TSH + TLH = RSH + RLH + OATH$$
$$= RSH + RLH + (OASH + OALH)$$

To find	Formula to be used
OASH	$0.02044\ v_1(t_{d1} - t_{d2})$
OALH	$50\ v_1\ (w_1 - w_2)$
OATH	$0.02\ v_1\ (h_1 - h_2)$

- Point 1 represents outside condition of air.
- Point 2 represent room air condition.
- Point 3 represent mixture condition of air which is entering the cooling coil.
- Point 4 represents leaving condition of air from cooling coil.
- To have economic operation of air conditioning plant the outside fresh air is mixed with recirculated air.

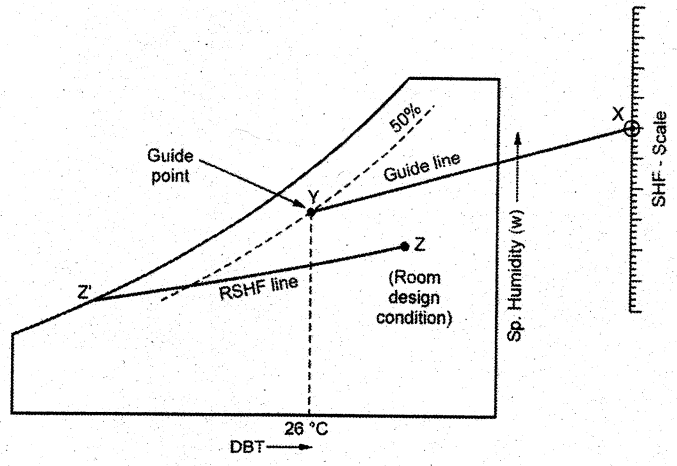


Fig. 2.44

- This air mixture is passed through cooling coil where it is cooled and dehumidified.
- In above figure, this is represented by point 4.
- When point 3 is joined with point 4 it gives one line which is Grand Sensible Heat Factor (GSHF line).
- If this line extended upto the saturation curve it gives Apparatus Dew Point (ADP).
- If we know mixture condition which is entering in cooling coil and Grand Sensible Heat Factor (GSHF) then GSHF line can be drawn in similar way as per the discussion for RSHF line.

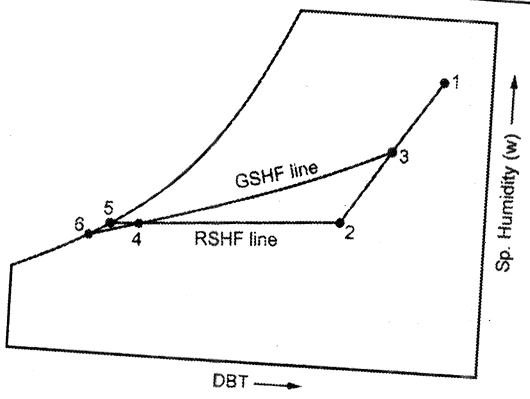


Fig. 2.45

2.10 Effective Room Sensible Heat Factor (ERSHF)

- ERSHF is the ratio of effective room sensible heat to effective room total heat.

$$\text{ERSHF} = \frac{\text{ERSH}}{\text{ERTH}} = \frac{\text{ERSH}}{\text{ERSH} + \text{ERLH}}$$

Where,

ERSH - Effective Room Sensible Heat = RSH + OASH × BPF

ERLH - Effective Room Latent Heat = RLH + OALH × BPF

ERTH - Effective Room Total Heat = ERSH + ERLH

To find	Formula to be used
ERSH	$RSH + 0.02044\ (t_{d1} - t_{d2}) \times BPF$
ERLH	$RLH + 50\ v_1\ (w_1 - w_2) \times BPF$

- The Effective Room Sensible Heat Factor line (ERSH line) is obtained by joining the point 2 and point 6 (which is on saturation curve i.e. ADP).
- Point 1 represents outdoor condition.
- Point 2 represents room design condition.
- Point 3 represents air mixture condition entering the coil.
- Point 4 represent room supply air condition.
- Point 6 represents ADP.
- From the point 4 draw a line 4 - 4′ which is parallel to line 3 - 2.
- Therefore from triangle 6 - 4 - 4′ and 6 - 3 - 2

We can write,

$$\text{BPF} = \frac{\text{Length 4 - 6}}{\text{Length 3 - 6}} = \frac{\text{Length 4′ - 6}}{\text{Length 2 - 6}}$$

OR,
$$\text{BPF} = \frac{t_{d4} - \text{ADP}}{t_{d3} - \text{ADP}} = \frac{t_{d4} - \text{ADP}}{t_{d2} - \text{ADP}}$$

- The mass of dehumidified air can be obtained by

$$m_d = \frac{\text{Room total heat}}{h_2 - h_4}$$

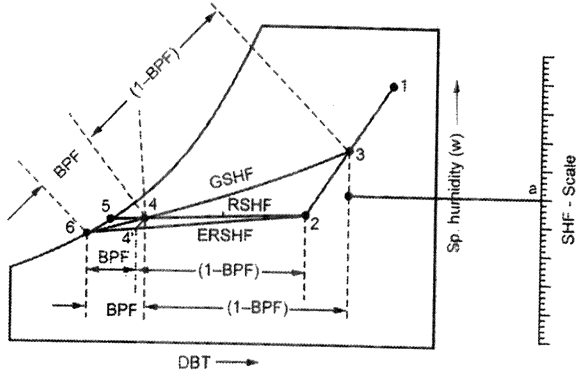


Fig. 2.46

2.11 Solved Examples

Ex. 2.34 : A room has sensible heat gain 15 kW and latent heat gain of 10 kW. The room is to be maintained at 20 °C and 40 % RH. Draw RSHF line.
Given data : RSH = 15 kW, RLH = 15 kW, t_{d1} = 20 C and 40 % RH

Sol. :

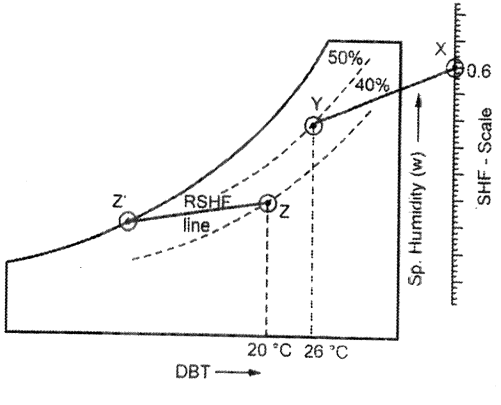


Fig. 2.47

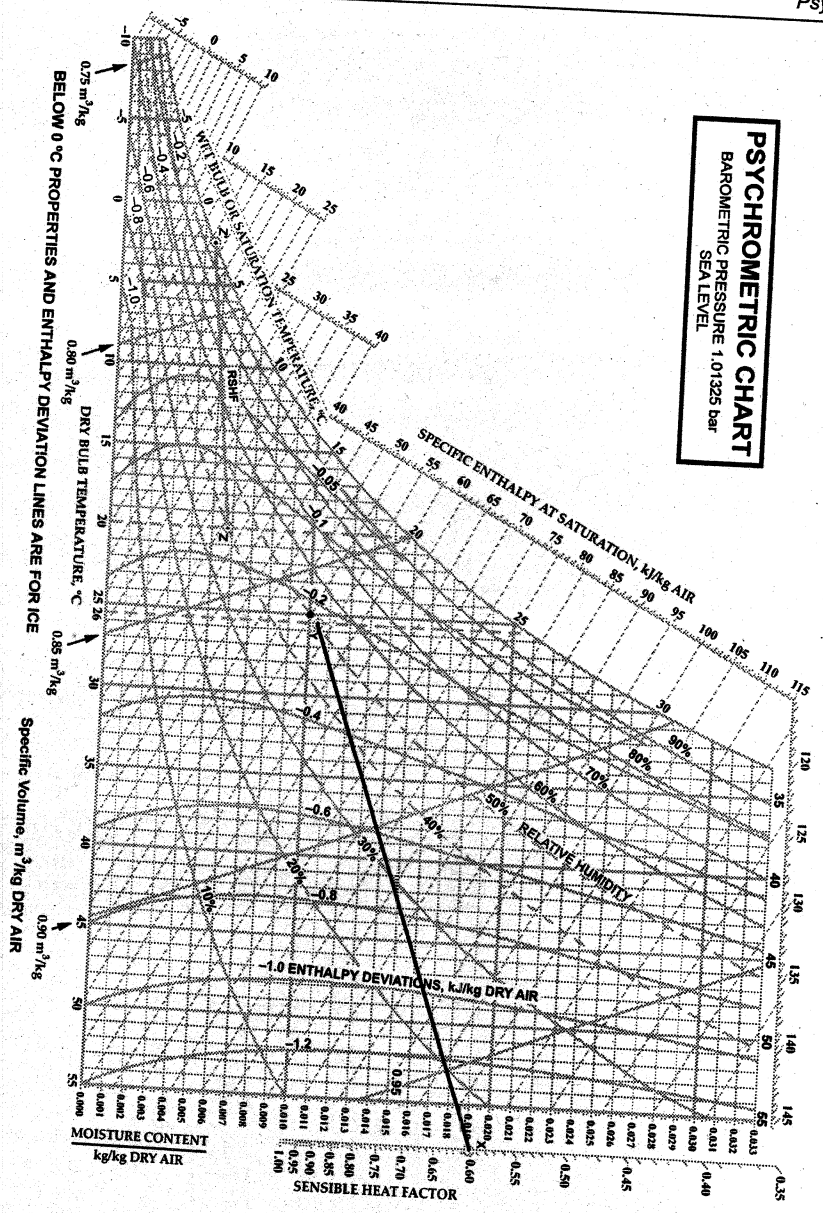


Fig. 2.47(a)

- Find out the RSHF.

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH} + \text{RLH}} = \frac{15}{15 + 10}$$

RSHF = 0.6

- Mark the point X on SHF scale (at 0.6)
- Locate point Y(Considering 26 °C DBT and 50 % RH) i.e. guide point.
- Draw XY line (which is guide line).
- Mark point Z (at 20 °C DBT and 40 % RH) ... given data
- From point Z draw a line which is parallel to guide line (XY).
- This line represents RSHF line.

Ex. 2.35 : A shop has sensible heat gain 24 kW and latent heat gain 3.5 kW and it has to be maintained at . DBT and 50 % RH. 190 m^3/min of air is delivered to room. Determine state of supply air.

Given data : RSH = 24 kW, RLH = 3.5 kW, t_{d1} = 26 C and 50 % RH, v = 190 m^3/min.

Sol. :

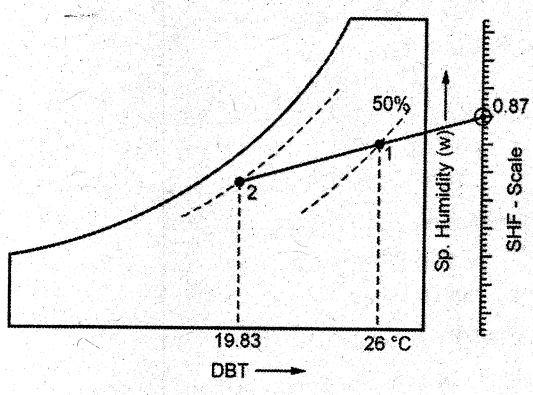


Fig. 2.48

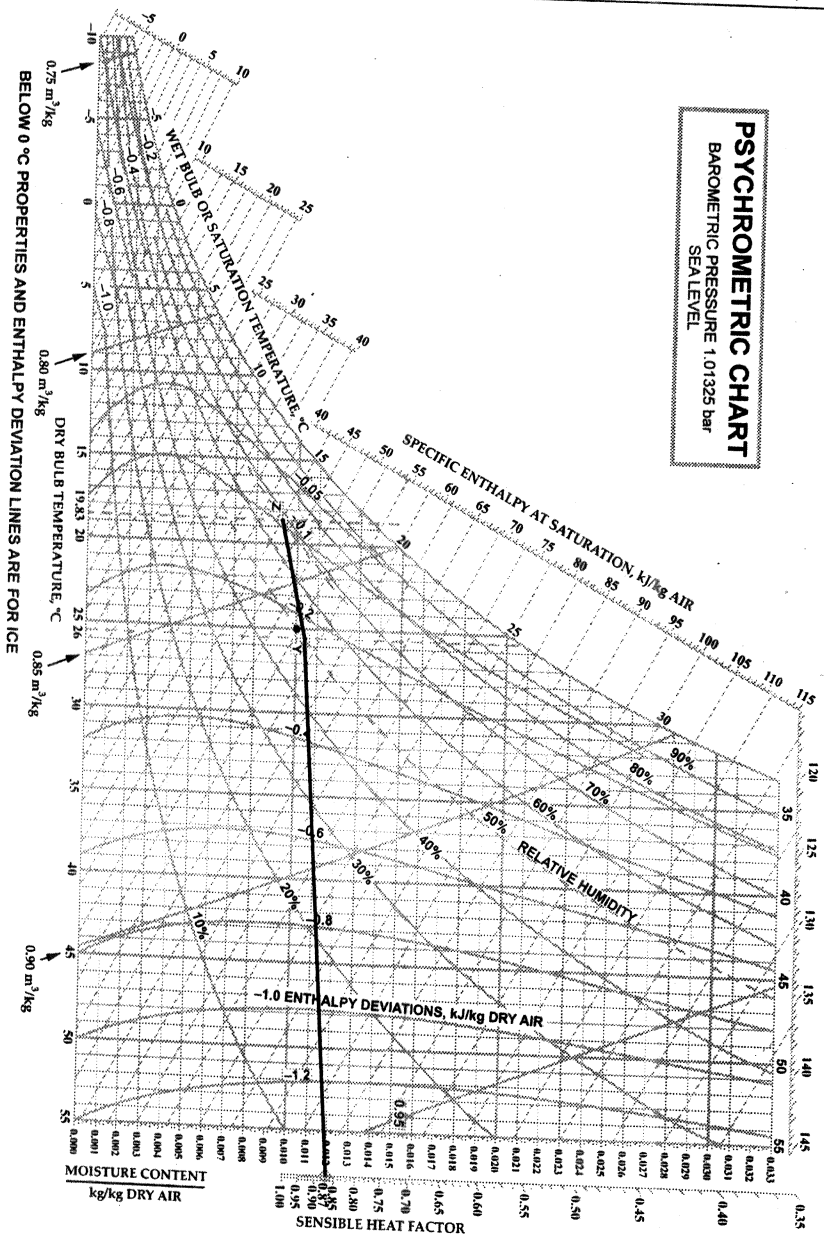


Fig. 2.48(a)

Refer Fig. 2.48 (a) on page 6-49.

$$RSH = 0.02044\ v\ (t_{d1} - t_{d2})$$

$$24 = 0.02044 \times 190\ (26 - t_{d2})$$

$$24 = 100.97 - 3.88\ t_{d2}$$

$$T_{d2} = 19.83\ °C$$

Now, $$RSHF = \frac{RSH}{RSH + RLH} = \frac{24}{24 + 3.5} = 0.87$$

State of supply air

$t_{d1} = 19.83°C$
$RH = 68\%$
$T_{d1} = 15.2\ °C$

} From psychrometry chart

Ex. 2.36 : Outside design conditions are 40 °C DBT and 30 % RH. Room design conditions are 25 °C DBT and 50 % RH Room sensible heat 50 kW and room latent heat is 10 kW. If outside air quantity is 50 m^3/mm and assuming BPF of cooling coil 0.1. Find GSHF and ESHF.

Given data

Outise condition	Design condition
40 °C DBT	25 °C DBT
30 °C RH	50 % RH

Room Sensible Heat (RSH) = 50 kW

Room Latent Heat (RLH) = 10 kW

Outside air quantity = 50 m^3/min.

BPF = 0.1

Sol. :

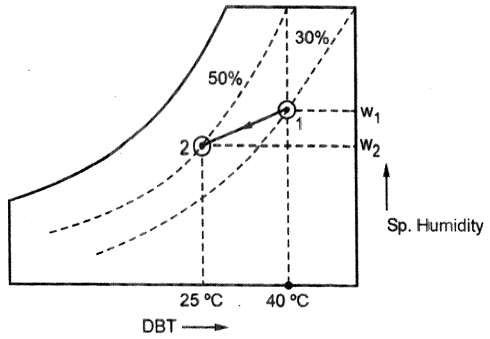


Fig. 2.49

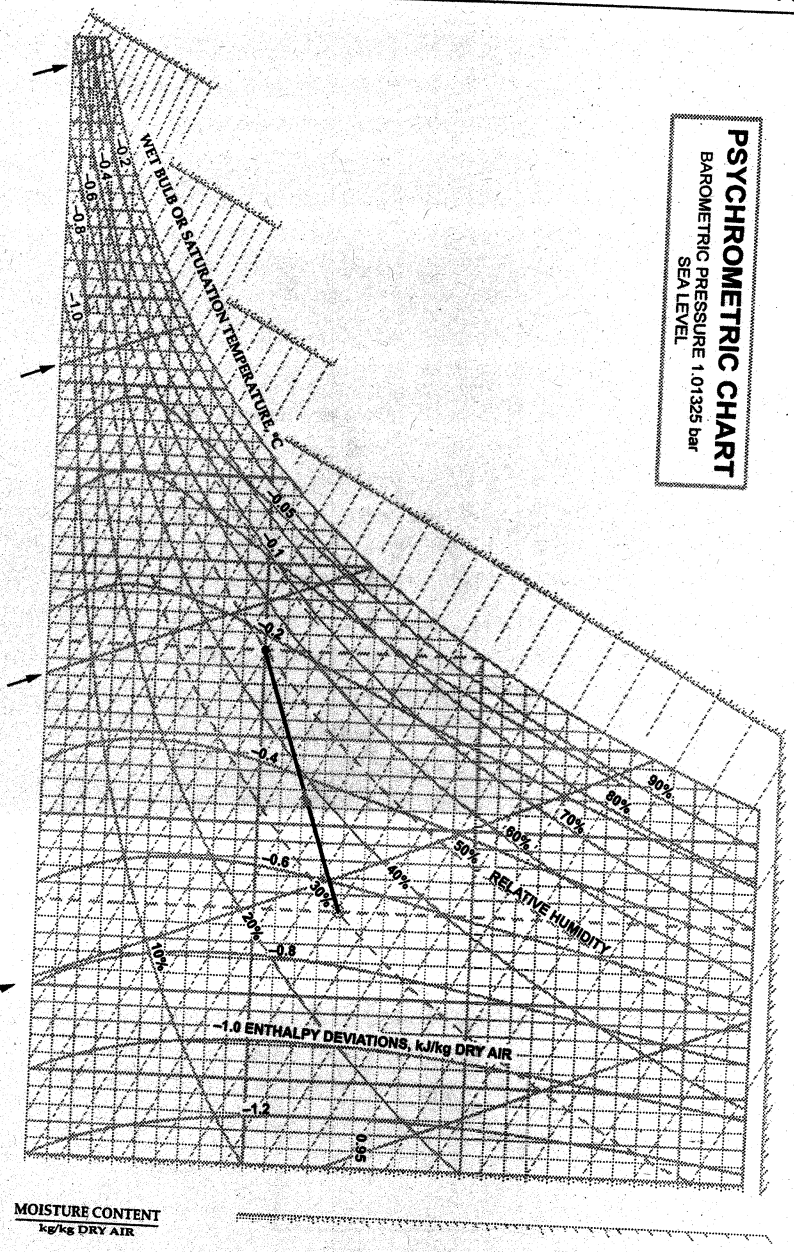


Fig. 2.49(a)

Locate the point 1 as 40 °C DBT and 30 % RH

Similarly locate point 2 a 30 °C DBT and 50 % RH.

- To find outside sensible heat load,

$$OASH = 0.02044\, v_1 (t_{d1} - t_{d2}) = 0.02044 \times 50 \times (40 - 25) = 15.33 \text{ kW}$$

- To find latent heat load,

$$OALH = 50 \times v_1 (w_1 - w_2) = 50 \times 50\,(0.014 - 0.010)$$

...[Values of w_1 and w_2 are from psychrometric chart]

$$= 10 \text{ kW}$$

- To find Grand Sensible Heat (GSH)

$$GSH = OASH + RSH = 15.33 + 50 = 65.3 \text{ kW}$$

- To find Grand Latent Heat (GLH)

$$GLH = OALH + RLH = 10 + 10 = 20 \text{ kW}$$

- To find GSHF

$$GSHF = \frac{GSH}{GSH + GLH} = \frac{65.3}{65.3 + 20} = 0.7655$$

- To find effective room heat,

$$ERH = ERSH = RSH + BPF \times OASH = 50 + 0.1 \times 15.3 = 51.53 \text{ kW}$$

- To find effective latent heat,

$$ERLH = RLH + BPF \times OALH = 10 + 0.1 \times 10 = 11 \text{ kW}$$

- To find ESHF,

$$ESHF = \frac{ERSH}{ERSH + ERLH} = \frac{51.53}{51.53 + 11} = 0.824$$

Ex. 2.37 : A room has to be maintained 25 °C DBT and 50 % RH. It has sensible heat gain 20 kW and latent heat gain of 4.8 kW. 190 m^3/min of air is supplied to the room. Estimate state of supply air.

Given data : T_{d1} = 25 °C RH = 50 %

RSH = 20 kW RLH = 4.8 kW v = 190 m^3/mm

Sol. : Refer Fig. 2.50 (a) on next page.

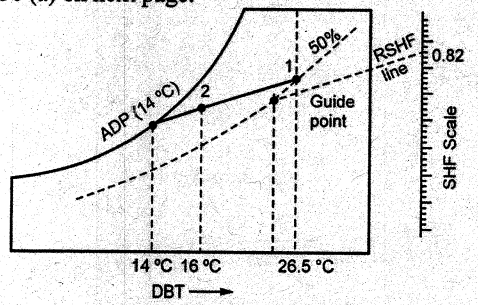


Fig. 2.50

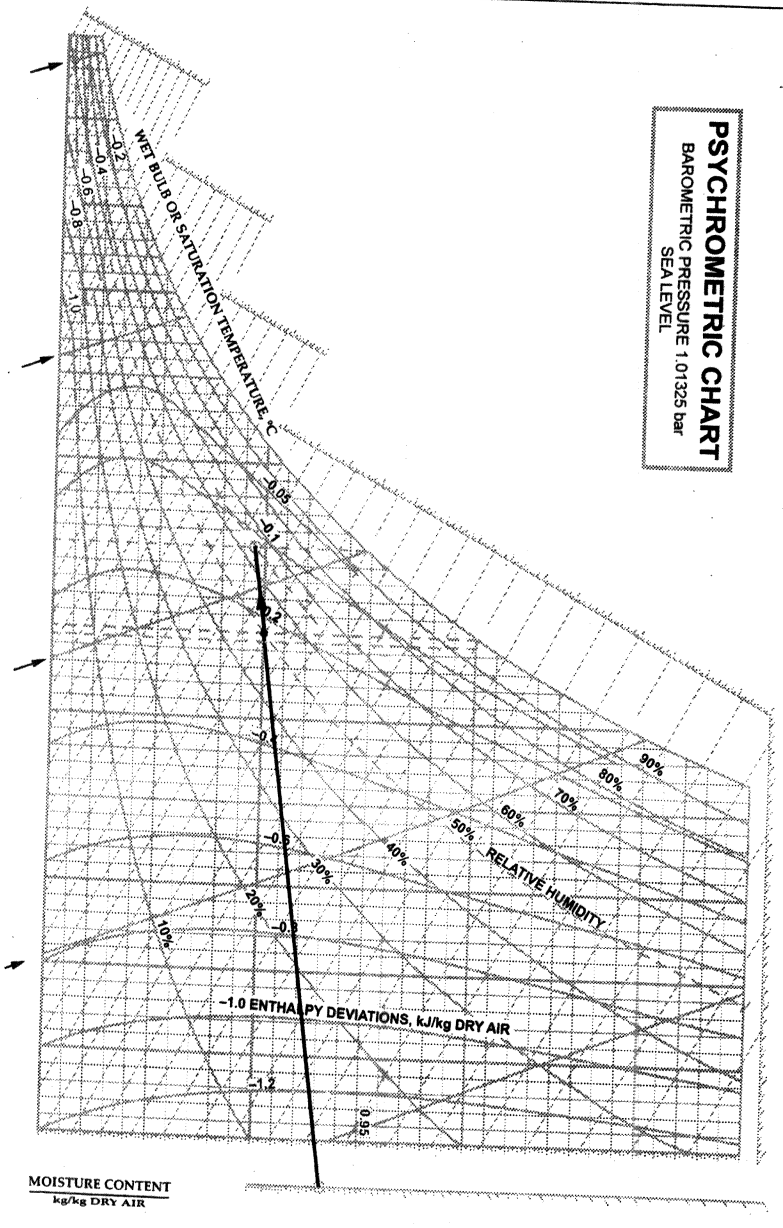


Fig. 2.50(a)

Calculate t_{d2}

$$RSH = 0.02044\ v_1\ (t_{d1} - t_{d2})\ ; \qquad 20 = 0.02044 \times 190\ (25 - t_{d2})$$

$$20 = 97.09 - 3.883\ t_{d2}\ ; \qquad T_{d2} = 19.85 \approx 25\ °C$$

- RSHF

$$RSHF = \frac{RSH}{RSH + RLH} = \frac{20}{20 + 4.8} = 0.80$$

- Locate the point 1 as 25 °C DBT and 50 % RH.
- Locate the guide point as 26 °C DBT and 50 % RH.
- Mark the value calculated value of RSHF on SHF-scale and join guide point and point '1'.
- To locate the point '2' extend the line upto $t_{d2} = 20°C$

From the chart state of supply air is,

$$t_{d2} = 20\ °C\ ; \qquad t_{w2} = 16.5\ °C$$

$$RH = 68\ \%$$

Ex. 2.38 : Following data refers to air conditioned room,
Room condition - 26.5 DBT and RH - 50 %
Room sensible heat gain = 27 kW
Room sensible heat factor = 0.82
Calculate
i) Room latent heat gain ii) ADP
iii) cmm of air if it is supplied to room at ADP.
iv) cmm of air if it is supplied to room at 16 °C.
v) Specific humidity at 16 °C
Given data
T_{d1} = 26.5 °C RH = 50 %
RSH = 27 kW RSHF = 0.82

Sol. :

i) To find RLH.,

$$RSHF = \frac{RSH}{RSH + RLH}$$

$$0.82 = \frac{27}{27 + RLH}\ ; \qquad RLH = 5.92\ kW$$

- Mark the point 1 as 26.5 DBT and 50 % RH.
- Mark the guide point as °C DBT and 50 % RH.

- Draw RSHF line (which joint guide point and RSHF value i.e. 0.82).
- Draw a line from point 1 which is parallel to RSHF line where it cuts the saturation curve this point indicates ADP (14 °C)
- Mark the point 2 on this line whose DBT is 17 °C

ii) The ADP = 14 °C from psychrometric chart.

iii) To find cmm of air at ADP

$$\text{cmm} = \frac{\text{RSH}}{0.0204\,(t_{d1} - \text{ADP})} = \frac{27}{0.0204\,(26.5 - 14)} = 105.88\ \text{m}^3/\text{min}$$

iv) To find cmm of air at 17 °C

$$\text{cmm} = \frac{\text{rSH}}{0.0204\,(t_{d1} - t_{d2})} = \frac{27}{0.0204\,(26.5 - 16)} = 126.05\ \text{m}^3/\text{min}$$

v) Specific humidity

$$w_2 = 0.011\ \text{kg/kg dry air}$$

Ex. 2.39 : An auditorium is to be maintained at temperature 23 °C and 60 % RH. The sensible heat load 130 kW and 84 kg/hr of moisture has to be removed. Air supplied to auditorium is at 15 °C. Calculate :

i) Mass of air to be supplied in kg/hr.

ii) ADP, DPT and RH.

Given data : T_{d1} = °C, RH = 60 %

RSH = 130 kW t_{d2} = 15 °C

Sol. : Find out RLH.

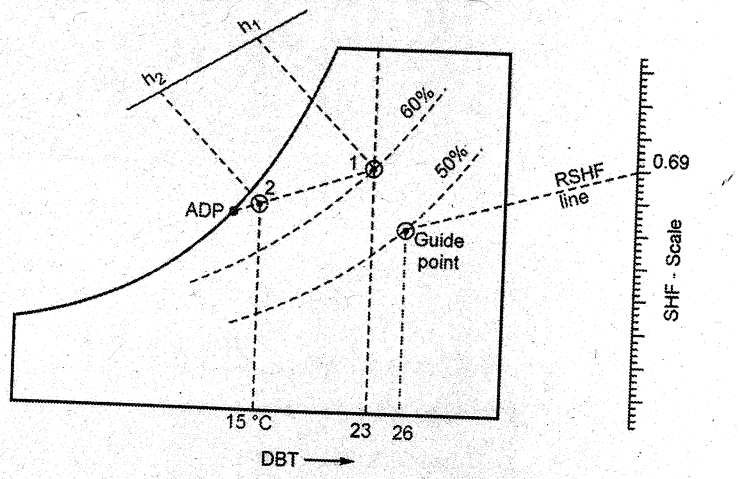


Fig. 2.52

$$\text{RLH} = \frac{84}{3600} \times 2447.2 \qquad (\because h_{fg} \text{ at } 23\ °\text{C } 2447.2)$$

$$= 57.10\ \text{kW}$$

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH} + \text{RLH}} = \frac{130}{130 + 57.10} = 0.69$$

- Room Total Load (RTL) = RSH + RLH

$$= 130 + 57.10 = 187.1\ \text{kW}$$

- Locate the point 1 as 23 °C DBT and 60 % RH.
- Locate the guide point at 26 °C DBT and 50 % RH.
- Draw the line from point 1 which is parallel to RSHF line.
- Extend this line upto saturation curve, it will indicate ADP.
- Locate the point 2 on this line which is at 15 °C DBT.
- Find out enthalpy and

$h_1 = 52$ kJ/kg ; $h_2 = 38$ kJ/kg

i) Mass of air supplied in kg/hr

$$m_a = \frac{\text{RTL}}{h_1 - h_2} = \frac{187.1}{(52 - 38)} = 13.36\ \text{kg/sec} = 48.09 \times 10^3\ \text{kg/hr}$$

ii) ADP = 12 °C

DPT = 14 °C at t_{d2} = 15 °C

RH = 90 % when t_{d2} = 15 °C

Ex. 2.40 : A small office hall of 25 person capacity is provided with summer air conditioning system with following data.

Outside condition = 34 °C DBT and 28 °C WBT

Inside condition = 24 °C DBT and 50 % RH

Volume of air supplied = 0.4 m^3 / min /person

Sensible heat load in room = 125600 kJ/hr

Latent heat load in room = 42000 kJ/hr

Find sensible heat factor of room.

Given data :

Outside condition - 34 °C DBT and 28 °C WBT

Inside condition - 24 °C DBT and 50 % RH

Sol. : Refer Fig. 2.53 (a) on next page.

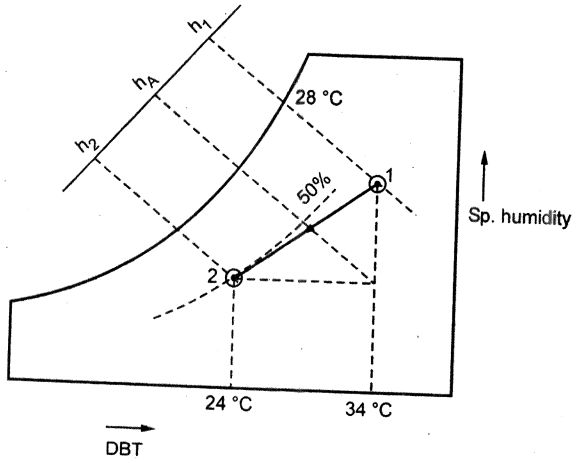


Fig. 2.53

- Locate point 1 (34 °C DBT and 28 °C WBT)

  Find out h_1 = 90 kJ/kg of dry air

- Locate point 2 (24 °C DBT and 50 % RH)

  Find out h_2 = 48 kJ/kg of dry air

- Locate point A by extending lines from point 1 and 2.
- Find out h_A = 58.5 kJ/kg of dry air.
- SH load = 0.4 m^3/min/person

  Specific volume at point 1 v_{s1} = 0.901 m^3 /kg

- SH load of supply air $= \frac{0.4 \times 25 \times 60}{0.901} (h_A - h_2)$

  $= \frac{0.4 \times 25 \times 60}{0.901} (58.5 - 48) = 6992.2$ kJ/hr

- LH load of supply air $= \frac{0.4 \times 25 \times 60}{0.901} (h_1 - h_A)$

  $= \frac{0.4 \times 25 \times 60}{0.901} (90 - 58.5)$

  = 20976 kJ/hr

- Total sensible heat load = 6992.2 + 125600 = 132592.2

  Total latent heat load = 20976 + 42000 = 62976

- $SHF = \frac{RSH}{RSH + TLH} = \frac{132592.2}{132592.2 + 62976} = 0.677$

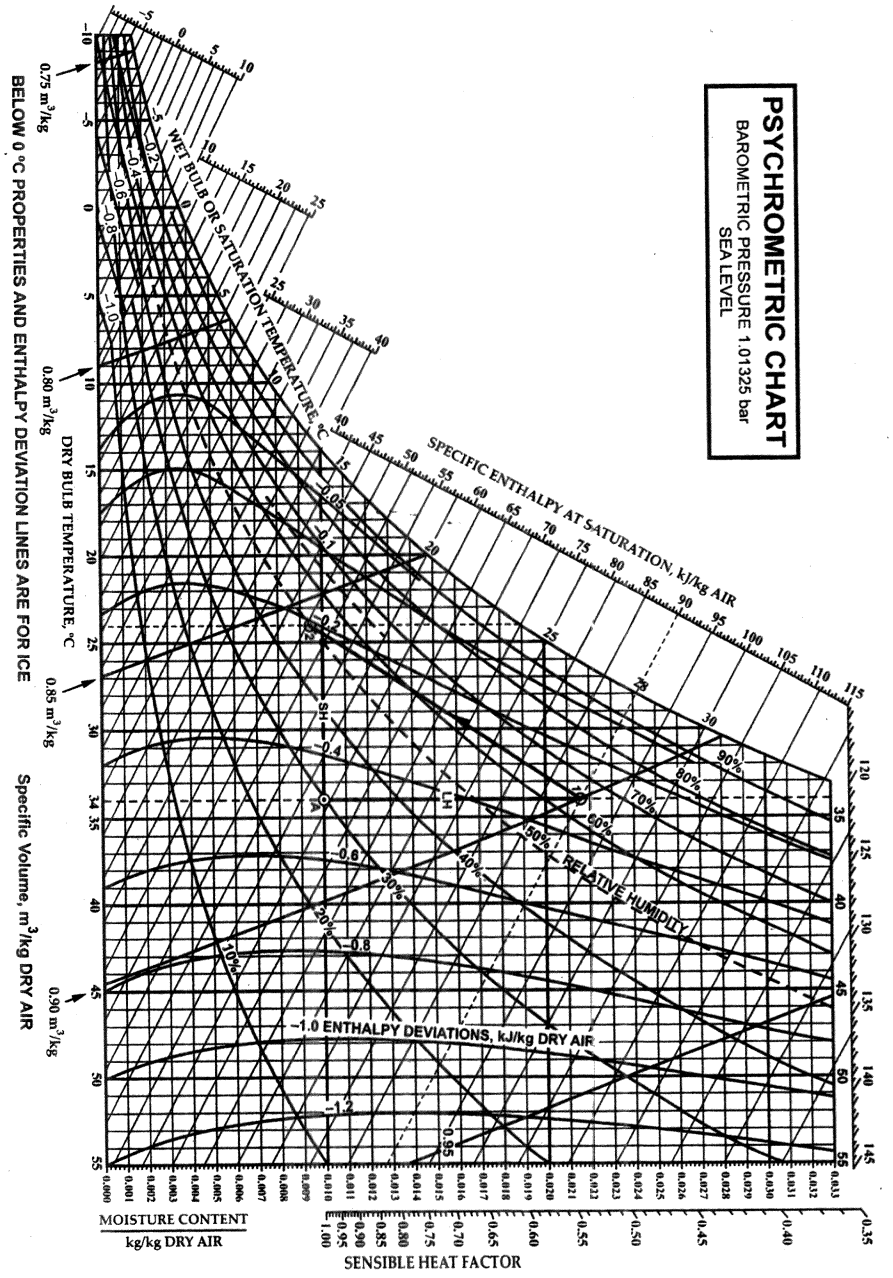


Fig. 2.53(a)

REVIEW QUESTIONS

Q. 1 Define - i) WBT ii) RH iii) DPT.

Q. 2 Explain BPF and ADP.

Q. 3 Explain any two psychrometric processes.

Q. 4 What is degree of saturation.

Q. 5 Explain GSHF and RSHF.

Q. 6 What is cooling load.

Q. 7 Explain factors contributing cooling load.

Q. 8 What is air conditioning ? Give classification of air conditioning systems.

Q. 9 What is psychrometry ? Explain in detail.

Q. 10 Define the following psychrometric terms :

a) DBT b) WBT

c) Humidity d) Degree of saturation

e) DPT f) RH

Q. 11 Explain Dalton's law of partial pressure.

Q. 12 Write short note on sling psychrometer.

Q. 13 Derive an expression,

$$w = 0.622\left(\frac{p_v}{p_b - p_v}\right)$$

Q. 14 What is psychrometric chart ? Explain the measurement of different lines on it.

Q. 15 Explain the following psychrometric process :

a) Sensible heating

b) Sensible cooling

c) Cooling and dehumidification

d) Heating and humidification

e) Adiabatic mixing of two air streams.

❑❑❑

CHAPTER - 3

REFRIGERANT AND REFRIGERATION CYCLES

Syllabus : Classification, nomenclature and desirable properties of refrigerant, secondary refrigerants, ODP and GWP, Compound compression with intercooler, flash gas removal and flash intercooler, Desirable characteristics of refrigerant absorbent pair for vapor absorption cycle, Simple H_2O-NH_3 cycle, $LiBr_2 - H_2O$ cycle and its working

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Refrigerant

3.1 Applications of Refrigeration and Air Conditioning

☞ [GTU : Winter-16]

3.1.1 Domestic Refrigerator

- Refrigerator is a common appliance used in domestic housing, vendor shop, offices, laboratories, chemists etc.
- The capacity of refrigerator is expressed in litres.
- The refrigerators are available in standard capacities of 65, 90, 165, 210, 240, 300, 380, 420, 540 litres etc.
- Refrigerant used in refrigerator is R-12 or R-22.
- Domestic refrigerator works on vapour compression system.
- The refrigerators may be single door or double door.
- Domestic refrigerator consists of following major components.

Compressor :

- In domestic refrigerator hermatically sealed compressor is used.
- It is located at the bottom and behind the compartment of refrigerator.
- Vapour refrigerators are sucked into the compressor for compression purpose.
- The power of the compressor vary according to the size of the compressor (i.e. 75 W, 92 W, 125 W, 180 W).

Evaporator

- Evaporator is mounted on top protion of the refrigerator.
- Refrigerating effect can be achieved in evaporator.
- Evaporator generally made in box type construction.
- In evaporator, evaporation will takes place i.e. liquid refrigerants can be convered into vapour refrigerants.
- In evaporator negative temperature is maintained to produce the ice.

Condenser

- Condenser which is used in domestic refrigerator is air cooled condenser.
- It is black coloured grills provided on the backside of the refrigerator.
- In order to increase heat transfer area wire meshing is provided to the condenser.

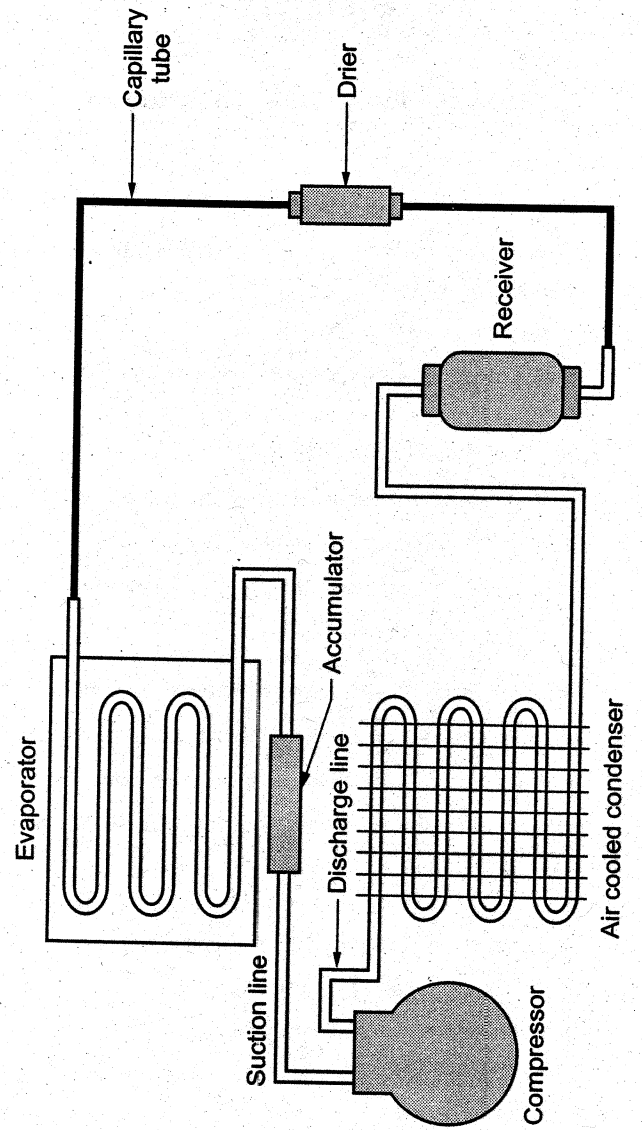


Fig. 3.1 Domestic refrigerator

Expansion valve

- Capillary tube can be used as an expansion device in domestic refrigerator.
- It is a tube of sufficient length having the internal diameter 1/16 to 1/8 inch.

- Capillary tube reduce the pressure of refrigerant by keeping enthalpy constant.
- It also provide constant flow of refrigerant to the evaporator.

3.1.2 Domestic Air Conditioner

- Domestic air conditioner is normally installed in a window or wall opening of the room and these are known as window type air conditioner.
- It works on vapour compression cycle.
- The domestic air conditioner consists of refrigeration system. Control system, electrical protection system, air circulation system, ventilation and exhaust system.
- Refrigeration system consists of hermatically sealed compressor, forced air cooled conderser, cooling coil, capillary tube and refrigerant drier.
- The refrigerant used for domestic air conditioner is R-12 or R-22.
- The condenser is a coil which is made up of copper tubing along with aluminium fins.
- These fins are used to increase the heat transfer rate.
- A propellar fan provides the air to cool the refrigerants.

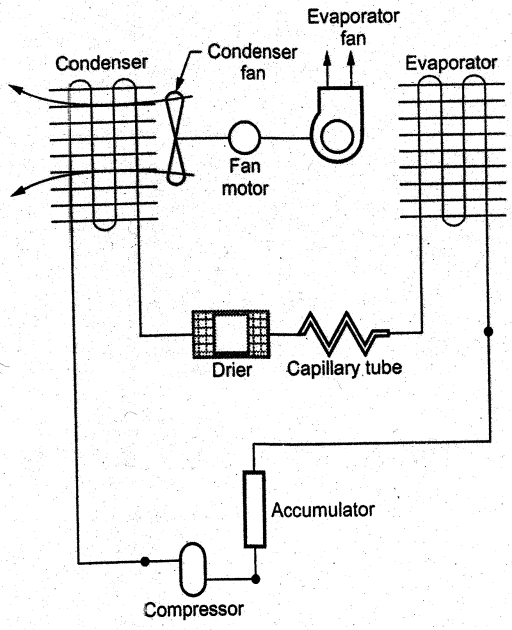


Fig. 3.2

- The room air conditioner is installed in such a way that evaporator faces the room.
- The blower is installed behind the cooling coil.
- The control system consists of thermostat and selector switch.
- Thermostat controls the operation of compressor.
- Selector switch controls the compressor motor, condenser fan motor and evaporator fan motor.
- Electrical protection system consists of motor overload switch which gives the protection to motor winding.
- Air circulation system consists of fan motor and centrifugal evaporator blower which helps in proper circulation of air.
- Ventilation unit consists of damper which helps to regulate the fresh air intake of room air-conditioner.
- The quality of fresh air may be varied by adjusting dampers.
- Exhaust system is used, if all the air in the room is to be exhausted.

3.1.3 Automotive Air Conditioner

- The automotive air conditioner can provide the adequate comfort cooling to the passanger in conditioned space.
- Cooling load can be affected by many factors such as outdoor temperature and humidity, number of occupants, quantity of fresh air taken in and sun load.
- Basic components used in automotive air conditioning are same as conventional air conditioning.
- It consists of compressor, magnetic clutch, condenser, drier, expansion valve, evaporator and air distribution system.
- Most of the automobile use 2-cylinder compressor which are driven by engine shaft pulley by the belts.
- Magnetic clutch is used for engage and dis-engage the compressor.
- The condenser is used for condensation purpose where the vapour form of refrigerant converted into liquid form.
- In automotive air conditioners condenser is located in front of radiator.
- Receiver is used to store the liquid refrigerants.
- In automobile air conditioner unit, there is large amount of chances of leakage, this is only because of the viabrations.

- The expansion valve is used for the process of expansion.
- Evaporator is used for evaporation purpose.
- In automotive air conditioner, the design of evaporator is very critical because of space limitations.
- Air distribution system is used to distribute the cool air into the conditioned space.

3.1.4 Evaporative Coolers

- Evaporative coolers are also called as desert coolers.
- The operating principle is same as that of desert coolers.
- When the air is brought in contact with water, the sensible heat of hot air is carried by the water in the form of latent heat.
- The basic components required for evaporative cooler are fan, water pumps and pads.
- From the pads water and air moves in cross-flow directions.
- The effective cooling depends upon the relative humidity of air, speed of the air and the time of contact between air and water.
- These evaporative coolers are used in summer, season.

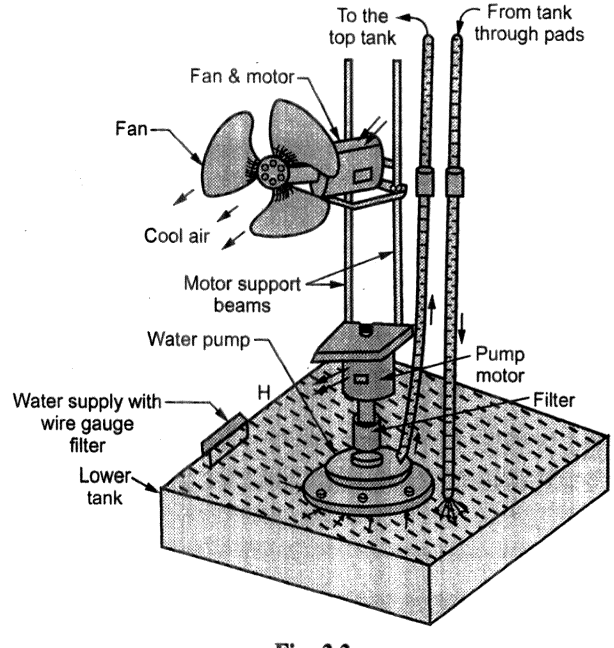


Fig. 3.3

3.1.5 Water Coolers

- The purpose of water cooling is to make the water available at constant temperature to quench the thirst.
- There are two types of water cooler.
  a) Storage type water cooler.
  b) Instantaneous water cooler.

a) Storage type water cooler

- In storage type water cooler the machine has to run for long time to bring down the temperature of water which is present in storage tank.
- Once the temperature of water touches to the set point of thermostat the machine cycle will be switched off automatically.
- Again temperature will rise slowly even when water is drawn from cooler because equal amount of fresh water is enters into the tank.

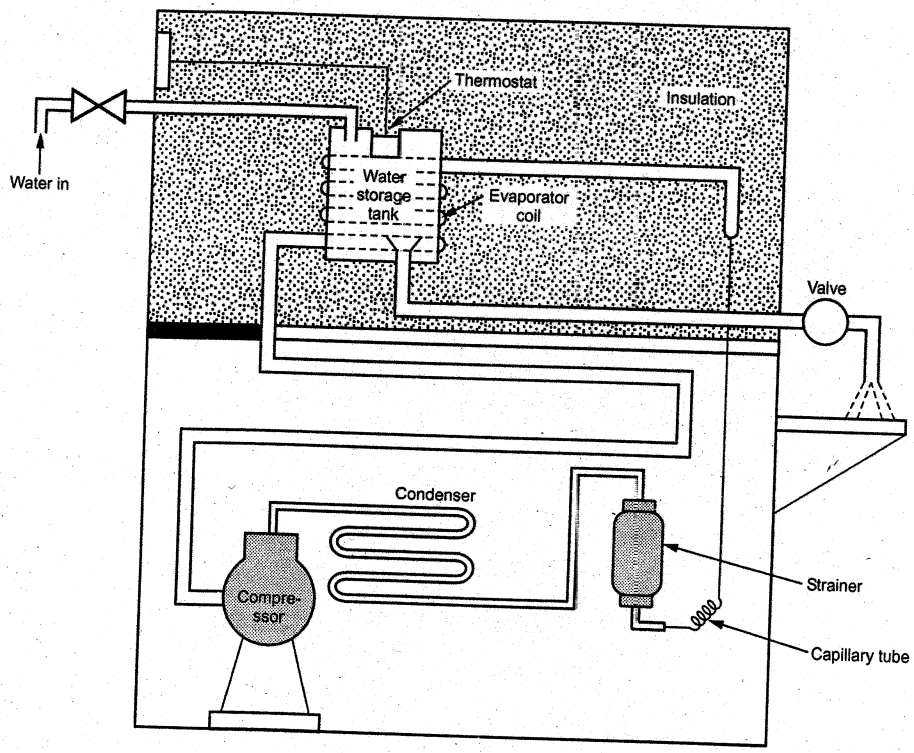


Fig. 3.4

- This process is controlled by float valve.
- In this way there will not be frequent cycling of the machine in case of storage type water cooler.

b) Instantaneous type water cooler

- In this type of water cooler. evaporator consists of two separate cylindrical wound coils made up of copper tube.

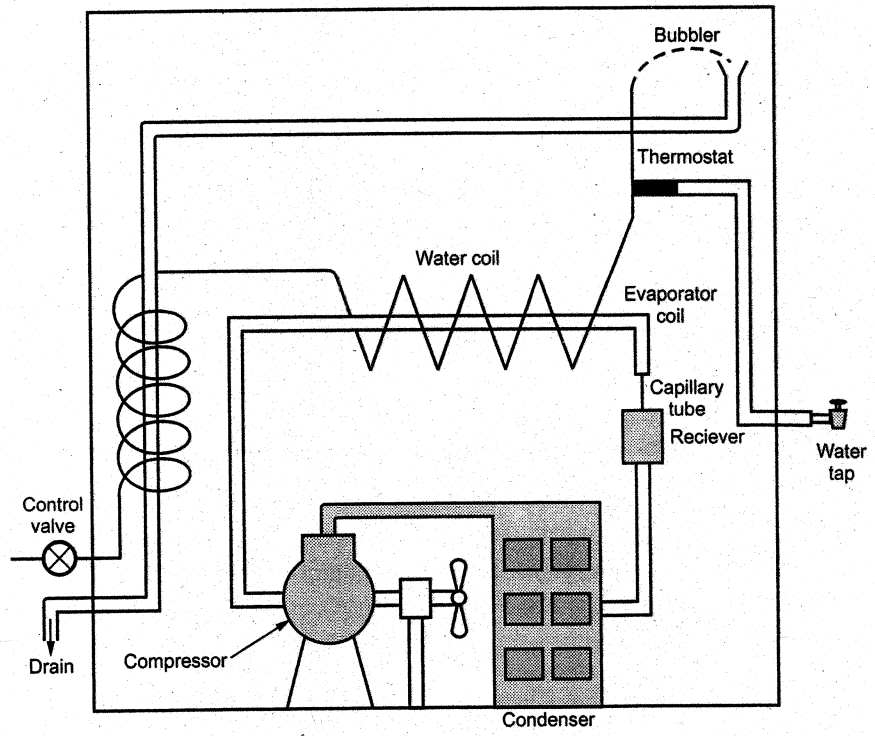


Fig. 3.5

- It uses the two tubes, in first coil evaporating refrigerant is allow to flow and in second coil water which is to be cooled is allow to flow.
- These two tube are bonded together by soldering.
- In case of instantaneous water cooler it is very important to adjust the rate of flow of water to the cooler to match the capacity with higher rate of flow.

3.1.6 Commercial Refrigeration Dairy (Milk Processing)

- Milk is a most perishable food.
- It gets spoiled, if it is not maintained at sufficient temperature because of growth of bacteria.
- To decrease the growth of bacteria it is required to cool the milk at 4 °C to 5 °C but still all the bacterias can not be killed.
- To eliminate bacteria processing on milk is required that process is called as milk pasteurization.
- In this process, the milk is heated upto the temperature of 62 °C to 65 °C and holding it at that temperature for about half an hour.
- There after to eliminate bacterial growth and preservation of milk, the milk is cooled up to 4 °C to 5 °C.
- This process of heating the milk and immediate cooling to control the bacterial growth is called as pasteurization.
- This process is done in batch type process.
- To control the fat content of the milk, it is desired to churn the milk.
- The removed fat is processed as butter and stored it at 4 °C to 5 °C.
- The chease is also another product of milk which is stored at about 4 °C.

3.1.7 Cold Storage

- Cold storage is an application of air conditioning.
- In the cold storage certain goods like food stuffs, fruits, dairy products can be stored with well defined temperature range and relative humidity.
- In cold storage air is cooled by passing it over a cooling coil.
- For the storage of different products different requirements are required.
- Hence, in cold storage temperature and humidity conditions are maintained depends upon the type of the product stored in it.
- There is a certain specific temperature range in cold storage to store the various products.
  - i) Cold storage for a product which are to be maintained at temperature 0 °C and above.
  - ii) Cold storage for product which are to be maintained below to 0 °C temperature.

3.1.8 Ice Plant

- Ice plants are used for preparation of ice.
- The cycle used for ice plant is vapour compression system.
- It uses two refrigerants primary and secondary.
- Ammonia can be used as primary refrigerant.
- While brine (H_2O + NaCl) can be used as secondary refrigerant.
- The brine temperature of refrigeration plant is maintained at –10 °C to –11 °C.
- The ammonia vapours are condensed in condenser.
- Due to condensation NH_3-vapours are converted into ammonia (NH_3) liquid.
- This liquid ammonia is expanded in the expansion valve.
- Due to expansion process pressure of liquid ammonia is reduced.
- Then this low pressure ammonia liquid passes into a evaporator.

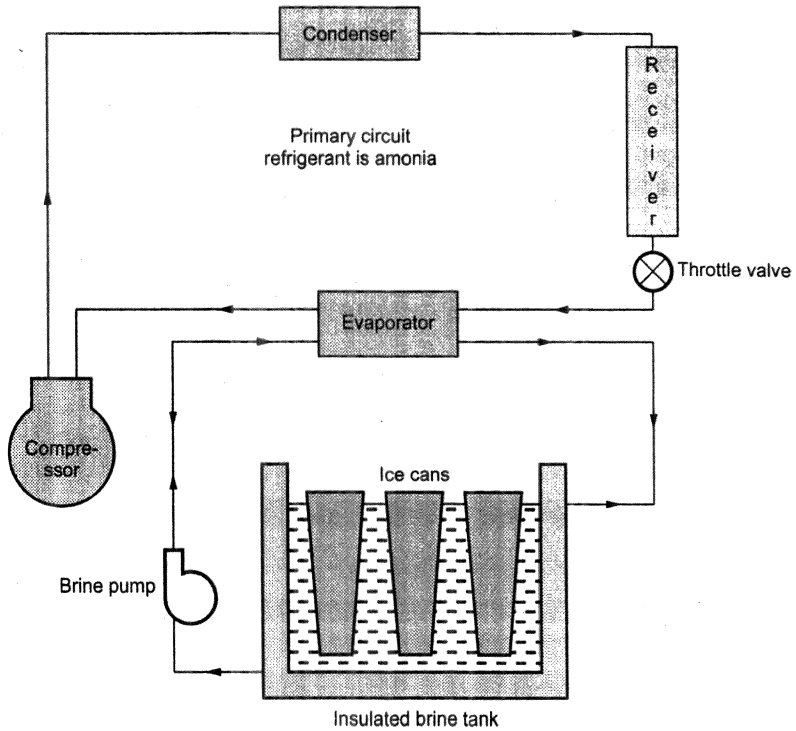


Fig. 3.6

- The evaporator coil surrounds the brine tank.
- The low pressure liquid ammonia absorbs the heat from brine solution and converted into ammonia vapour.
- In this way, bring the temperature of brine below to −10 °C to −11 °C.
- Now, in the brine tank incert the ice can which contains water
- These ice can's are fabricated from galvanised steel sheets
- Brine solution absorbs the heat from water which is present in ice can till the water to be converted into ice.

3.1.9 Commerical Air Conditioning - Multiplex

- The main load on air conditioning system is in theaters.
- The air conditioning requires higher capacity during day shows as compare to night shows.
- Per person outdoor air required is 0.15 cu.m.
- Per person ventilating air is 0.5 cu.m.
- The common design conditions are 26 °C DBT and 55 % RH.
- Because of the presence of large number of people in the multiplex increases the cooling load.
- But due to comparatively short time occupancy precooling of the structure can be used to minimize the cooling load.
- The air distribution can be achieved by overhead diffuser.
- The ventilation of outdoor air can be as low as 0.2 m^3/min per person.

3.1.10 Air Conditioning of Hospitals

- The major problem associated with air conditioning of hospital is rapid infection.
- To avoid the infection there must be sufficient circulation of air to remove bacteria.
- In hospital also there is various sections.
- The requirements of each section is different.
- The hospital contain operation theatre, delivery room, recovery room, these sections are served by individual A.C. systems.

- The various sections in the hospital and there requirements are as given below-

Section	Requirement
1. Diagnostic and treatment department	• It contain various labs, pathalogy, radiology • Ventilation rate of air 10 air-changes per hour.
2. Nursery department	• Temperature of 24 °C and 50 % RH is recommended in full time nursery section. • Temperature of 27 °C and 60 % RH is recommended for premature nursery.
3. Surgical department	• Temperature 23 °C to 25 °C • Relative humidity 50 % • Ventilation rate 15 to 20 air change/hr. • 100 % fresh air.
4. Emergency room	• Same requirment as that of surgical department.
5. Recovery room	• Temperature 22 to 24 °C • Relative humidity 55 to 60%

3.2 Refrigerants

☞ [GTU : Summer-18]

- A refrigerant is a medium of heat transfer, which absorbs the heat by evaporating as low temperature and gives out heat by condensing at high pressure and high temperature conditions.
- Refrigerant removes the heat in the form of sensible heat as in case of air refrigeration cycle or sensible heat and latent heat as in case of vapour compression cycle.
- The natural ice and mixture of ice and salt were the first refrigerants.
- In 1834, ether, ammonia, sulphur dioxide, methyl chloride and carbondioxide came into use as refrigerants in compression cycle refrigeration machines.
- A refrigerant must meet certain chemical, physical and thermodynamic properties so that it can be used suitably in vapour compression cycle.

3.3 Classification of Refrigerants

- Refrigerants are classified as shown in Fig. 3.7.

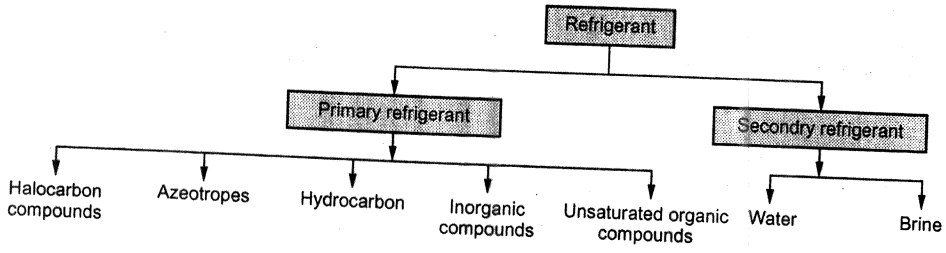


Fig. 3.7

3.3.1 Primary Refrigerant

- The refrigerants which directly takes part in the refrigeration cycle or system are called as primary refrigerants.
- Primary refrigerants includes halocarbon compound, azeatropes, hydrocarbons, inorganic compound and unsaturated organic compound.

Halocarbon Compounds

- Halocarbon compounds are commercially known as Freon, Frigen, Arcton etc.
- Such refrigerants are fluorinated hydrocarbons derived from methane, ethane etc. OR
- Refrigerants, which contain one or more of three halogens, chlorine, fluorine and bromine are halocarbon compounds.
- Halocarbon refrigerants have wide range of boiling points of one atmospheric pressure, hence they satisfy the requirements of domestic, commercial and industrial applications.
- The presence of fluorine in the compound makes them non toxic.
- Halocarbons are ozone-unfriendly and because of that, use of these refrigerants should be totally stopped.

Table 3.1 : Commonly used halocarbon refrigerants

Refrigerant No.	Chemical Name	Chemical Formulae
R-11	Trichloro-monofluro-methane	CCl_3F
R-12	Dichloro-difluoro-methane	CCl_2F_2

R-13	Monochloro-trifluoro-methane	$CClF_3$
R-14	Carbon tetrafluoride	CF_4
R-21	Dichloro-monofluro-methane	$CHCl_2F$
R-22	Monochloro-difluoro-methane	$CHClF_2$
R-30	Methylene chloride	CH_2Cl_2
R-40	Methyl chloride	CH_3Cl
R-100	Ethyl chloride	C_2H_5Cl
R-113	Trichloro -trifluoro-ethane	$C_2Cl_3F_3$

Refrigerant No.	Chemical Name	Chemical Formulae
R-114	Dichloro-Tetrafluoro-ethane	$C_2Cl_2F_4$
R-115	Monochloro-Pentafluoro-ethane	C_2ClF_5
R-123	Dichloro-Trifluoro-ethane	CF_3CHCl_2
R-124	Monochloro-tetrafluoro-ethane	CF_3CHClF
R-134a	Tetrafluoro-ethane	CF_3CH_2F
R-152a	Difluoro-ethane	CH_3CHF_2

Azeotropes

The azeotropes consists of mixture of different refrigerants.

- This is a group of refrigerants whose code starts with digit 5 (for example R500)
- This is a mixture of different refrigerants that cannot be separated under pressure and temperature They have fixed thermodynamic properties.

Table 3.2 : Azeotrope refrigerant

Refrigerant No.	Azeotropic Mixing Refrigerants	Chemical Formulae
R-500	73.8% R-12 and 26.2% R-152	CH_3CHF_2
R-502	48.8% R-22 and 51.2% R-115	$CHClF_2$
R-503	40.1% R-23 and 59.9% R-13	CHF_3
R-504	48.2% R-32 and 51.8% R-115	CH_2F_2 / $CClF_2CF_3$

Hydrocarbons-Refrigerants

- Most of the hydrocarbon refrigerants are successfully used in industrial and commercial applications.
- They passes satisfactory thermodynamic properties but are highly flamable and explosive.

Table 3.3 : Hydrocarbon Refrigerant

Refrigerant No.	Chemical Name	Chemical Formulae
R-170	Ethane	C_2H_6
R-290	Propane	C_3H_8
R-600	Butane	C_4H_{10}
R-6000	ISO-butane	C_4H_{10}
R-1120	Trichloro ethylene	$C_2H_4Cl_3$
R-1130	Dichloro ethylene	$C_2H_4Cl_2$
R-1150	Ethylene	C_2H_4
R-1270	Propylene	C_3H_6

Inorganic - Compounds

- The in-organic refrigerants were exclusively used before the introduction of halo-carbon refrigerants.
- These refrigerants are still in use due to their inherent thermodynamic and physical properties.

Table 3.4 : Inorganic Refrigerants

Refrigerant No.	Chemical Name	Chemical Formulae
R-717	Ammonia	NH_3
R-729	Air	–
R-744	Carbon dioxide	CO_2
R-764	Sulphur dioxide	SO_2
R-118	Water	H_2O

Unsaturated Organic Compounds :

- These are the hydro-carbons with ethylene and propylene base.

Table 3.5 : Unsaturated organic compounds

Refrigerant No.	Chemical Name	Chemical Formulae
R-1120	Trichloro ethylene	$C_2H_4Cl_3$
R-1130	Dichloro ethylene	$C_2H_4Cl_2$
R-1150	Ethylene	C_2H_4
R-1270	Propylene	C_3H_6

3.3.2 Secondary Refrigerants

- The refrigerants which are first cooled by primary refrigerants and then used for cooling purpose are known as secondary refrigerants.
- Secondary refrigerants are used in ice plant and many big installations to have advantages like-
  - i) Different temperature in different zone can be maintained by varying circulation rate of secondary refrigerants.
  - ii) Use of secondary refrigerant is advisable in order to keep toxic refrigerant away from load places.

Commonly used secondary refrigerants are

i) Water ii) Brine

i) Water : When required temperature is above the freezing temperature of water then water is used as secondary refrigerant.

- In air conditioning plants, chilled water is used for cooling and dehumidifying air.

ii) Brines : When required temperature to be maintained is less than freezing point of water then brine solution can be used as secondary refrigerant.

- Brine is a solution containing salt in dissolved condition in water.
- Salt used is sodium chloride (NaCl) or calcium chloride ($CaCl_2$).
- Freezing temperature of brine is – 20 °C.

3.4 Designation of Refrigerants

- The refrigerants are internationally designated as R followed by certain number such as R-11, R-12 etc.
- A refrigerant followed by two-digit number indicates that a refrigerant is derived from methane base while three digit number represents ethane base.
- The numbers assigned to hydro-carbon and halocarbon refrigerants have special meaning
- Example - R-12 can be designated as R012.
- Three numerals of this code follow the formula.
- First number of carbon atoms minus 1.
  - Second number of hydrogen atoms plus 1 and
  - Finally the number of fluorine atoms in that order.
  - Any unfilled positions are normally occupied by chlorine atoms.
- Thus chemical formula is $C_mH_nCl_oF_p$ in which $n + p + o = 2m + 2$
- The chemical formula for any compound derived from a hydrocarbon is denoted by $R - (m - 1)\ (n + 1)\ (P)$

Where, m = Number of carbon atoms

n = Number of hydrogen atoms

p = Number of fluorine atoms

o = Number of chlorine atoms

3.4.1 Examples to Study Designation of Refrigerants

Example 1 : R-22 or R-022

$m - 1 = 0 \therefore m = 1$ which means number of carbon atoms is '1'

$n + 1 = 2 \therefore n = 1$ which means number of hydrogen atoms is '1'

$P = 2$ it means number of fluorine atoms.

We know,

$$n + P + O = 2m + 2$$

$$1 + 2 + O = (2 \times 1 + 2)$$

$$3 + O = 4$$

$$O = 1$$

$\therefore$ Number of chlorine atoms = 1

$\therefore$ We can write chemical formula for R-22 as,

$$C_mH_nCl_oF_p \rightarrow \mathbf{CHClF_2}$$

Example 2 : R-12 or R-012

$m - 1 = 0$ $\therefore m = 1$ which means number of carbon atoms is '1'

$n + 1 = 1$ $\therefore n = 0$ which means number of hydrogen atom '0'

$P = 2$ It means number of fluorine atoms '2'

We know,

$$n + P + O = (2m + 2)$$

$$0 + 2 + O = (2 \times 1 + 2)$$

$$2 + O = 4$$

$$O = 2$$

$\therefore$ Number of chlorine atoms = 2

$\therefore$ Chemical formula for R-12

$$C_mH_nCl_oF_p \rightarrow \mathbf{CCl_2F_2}$$

Example 3 : R-114

$m - 1 = 1$ $\therefore m = 2$ it means number of carbon atoms '2'

$n + 1 = 1$ $\therefore n = 0$ it means number of hydrogen atoms '0'

$P = 4$ It means number of fluorine atoms '4'

We know,

$$n + P + O = (2m + 2)$$

$$0 + 4 + O = (2 \times 2 + 2)$$

$$4 + O = 6$$

$$O = 2$$

$\therefore$ Number of chlorine atoms '2'

$\therefore$ Chemical formula for R-114

$$\mathbf{C_mH_nCl_oF_p \rightarrow C_2Cl_2F_4}$$

3.5 Desirable Properties of Refrigerants

- The desirable properties of refrigerants are shown in Fig. 3.8.

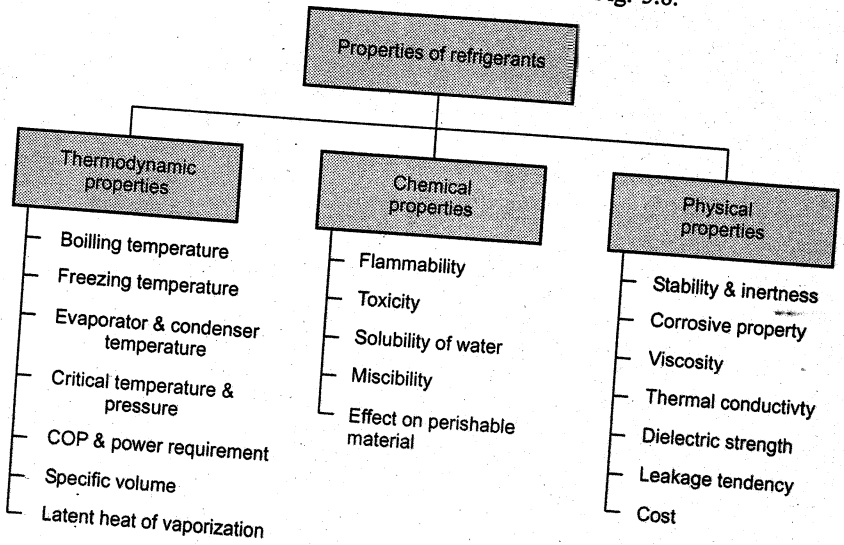


Fig. 3.8

3.5.1 Thermodynamic Properties

i) Boiling Temperature :

- The boilling temperature of refrigerant at atmospheric pressure should be low.
- If the boiling temperature of refrigerant is high at atmospheric pressure, compressor should be operated at high vacuum.

Table 3.6 : Boiling temperature of refrigerant

Refrigerants	R-11	R-12	R-21	R-22	R-30	R-40	R-134a	R-717
Boiling temp (°C)	23.77	– 29	9	– 41	39.8	– 23.7	– 26.15	– 33.3

ii) Freezing Temperature :

- The freezing temperature of refrigerant should be well below the operating evaporator temperature.
- Freezing temperature of most of the refrigerants are below –35 °C

Table 3.7 : Freezing temperature of refrigerants

Refrigerants	R-11	R-12	R-21	R-22	R-30	R-40	R-134a	R-717
Freezing temp (°C)	–111	– 157.5	–135	– 160	–96.6	– 97.5	– 101.0	– 77.8

iii) Evaporator and Condenser Pressure :

- Both the evaporator (low side) and condenser pressure (high side) should be positive (i.e. above the atmospheric) and it should be as near to the atmospheric pressure as possible.
- The positive pressures are necessary in order to prevent leakage of air and moisture into the refrigerating system.
- A small difference between suction pressure and discharge pressure or low pressure ratio (P_C / P_L) (Ratio of condenser pressure to evaporator pressure is desirable for good efficiency.

Table 3.8 : Evaporator and condenser pressure

Refrigerant	Evaporator pressure (P_E) at – 15° C in bar	Condenser pressure (P_C) at +30° C in bar	Compression ratio P_C / P_E
R-11	0.2021	1.2607	6.24
R-12	1.8262	7.4510	4.08
R-21	0.3618	2.1540	5.95
R-22	2.9670	12.0340	4.05
R-30	0.0807	0.7310	9.06

iv) Critical Temperature and Pressure :

- It is temperature above which there will not be any phase change.
- The critical temperature of the refrigerant should be well above the condensing temperature for easy condensation of vapour.

Table 3.9 : Critical temperature and pressure

Refrigerant	R-11	R-12	R-21	R-22	R-30	R-40	R-134a
Critical temperature °C	198	112	178.5	96	216	14.3	101.06
Critical pressure bar	43.8	41.2	51.6	49.3	44.14	66.8	40.56

v) COP and Power Requirement :

- For an ideal refrigerant operating between –15 °C evaporator temperature and 30 °C condenser temperature the theoretic COP for reversed carnot cycle is 5.74
- Following table shows values of theoretical COP and power per tonne of refrigeration for some common refrigerants operating between –15 °C evaporator temperature and 30 °C condenser temperature.

Table 3.10 : COP and power per TR

Refrigerant	R-11	R-12	R-22	R-30	R-40	R-113	R-134 a
COP	5.09	4.70	4.66	4.90	4.90	4.92	4.61
kW/TR	0.694	0.74	0.75	0.71	0.716	0.716	0.762

vi) Latent Heat of Vaporisation :

- A refrigerant should have high latent heat of vaporisation at the evaporator temperature.
- A small quantity of refrigerant produces a large cooling effect which requires less power for circulation.

Table 3.11 : Latent heat of vaporisation

Refrigerant	R-11	R-12	R-22	R-23	R-134 a	R-502
Latent heat of vaporisation (h_{fg})	157.28	157.28	213.12	182.38	204.85	153.45

vii) Specific Volume :

- Specific volume of refrigerant vapour at evaporator temperature indicates the theoretical displacement of compressor.
- Reciprocating compressors are used with refrigerant having high pressure and low volume of suction vapour.
- The centrifugal or turbo compressors are used with refrigerants having low pressure and high volume of suction vapour.
- The rotary compressors are used with refrigerants having intermediate pressure and volume of suction vapour.

Table 3.12 : Specific Volume of refrigerants

Refrigerant	R-11	R-12	R-22	R-30	R-40	R-134 a
Specific volume in m^3/kg	0.77	0.091	0.078	3.12	0.28	0.121

3.5.2 Chemical Properties

i) Flammability :

- Refrigerants should not be inflammable in presence of air or lubricating oils.
- CO_2 is most inert while ethane and butane are highly inflammable.
- Hydrocarbons such as methane, ethane, propane, butane are highly explosive and inflammable.
- Fluorocarbons are non explosive and non flammable but have high ODP.

ii) Toxicity

- A refrigerant should not be toxic, poisonous and injurious.
- Ammonia is toxic as well as flammable so it is not used in domestic refrigerators.

iii) Solubility of Water :

- The presence of moisture is very critical in refrigeration systems.
- If it is carried along with refrigerants, it becomes ice at expansion valve and results into chocking of valve. This is called moisture chocking.
- It is avoided by proper dehydration of system or by placing silica gel in pipeline before the expansion valve.
- Ammonia is soluble in water. R134a is also very hygroscopic in nature.

iv) Miscibility :

- The ability of refrigerant to mix with oil is called as miscibility
- The degree of miscibility depends upon the temperature of the oil and pressure of refrigerating vapour.
- The freon group of refrigerants are highly miscible while NH_3, CO_2, SO_2, CH_3Cl are relatively non miscible.
- The miscible refrigerants are advantageous from heat transfer point of view.

v) Effect on Perishable Material :

- The refrigerant should not have any effect on perishable material.
- The freon group of refrigerants does not have any effect on dairy products, meats, vegatables, flowers.
- There will not be any change in colour, taste or texture of material when exposed to freon.

3.5.3 Physical Properties of Refrigerants

i) Stability and Inertness :

- An ideal refrigerant should not decompase at any temperature normally encountered in the refrigerating system.
- Refrigerants should be inert with respect to all materials which are used in refrigerating system.
- The freon group refrigerants are stable upto temperature of 535 °C.

ii) Corrosive Properties :

- The corrosive property of refrigerant must be taken into consideration while selecting the refrigerants.
- Freon group refrigerants are non corrosive with practically all metals.
- Ammonia is used only with iron and steel.
- Sulphur dioxide is non corrosive to all metals in the absence of water because it will react with water and form sulphuric acid.

iii) Viscosity

- It is defined as a measure of fluid friction or as measure of resistance that fluid offers to flow.
- The refrigerants in liquid and vapour state should have low viscosity.

iv) Thermal Conductivity :

- The refrigerants must have high thermal conductivity in liquid as well as vapour phase.

v) Dielectric Strength :

- The refrigerants should have high electric strength otherwise electrical short circuits are likely to occur in hearmatically sealed compressors.

vi) Leakage Tendency :

- The leakage tendency of the refrigerant should be low.
- If there is leakage of refrigerant, it should be easily detectable.
- Ammonia leakage is easily detected due to pungent odour.

vii) Cost :

- The refrigerant should be available at low cost.

Example : NH_3 is cheap and R-12, R-22, are more expensive.

3.5.4 Properties of an Ideal Refrigerant

- Refrigerant should be non toxic.
- It should be non flammable.
- It should posses high latent heat.
- It should have high critical temperature.
- It should be easily available.
- It should not be miscible with oil.
- It should posses high critical pressure and temperature.
- It should be non corrosive.
- It should have zero ODP.

3.6 Environmental Issues

- With all human activities, refrigeration and air conditioning applications have an impact upon the environment.
- It contributes to ozone depletion and global warming.
- Now a days, refrigeration and air conditioning industries are focused on reducing environmental impact in two important aspects.
  1) Tries to minimize the direct emission of refrigerant gas into the atmosphere.
  2) Minimizing the indirect emission of greenhouse gases caused by refrigeration equipment power consumption which can lead to global warming.

- Overview of refrigerant group

Table 3.13

Substance group	Name of refrigerant	ODP	GWP	Remark
CFC	R-11	1.00	4750	Very high ODP and GWP
	R-12	0.82	10,900	No longer sold
HCFC	R-22	0.06	1810	Medium ODP and GW
	R-124	0.03	610	Out of phase
HFC	R-134 a	0	1430	Zero ODP and medium GWP
	R-404 A	0	3920	Zero ODP and medium GWP
Natural refrigerant	R-717	0	0	Zero ODP and low GWP

3.7 Ozone Depletion

- Nearly 50 km from the surface of the earth, there is a stratosphere where the layer of the ozone gas is exist.
- This ozone layer absorbs the band of ultraviolet radiations which is harmful to the living organisms.
- Therefore this ozone layer protect us from such harmful UV-rays which may cause skin cancer.
- Ozone is formed from three oxygen atoms.
- But some of the gases like CFC which are given out by various processes, reach upto the ozone layer.
- And react with ozone gas to form different compounds.
- This causes the depletion of ozone, at various places which is called as ozone holes.
- The chlorine and bromine are the major reasons for the depletion of ozone.
- As the ozone (O_3) absorbs ultraviolet radiation following reaction will takes place.

$$O_3 \rightarrow O_2 + O$$

- This free oxygen atom again reacts with ozone molecule which results in destruction of ozone.

$$O + O_3 \rightarrow O_2 + O_2$$

- When the ultraviolet radiations strikes the CFC it breaks the chlorine atom.

$$CFC1_3 \rightarrow CFC1_2 + C1$$

- The chlorine atom reacts with ozone molecule and forms chlorine monoxide and leave the oxygen.

$$C1 + O_3 \rightarrow C1O + O_2$$

- In this way ozone (O_3) is depleted to O_2 gas.

3.7.1 Effect of Ozone Depletion

- UV radiation causes skin cancer.
- Due to UV-radiation eyes may get damaged.
- It increases global warming.
- Lower yield of the crop.
- It leads to increase in UV-radiations at ground level.

3.7.2 Ozone Depletion Potential (ODP)

- The ozone depletion potential (ODP) is the relative amount of degradation to ozone layer it can cause with trichlorofluromethane (R-11 or CFC-11) which is being fixed at an ODP of 1.0.
- ODP of given substance is defined as ratio of global loss of ozone due to given substance to global loss of ozone due to CFC-11 of the same mass.
- ODP can be estimated from molecular structure of given substance.
- CFC have ODP roughly equal to 1.
- Brominated substances have usually higher ODP in the range of 5-15.
- Hydroflurocarbon have ODP in the range of 0.005-0.2.

3.8 Green House Effect

- The term green house effect indicates physical property of earth atmosphere.
- Sun can emit the energy in the form of short wavelength.
- The earth can absorb the heat from the sun.

- Some of the heat which is absorbed by earth from the sun is reflected back to the space.
- Because of this earth's temperature does not rise above the certain level.
- The film of gas envelopes the earth's atmosphere.
- But there are certain gases known as green house gases traps some reflected heat, preventing it to reflect back to space which results in increase in average temperature of earth, which leads global warming.
- Thus higher the concentration of green house gases in the atmosphere, it leads to increase the temperature of earth surface.
- Carbon dioxide (CO_2), methane (CH_4), nitrous oxide are the main green house gases.
- All the hologenated refrigerants (CFC, HFC) are green house gases.

3.9 Global Warming Potential (GWP)

- The refrigeration and air conditioning system contributes in global warming.
  1) While servicing of the equipment the refrigerant may get leak and spread into atmosphere.
  2) Indirectly by the way of consumption of energy for its operation as carbon dioxide is relead to generate the energy.
- The ODP and GWP are the common parameters that describes the potential of refrigerant on global warming.
- To minimize the global warming following care should be taken -
  i) Refrigeration system should be leak proof.
  ii) While handling the refrigerant proper care should be taken, so that there will not be a leakage of refrigerant.
- Global warming increases the temperature which causes the melting of ice which rises the sea level.
- Global warming has increased ocean level by more than 1 mm a year in past century.
- But since 1960 ocean level is increasing 1.3 mm annually.

3.10 Life Cycle Climate Performance (LCCP)

- Life cycle climate performance is an evaluation method by which heating ventilation, air-conditioning and refrigeration (HVAC and R) systems can be evaluated.
- Life cycle climate performance is calculated as the sum of direct and indirect emissions generated over the lifetime of the system.
- There are two types of emissions
  1) direct emission and
  2) indirect emissions
- Direct emissions include all effects from the release of refrigerant into the atmosphere during the lifetime of the system.
- Annual leakage and leakage during the disposal of the unit comes under direct emission.
- The indirect emissions include emissions from the manufacturing process, energy consumption and disposal of the system.
- To reduce the environmental impacts of air conditioning and refrigeration systems the heating, ventilation, air conditioning and refrigeration (HVAC and R) industry is spending a large amount of effort.
- Several indices are developed to measure this impact.
- It includes Total Equivalent Warming Impact (TEWI), Life Cycle Analysis (LCA) and Life Cycle Climate Performance (LCCP).

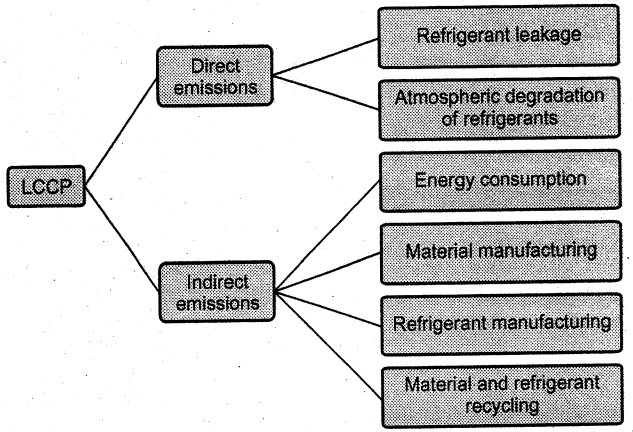


Fig. 3.9

- LCCP is an important tool in analyzing refrigeration systems and needs a globally harmonized approach.

3.10.1 Limitations of LCCP

- Numbers of assumptions are need to be consider while calculating the value of LCCP.
- LCCP should be used as comparison tool for systems with similar performance and function.
- It can not be used as definitive estimate of lifetime emissions.

3.11 Total Equivalent Warming Impact (TEWI)

- TEWI (Total Equivalent Warming Impact), was developed at Oak Ridge National Laboratory in the early 1990s.
- It is defined as sum of the direct emissions (chemical) and indirect emission (energy use) of greenhouse gases.
- Total Equivalent Warming Impact (TEWI) is used to measure the global warming impact of a HVAC and R system.
- The result of a TEWI calculation is useful in a complete life cycle assessment of an industry, supermarket and all types of buildings.
- **To calculate TEWI for a unit, the following are required.**
- Power draw at various ambient temperatures.
- Generated emission factors for the power supplies to be used.
- Temperatures and types of power supply.
- Refrigerant leakage in use and at decommissioning.
- Global warming potential of the refrigerant fluid.

3.12 Selection of Environment Friendly Refrigerants

Selection of refrigerant for a particular application is based on following requirements :

i) Thermodynamic and thermophysical properties

ii) Environmental and safety properties

ii) Economics

Table 3.14

HFC used	Possible eco friendly refrigerant	Application
R134 a, R152 a	HC 600 a and blends	Domestic refrigeration
R134 a, R404 A, R407 C	HC blends, NH_3, CO_2	Commercial refrigeration Cold storage Food processing
R134 a, R404 A, R507 A	NH_3, HCS, CO_2	Industrial refrigeration
R410 A, R407 C	CO_2, HCS	Unitary air conditioners
R134 a, R410 A, R407 C	NH_3, HCS, CO_2, water	Centralized AC (chillers)
R134 a, R404 A	CO_2	Transport refrigeration
R134 a	CO_2, HCS	Mobile air conditioner
R134 a, R152 a, R404 a, R407 C, R410 A	NH_3, HCS, CO_2, water	Heat pumps
CFC used	**Possible alternative refrigerant**	**Application**
CFC - 11 CFC - 114 CFC - 12	Helium HCFC - 134 a ammonia	Refrigeration
CFC - 11	HCFC - 22 + HCFC 142 b HFC - 134 a	Air conditioning
CFC - 11 RCF - 12	water - bloom foam HCFC - 141 b HCFC - 22	Foam blowing
CFC - 12	CO_2, HCFC - 22	Sterilization

3.13 Anti-Freeze Solution

- The compounds which are soluble in water, when added into water but because of this the freezing point of water gets decreased such compounds are known as anti freeze compounds.
- Glycerine can be used as antifreeze compound.
- This glycerine is obtained from carbohydrates.

- Ethelene glycol, propelene glycol can also glycol can also be used as antifreeze solution.
- The use of glycol has certain advantages.
  i) These are extremely stable
  ii) They will not evaporate during service.
  iii) These are non corrosive.
  iv) These are used as lubricants.

3.14 Zeotropic and Azeotropic Refrigerants

- Azeotropic mixture is one whose composition in liquid phase differs to that in vapour phase.
- Zeotropic refrigerant do not boil at constant temperature.

Ex.

R404a :- R125/143a/134a (44%, 52%, 4%)

R407a :- R32/125/134a (23%, 25%, 52%)

R410a :- R32/125 (50%, 50%)

- Azeotropic refrigerants are stable mixture of two or several refrigerants whose vapour and liquid phases retain identical composition over wide range of temperature.

Ex.

R500 :- 73.8% R12 and 26.2% R152

R502 :- 8.8% R22 and 51.2% R115

R503 :- 40.1% R23 and 59.9% R13

3.15 Montreal and Kyoto Protocol

Montreal protocol

- The Montreal protocol was happened in year 1990. The agenda of this meeting is to discuss about depletion of ozone layer by refrigerants.
- The Montreal Protocol on substances that deplete the ozone layer is a landmark international agreement designed to protect the stratospheric ozone layer.
- The treaty was originally signed in 1987 and substantially amended in 1990 and 1992.

- The Montreal Protocol stipulates that the production and consumption of compounds that deplete ozone in the stratosphere-chlorofluorocarbons (CFCs), halons, carbon tetrachloride and methyl chloroform are to be phased out by 2000 (2005 for methyl chloroform).
- Scientific theory and evidence suggest that, once emitted to the atmosphere, these compounds could significantly deplete the stratospheric ozone layer that shields the planet from damaging UV-B radiation.
- The United Nations Environment Programme (UNEP) has prepared a Montreal protocol handbook that provides additional detail and explanation of the provisions.

Kyoto protocol

- The Kyoto protocol is an international agreement linked to the United Nations framework convention on climate change, which **commits** its parties by setting internationally binding emission reduction targets.

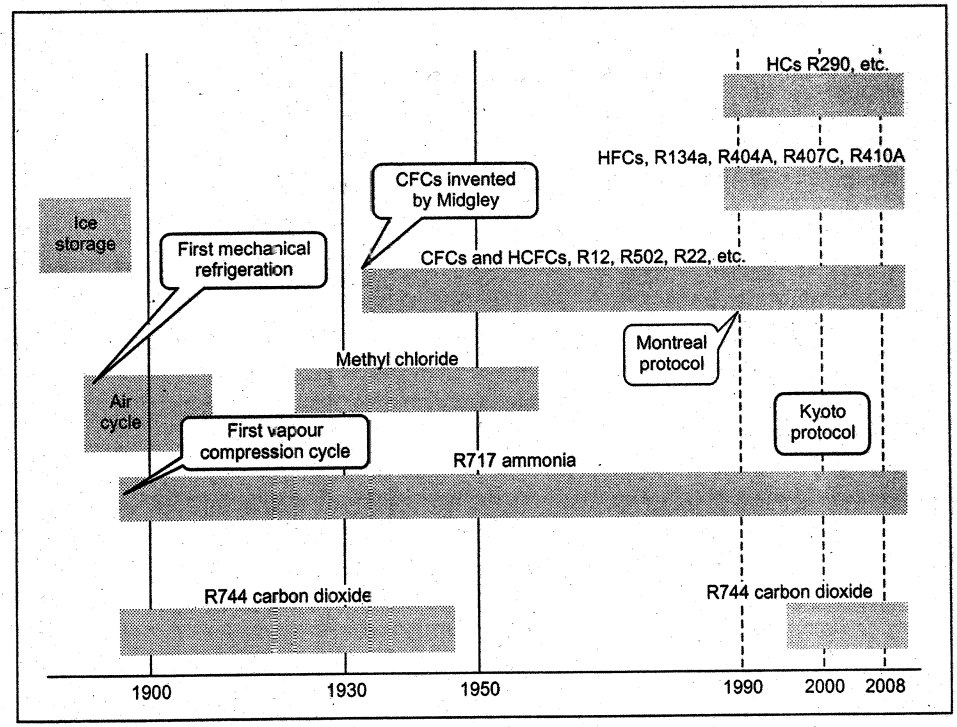


Fig. 3.10

- Recognizing that developed countries are principally responsible for the current high level of GHG emissions in the atmosphere as a result of more than 150 years of industrial activity, the protocol places a heavier burden on developed nations under the principle of "common but differentiated responsibilities".
- The Kyoto protocol was adopted in Kyoto, Japan, on 11 December 1997 and entered into force on 16 February 2005. The detailed rules for the implementation of the protocol were adopted at COP 7 in Marrakesh, Morocco, in 2001, and are referred to as the "Marrakesh Accords". Its first commitment period started in 2008 and ended in 2012.

3.16 Recovery, Recycling and Reclaiming of Refrigerant (RRR)

- RECOVERY of refrigerant involves the removal of refrigerant from system and placing that refrigerant into container.
- Whenever technicians need to open or dispose of air conditioning and refrigeration equipment then this recovery process is conducted.
- There are so many environmental and economic advantages gained from recovery, recycling and reclaiming of refrigerants.
- Some advantages are listed below -
- Reduced environmental compliance cost.
- It reduces the need of new refrigerants.
- It helps to minimize the atmospheric emissions.
- RECYCLING of refrigerant involves processing of refrigerants ro reduce contaminants.
- Then these refrigerants can be reused.
- Result of contamination is failure of system.
- Contaminants includes oil, moisture, acid, particulars etc.
- RECLAMATION of refrigerant involves purifying used refrigerant to meet industry product specification.
- The reclamation is used to identify bad or mixed refrigerant which could results in damage of the equipment.
- The reclamation process includes filtering, separation, distillation, dilution of recovered refrigerant.
- The reclamation facilities are designed to minimize the emission.

3.17 Recharging of Refrigerant

- It is a process in which fresh refrigerant is added in the existing refrigeration or air conditioning system.
- The actual task of replacing the refrigerant in an existing system is not simple.
- Following caution should be taken while charging the refrigerant
  1) Before charging the refrigerant, evacuate the system to remove the moisture.
  2) Close the valves on low or high pressure sides of manifold gauge.
  3) Install low/high pressure hoses to corresponding service part.
  4) Connect the center hose of manifold gauge set with vacuum pump.
  5) Open the low/high pressure side valves to activate vacuum pump.
  6) Evacuate the system approximately for 15 min when low pressure gauge reaches to 100 kPa.
  7) After evacuation of 15 min, if reading shows 100 kPa or higher then close the valves on both sides to stop the vacuum pump.
  8) Note down the low-pressure gauge reading.
  9) Wait for 5 min and check reading of low pressure gauge when gauge indicator shows near to zero point this is the sign of leakage.
  10) Disconnect center manifold hose from vacuum pump and connect hose to the tap valve.
  11) Confirm all the 3 hoses are tightly connected to manifold gauge.
  12) Open the valve on HFC-134a source.
  13) Loosen the center hose connection on manifold gauge only for a couple of seconds to allow the air in center hose to escape by refrigerant.

Refrigeration Cycle

3.18 Introduction to Refrigeration

- Refrigeration is a process of providing and maintaining temperature below to the atmosphere.
- The medium used for refrigeration process is called as refrigerants.
- The device which is used for the process of refrigeration is called as refrigerator.
- In air cycle refrigeration, air is used as working fluid.

- The basic elements of air refrigeration systems are compressor, condenser, expansion valve and evaporator.

3.19 Reverse Carnot Cycle

☞ [GTU : Summer-16]

- Carnot cycle is considered as reversed carnot cycle in refrigeration system.
- This cycle is only a theoretical cyclel but it serves as an ideal cycle.
- The refrigeration systems which works on reverse carnot cycle has maximum COP [Coefficient Of Performance].
- The carnot cycle is independent on physical properties of working medium.
- The reversed carnot cycle which work by considering air as a working medium is shown in Fig. 3.11.

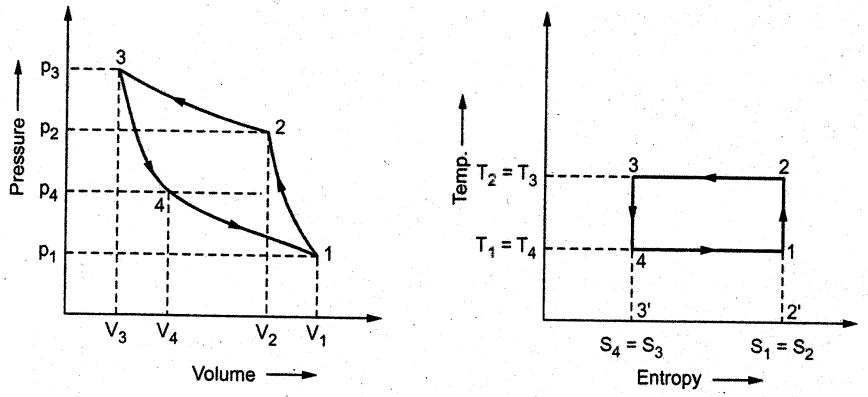


Fig. 3.11

Process 1-2 : Isentropic compression

- In this process, air is compressed isentropically.
- Because of compression pressure increases from p_1 to p_2 and temperature increases from T_1 to T_2.
- Where as the specific volume decreases from V_1 to V_2.

Process 2-3 : Isothermal compression process

- In this process, by maintaining temperature constant i.e. $(T_2 = T_3)$ air is compressed.

- During this process pressure increases from p_2 to p_3 and specific volume decreases from V_2 to V_3.
- Heat rejected during this process can be calculated as,

$$Q_{2\text{-}3} = T_3 (S_2 - S_3) \text{ or}$$

$$Q_{2\text{-}3} = T_2 (S_2 - S_3) \quad \ldots\ldots (\because T_2 = T_3).$$

Process 3-4 : Isentropic expansion process

- In this proces expansion of air is takes place isentropically
- During this process pressure and temperature decreases from p_3 to p_4 and T_3 to T_4 respectively.
- Where as specific volume increases from V_3 to V_4.

Process 4-1 : Isothermal expansion process

- In this process air is expanded isothermally $(T_1 = T_4)$.
- During this process pressure decreases from p_4 to p_1 and temperature remains constant $(T_1 = T_4)$.
- Specific volume gets increased from V_4 to V_1
- Heat absorbed by air during this process can be calculated as,

$$Q_{4\text{-}1} = T_4 (S_1 - S_4) \text{ or}$$

$$= T_4 (S_2 - S_3) \text{ or } T_1 (S_2 - S_3).$$

- Work done during the cycle,

$$W = \text{Heat rejected} - \text{Heat absorbed.} = T_2 (S_2 - S_3) - T_1 (S_2 - S_3)$$

$$= (T_2 - T_1)\ (S_2 - S_3)$$

- Coefficient of reversed carnot cycle

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$COP = \frac{T_1\ (S_2 - S_3)}{(T_2 - T_1)\ (S_2 - S_3)}$$

$$\boxed{COP = \frac{T_1}{T_2 - T_1}}$$

- No any refrigerator has been made by using reversed carnot cycle because of following reasons.
- Isentropic process requires high speed while isothermal process is extremely slow speed.
- This variation in the speed is not practicable.
- COP of heat pump,

$$(COP)_{H.P.} = (COP)_R + 1$$

$$= \frac{T_1}{T_2 - T_1} + 1$$

$$\boxed{(COP)_{H.P.} = \frac{T_2}{T_2 - T_1}}$$

- COP or efficiency of Heat engine,

$$(COP)_{H.E.} = \frac{1}{(COP)_{H.P.}}$$

$$(COP)_{H.E.} = \frac{1}{\frac{T_2}{T_2 - T_1}}$$

$$\boxed{(COP)_{H.E.} = \frac{T_2 - T_1}{T_2}}$$

3.20 Block Diagram of Heat Engine, Refrigerator and Heat Pump

- The performance of heat engine is always expressed in terms of efficiency.
- Heat engine is a work producing device.

$$\eta_{H.E.} \text{ or } (COP)_{H.E.} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$\boxed{(COP)_{H.E.} = \frac{Q_2 - Q_1}{Q_2}}$$

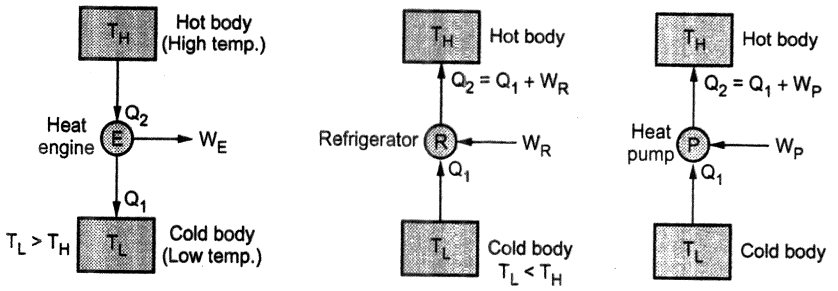


Fig. 3.12

- Refrigerator is a work absorbing device.
- Refrigerator is reversed heat engine which either cool or maintain the temperature of body.
- Refrigerator extract the heat (Q_1) from cold body which is at a temperature of (T_1) and supplied to hot body which is at a temperature of (T_2)

$$\boxed{(COP)_R = \frac{Q_1}{Q_2 - Q_1}}$$

- The performance of heat pump is expressed by the ratio of amount of heat supplied to hot body (Q_2) to amount of work required to be done on the system (Wp).
- This ratio is also called as energy performance ratio.

$$\boxed{(COP)_{HP} = \frac{Q_2}{Q_2 - Q_1}}$$

3.20.1 Solved Examples

Example 3.1 : *A heat pump is used to keep a room at 25°C by rejecting heat to an environment at 5°C. The total heat loss from the room to the environment is estimated to be 45,000 kJ/hr and the power input to the compressor is 4.5 kW. Determine :*

i) The rate of heat absorbed from the environment in kJ/h.

ii) The COP of the heat pump

iii) The maximum rate of heat supply to the room for the given power input and

iv) The second - law efficiency of the cycle.

Solution : Given :

$T_1 = 25°C = 298\ K$, $T_2 = 5°C = 278\ K$,

$W = 4.5\ kW$, $Q_2 = 45{,}000\ kJ/hr = 12.5\ kW$

To Find : 1) Q1 ii) COP iii) $(Q_2)_{max}$ iv) η_{II}

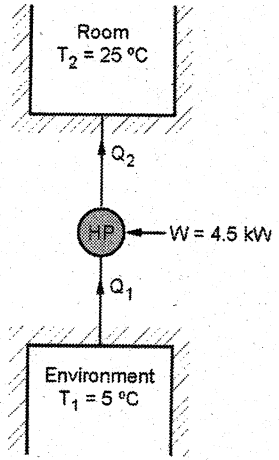


Fig. 3.13

Step 1 : Calculate COP

COP of heat pump is given as,

$$COP = \frac{Q_2}{W} = \frac{12.5}{4.5}$$

$$= \mathbf{2.7778} \qquad \textbf{...Ans.}$$

Step 2 : Calculate rate of heat absorbed

We know that,

$$W = Q_2 - Q_1 \qquad \therefore\ 4.5 = 12.5 - Q_1$$

$$\therefore \quad Q_1 = 8\ kW = 8 \times 3600\ kJ/hr$$

$$\therefore \quad \mathbf{Q_1 = 28.800\ kJ/hr} \qquad \textbf{...Ans.}$$

Step 3 : Calculate rate of heat supply

Carnot COP is given as,

$$(COP)_{carnot} = \frac{T_2}{T_2 - T_1} = \frac{298}{298 - 278} = 14.9$$

But it can also be given as,

$$(COP)_{carnot} = \frac{(Q_2)_{max}}{W} \qquad \therefore\ 14.9 = \frac{(Q_2)_{max}}{4.5}$$

$$\therefore \quad \mathbf{(Q_2)_{max} = 67.05\ kW} \qquad \textbf{...Ans.}$$

Step 4 : Calculate second law efficiency

Second law efficiency of cycle is given as,

$$\eta_{II} = \frac{(COP)}{(COP)_{carnot}} = \frac{2.7778}{14.9}$$

$$\eta_{II} = 0.1864 = \mathbf{18.64\ \%} \qquad \textbf{...Ans.}$$

Example 3.2 : *A Carnot refrigerator requires 1.2 kW per ton of refrigeration to maintain a temperature of − 35 °C. Determine.*
i) COP of the refrigerator ii) The temperature at which the heat is rejected iii) COP, if the cycle is used as a heat pump

Solution : Given data : $Q_1 = 1\ TR = 3.52\ kW$, $W = 1.2\ kW$

To find : $(COP)_{Heat\ pump}$

$$(COP)_{Refrigerator} = \frac{Q_1}{W} = \frac{3.52}{1.2}$$

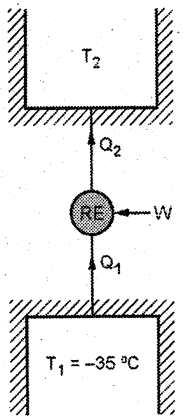


Fig. 3.14

$$\mathbf{(COP)_{Refrigerator} = 2.93} \qquad \textbf{...Ans.}$$

$$COP = \frac{T_1}{T_2 - T_1}$$

$$2.93 = \frac{-35 + 273}{(T_2 + 273) - (-35 + 273)}$$

$$2.93\ ((T_2 + 273) - 238) = 238$$

$$2.93\ (T_2 + 273) - 697.34 = 238$$

$$T_2 + 273 = 319.22$$

$$\mathbf{T_2 = 46.22\ °C}$$

$$(COP)_{Heat\ pump} = \frac{T_2}{T_2 - T_1} \quad \frac{46.22 + 273}{(46.22 + 273) - (-35 + 273)}$$

$$= \mathbf{3.93} \qquad \textbf{...Ans.}$$

Example 3.3 : *The COP of a refrigerator is 6, when it maintains the temperature of −3 °C in the evaporator. Determine the condenser temperature and refrigerating effect if the power required to run the refrigerator is 7.5 kW.* ☞ **[GTU : Summer-16]**

Solution : Given data : $(COP)_{ref.} = 6$,

$T_2 = -3\ °C = -3 + 273 = 270\ K$, $W = 7.5\ kW$

To find :

i) Condenser temperature T_1 and ii) Refrigerating effect.

Step - 1 : Calculate the condenser temperature T_1

COP of refrigerator is given by,

$$(COP)_{ref.} = \frac{T_2}{T_1 - T_2}$$

$$\therefore \quad 6 = \frac{270}{T_1 - 270}$$

$$\therefore \quad T_1 = \mathbf{315\ K}$$ **... Ans.**

Step - 2 : Calculate the refrigerating effect

Power required to run the refrigerator

$$= 7.5\ kW = 7500\ W = 7500\ J/sec$$

We know that,

$$(COP)_{ref.} = \frac{Q_2}{W} = \frac{\text{Work output}}{\text{Work input}}$$

$$(COP)_{ref.} = \frac{\text{Refrigerating effect}}{\text{Input power}}$$

$$\therefore \quad 6 = \frac{\text{Refrigerating effect}}{7500}$$

$$\therefore \quad \text{Refrigerating effect} = 45000\ J/sec.$$

$$= 45000\ W = \mathbf{45\ kW}$$ **... Ans.**

Example 3.4 : *A reversible heat engine takes 900 kJ heat from a source at 700 K. The engine develops 350 kJ of network and rejects heat to two low temperature reservoirs at 600 K and 500 K. Determine engine thermal efficiency and heat rejected to each low temperature reservoir by using Clausius inequality.*

Solution : Given data : $T_1 = 700\ K$, $Q_1 = 900\ kJ$,

$W_{net} = 350\ kJ$, $T_{2(Reservoir\ -1)} = 600\ K$,

$T_{2(Reservoir\ -\ 2)} = 500\ K$

To find : 1) η_E 2) Q_2

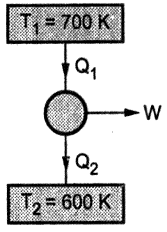


Fig. 3.15 (a)

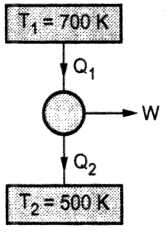


Fig. 3.15 (b)

Thermal Efficiency

Step - 1 : Calculate thermal efficiency

For Reservoir - 1

$$\eta_E = 1 - \frac{T_2}{T_1} = 1 - \frac{600}{700} = 0.143$$

$$= \mathbf{14.30\ \%}$$ **... Ans.**

For Reservoir - 2

$$\eta_E = 1 - \frac{T_2}{T_1} = 1 - \frac{500}{700} = 0.2357$$

$$= \mathbf{23.57\ \%}$$

Step - 2 : Calculate heat rejected (Q_2)

$$W_{net} = Q_1 - Q_2$$

$$\therefore \quad Q_2 = Q_1 - W_{net} = 900 - 350$$

$$\therefore \quad \boxed{Q_2 = 550\ kJ}$$ **... Ans.**

Example 3.5 : *A reversible heat pump is used to maintain a temperature of 0 °C in a refrigerator when it rejects the heat to the surroundings at 27 °C. Determine COP of the machine and work input required if the heat removal rate is 25 kW. If the required input to run the pump is developed by reversible engine which receives heat at 673 K and rejects heat to the atmosphere, determine overall COP of the system.*

Solution : **Given data :** $T_1 = 27 + 273 = 300$ K, $T_2 = 0 + 273 = 273$ K

Device : Heat pump, $Q_1 = 25$ kW $= 25 \times 10^3$ W

To find :

i) COP ii) W_{net}

Step - 1 : Calculate COP

$$COP = \frac{T_2}{T_1 - T_2}$$

$$\textbf{COP} = \frac{293}{300 - 273} = \textbf{10.11} \quad \textbf{... Ans.}$$

Step - 2 : Calculate work input

We have $Q_1 = 25 \times 10^3$ W

$$COP = \frac{Q_1}{Q_1 - Q_2} = \frac{Q}{W_{net}}$$

$$\therefore \quad \textbf{W}_{\textbf{net}} = \frac{Q_1}{COP} = \frac{25 \times 10^3}{10.11} = \textbf{2472.80 W} \quad \textbf{... Ans.}$$

3.20.2 Tons of Refrigeration

- The capacity of refrigeration system is given in **Tons of Refrigeration (T.R).**
- A ton of refrigeration is defined as the quantity of heat required to be removed, to form one ton of ice at 0 °C in 24 hours when initial condition of water is 0 °C because same cooling effect will be obtained by melting the same ice.
- S.I. unit of 1 ton of refrigeration is,

  1 TR = 12660 kJ/hr = 211 kJ/min

  or 1 TR = 3.517 kJ/sec = 3.517 kW

  $\therefore$ 1 TR = 3.517 kW
- TR actually measures the rate of heat transfer.

3.21 Modified Reversed Carnot Cycle / Bell Coleman Cycle

☞ [GTU : Summer-16]

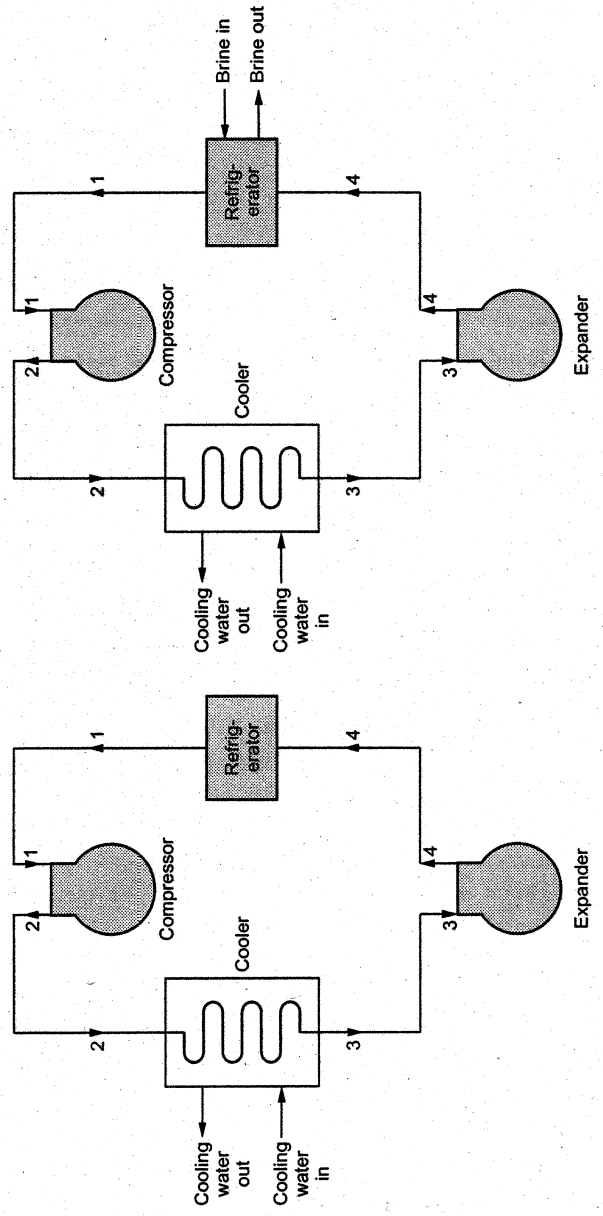


Fig. 3.17 Bell Coleman cycle

- Bell coleman cycle is also called as **Brayton or Joul cycle.**
- In this cycle the isothermal processes of carnot cycle are replaced by constant pressure process.
- This cycle cosists of compressor, coolar, expander and refrigerator.
- The four processes of this cycle are described below,

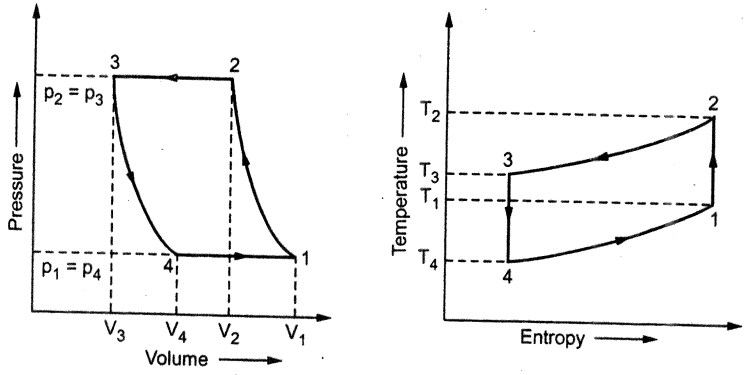


Fig. 3.18

Process 1-2 : Isentropic compression

- In this process, compression of air takes place isentropically.
- During this process no heat is added or removed.
- The pressure and temperature increases from p_1 to p_2 and T_1 to T_2.
- Where as the specific volume decreases from V_1 to V_2.

Process 2-3 : Constant pressure cooling process

- In this process hot air from the compressor is passed into cooler where it is cooled at constant pressure $(p_2 = p_3)$.
- During this process temperature is reduced to T_2 to T_3 and specific volume reduces from V_2 to V_3.
- Heat rejected during this process,

$$Q_R = C_p(T_2 - T_3)$$

Process 3-4 : Isentropic expansion process

- The cooled air from coolar is drawn into the expander for the expansion process.

- During this process pressure and temperature decreases from p_3 to p_4 and T_3 to T_4.
- Where as the specific volume increases from V_3 to V_4.

Process 4-1 : Constant pressure expansion process

- The cold air from expander is passes into the refrigerator. where it is expanded at constant pressure $(p_4 = p_1)$.
- The temperature of air increases from T_4 to T_1.
- Specific volume changes from V_4 to V_1.
- Heat absorbed during this process,

$$Q_A = C_p(T_1 - T_4)$$

- Work done by the cycle,

$$W = \text{Heat rejected} - \text{heat absorbed}$$

$$= C_p(T_2 - T_3) - C_p(T_1 - T_4)$$

- Coefficient of performance of cycle,

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$= \frac{C_p(T_1 - T_4)}{C_p(T_2 - T_3) - C_p(T_1 - T_4)}$$

$$COP = \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4\left(\frac{T_1}{T_4} - 1\right)}{T_3\left(\frac{T_2}{T_3} - 1\right) - T_4\left(\frac{T_1}{T_4} - 1\right)} \quad \ldots (3.1)$$

- Process 1-2 is isentropic process,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} \quad \ldots (3.2)$$

- Process 3-4 is isentropic expansion.

$$\frac{T_1}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots (3.3)$$

But, $p_2 = p_3$ and $p_1 = p_4$

We can write,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \text{ or } \frac{T_2}{T_3} = \frac{T_1}{T_4} \quad \ldots (3.4)$$

Put the above value in equation (3.1)

We get, $$COP = \frac{T_4}{T_3 - T_4}$$

$$COP = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$COP = \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$COP = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$\boxed{COP = \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}} - 1}}$$

... r_P - compression ratio or expansion ratio

- But sometimes compression and expansion processes are takes place as per the law PV^n = Constant.
- Work done by compressor during process 1-2

$$W_1 = \frac{n}{n-1}\left(p_2 V_2 - p_1 V_1\right)$$

$$= \frac{n}{n-1}\left(RT_2 - RT_1\right)$$

- Work done by expander during process 3-4.

$$W_1 = \frac{n}{n-1}\left(p_3 V_3 - p_4 V_4\right)$$

$$= \frac{n}{n-1}\left(RT_3 - RT_4\right)$$

- Net work done can be find out as,

$$W = \frac{n}{n-1} \times R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right] \quad \ldots (3.5)$$

$$COP = \frac{\text{Heat absorbed}}{\text{Work done}}$$

$$= \frac{C_p\left(T_1 - T_4\right)}{\frac{n}{n-1} \times R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]} \quad \ldots (3.6)$$

We know, $C_P - C_V = R = C_V(r - 1)$

Put value of R in equation (3.6)

$$COP = \frac{C_p\left(T_1 - T_4\right)}{\frac{n}{n-1} \times (\gamma - 1) C_v\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]}$$

$$COP = \frac{\gamma\left(T_1 - T_4\right)}{\frac{n}{n-1}(\gamma - 1)\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]}$$

$$\boxed{COP = \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{\gamma-1}{\gamma}\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]}} \quad \ldots (3.7)$$

For isentropic procss n = r. then equation (3.7) will become :

$$\boxed{COP = \frac{T_1 - T_4}{\left(T_2 - T_3\right) - \left(T_1 - T_4\right)}}$$

3.21.1 Advantages of Bell - Coleman Cycle

- It uses air as a working medium which is freely available.
- As the air is non flammable this system is safe.
- The weight of air refrigeration system per ton of refrigeration is low compared with other refrigeration system.

3.21.2 Disadvantages of Bell - Coleman Cycle

- COP of system is low as compare to VCC.
- For 1TR large volume of air is required which results in large size of compressor and expander.

3.21.3 Solved Examples on Bell - Coleman Cycle

Example 3.6 : *In refrigeration plant working on Bell - coleman cycle, air is compressed to 1 bar from 7 bar. Its initial temperature is 10 °C. After compression air is cooled upto 20 °C in cooler before expanding back to pressure of 1 bar. Determine theoretical COP of the plant and net refrigerating effect, Take* $C_P = 1.005\ kJ/kg\ K$ *and* $C_V = 0.718\ kJ/kg\ K$.

Solution : Given data :

$p_1 = 1$ bar, $p_2 = 7$ bar

$T_1 = 10\,°C = 10 + 273 = 283$ K

$T_3 = 20\,°C = 20 + 273 = 293$ K

$C_P = 1.005$ kJ/kg K

$C_P = 0.718$ kJ/kg K

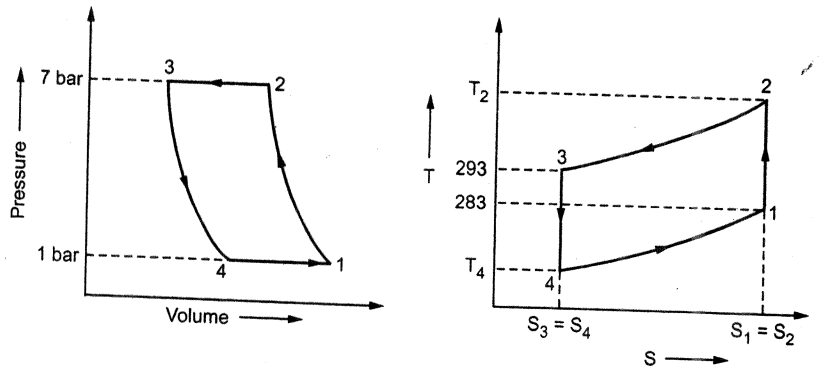


Fig. 3.19

We know,

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.4$$

The process 1-2 is isentropic compression process,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$T_2 = 283 \times \left(\frac{7}{1}\right)^{\frac{1.4 - 1}{1.4}}$$

$$T_2 = 493.44 \text{ K}$$

Process 3-4 is isentropic expansion

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{293}{T_4} = \left(\frac{7}{1}\right)^{\frac{1.4 - 1}{1.4}}$$

$$\frac{293}{T_4} = 1.743$$

$$\mathbf{T_4 = 168.03}$$

Theoretical COP of the plant,

$$\text{COP} = \frac{T_4}{T_3 - T_4} = \frac{168.03}{293 - 168.03} = 1.34$$

Net refrigerating effect,

$$\text{RE} = C_p \left(T_1 - T_4\right) = 1.005\ (283 - 168.03) = 115.54 \text{ kJ/kg}$$

Example 3.7 : *A refrigerator working on Bell - coleman cycle operates between pressure limit of 1.06 and 8.7 bar. Air is drawn from cold chamber at 10 °C. The air coming out of the compressor is cooled to 30 °C before entering to expansion cylinder. Expansion and compression follows the law* $PV^{1.33} = C$. *Determine COP of the system.* $[C_p = 1\ kJ/kg\ K]$ $[\gamma = 1.4]$ ☞ **[GTU : Summer-14, Winter-16]**

Solution : Given data :

$p_1 = 1{\cdot}06$ bar, $p_2 = 8{\cdot}7$ bar

$T_1 = 10\,°C = 10 + 273 = 283$ K

$T_3 = 30\,°C = 30 + 273 = 303$K

$C_p = 1$ kJ/kg K

$\gamma = 1.4$

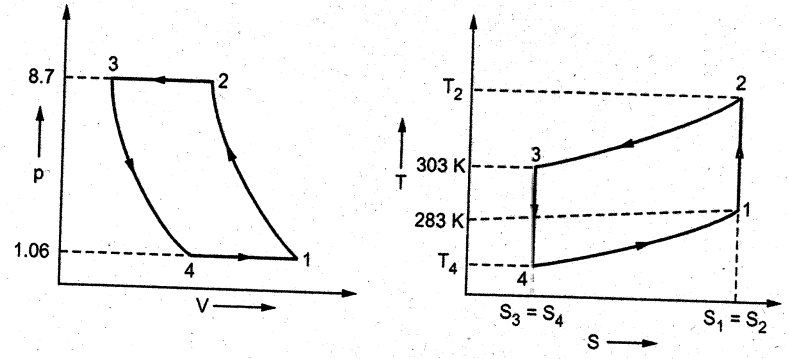


Fig. 3.20

The compression follows the law $pV^{1.33} = C$.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 283 \times \left(\frac{8{\cdot}7}{1{\cdot}06}\right)^{\frac{1{\cdot}33-1}{1{\cdot}33}}$$

$$T_2 = 477.11\ K$$

Similarly expansion follows the law $pV^{1.35} = C$.

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}$$

$$\frac{303}{T_4} = \left(\frac{8{\cdot}7}{1{\cdot}06}\right)^{\frac{1{\cdot}33-1}{1{\cdot}33}}$$

$$\frac{303}{T_4} = 1.685$$

$$\mathbf{T_4 = 179.72\ K}$$

To find out net work done, (W_{net})

$$W_{net} = W_C - W_E$$

$$W_{net} = \frac{n}{n-1}\left(p_2V_2 - p_1V_1\right) - \frac{n}{n-1}\left(p_3V_3 - p_4V_4\right)$$

$$= \frac{n}{n-1} R\left[\left(T_2 - T_1\right) - \left(T_3 - T_4\right)\right]$$

To find value of 'R'

$$R = C_p - C_v \text{ and } \gamma = \frac{C_P}{C_v}$$

$$C_v = \frac{C_p}{\gamma} = \frac{1}{1.4} = 0.70\ kJ/kg\ K$$

$$R = C_p - C_v = 1 - 0.70 = 0.3\ kJ/kg\ K$$

Now,

$$W_{net} = \frac{1{\cdot}33}{1{\cdot}33-1} \times 0{\cdot}3\left[\left(477{\cdot}11 - 283\right) - \left(303 - 179{\cdot}72\right)\right]$$

$$= 85.63\ kJ/kg\ K$$

$$COP = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{C_p\left(T_1 - T_4\right)}{(W)_{net}} = \frac{1\left(283 - 179{\cdot}72\right)}{85{\cdot}63}$$

$$= 1.20$$

Example 3.8 : *An air refrigerator works between pressure limit of 1 bar and 5 bar. The temperature of air entering the compressor is 15 °C and entering the expansion cylinder is 30 °C*

The expansion follows the law $p V^{1.25} = C$

The compression follows the law $p V^{1.35} = C$

Take $C_p = 1\ kJ/kg\ K$ *and* $C_v = 0.7 kJ/kg\ K$ *find.*

i) COP of the cycle

ii) If air circulation through system is 25 kg/min find refrigerating capacity of system.

☞ **[GTU : Summer-14, 16]**

Solution : Given data :

$p_1 = 1$ bar $p_2 = 5$ bar

$T_1 = 15\,°C = 15 + 273 = 288$ K

$T_3 = 30\,°C = 30 + 273 = 303$ K

$C_p = 1$ kJ/kg K $C_v = 0{\cdot}7$ kJ/kg K

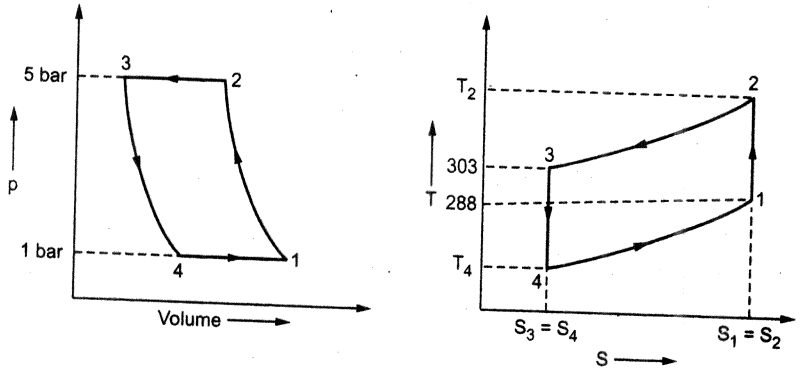


Fig. 3.21

Compression follows the law $pV^{1{\cdot}35} = C$.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1{\cdot}35 - 1}{1{\cdot}35}}$$

$$T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{1{\cdot}35 - 1}{1{\cdot}35}}$$

$$T_2 = 288 \times \left(\frac{5}{1}\right)^{\frac{1{\cdot}35 - 1}{1{\cdot}35}}$$

$$T_2 = 437.12\ K$$

Expansion follows the law $pV^{1{\cdot}25} = C$.

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{1{\cdot}25 - 1}{1{\cdot}25}}$$

$$\frac{303}{T_4} = \left(\frac{5}{1}\right)^{\frac{1{\cdot}25 - 1}{1{\cdot}25}}$$

$$T_4 = 219.60\ K$$

Now, To find out the refrigerating effect.

$$R.E. = C_p \left(T_1 - T_4\right) = 1\ (288 - 219{\cdot}60) = 68.39\ kJ/kg$$

$$R = C_p - C_v = 1 - 0{\cdot}7 = 0.3\ kJ/kg\ K$$

To find out net work done,

$$W_{net} = \left[\frac{1{\cdot}35}{1{\cdot}35 - 1} \times R\left(T_2 - T_1\right) - \frac{1{\cdot}25}{1{\cdot}25 - 1} \times R\left(T_3 - T_4\right)\right]$$

$$= R\left[\frac{1{\cdot}35}{0{\cdot}35}\left(T_2 - T_1\right) - \frac{1{\cdot}25}{0{\cdot}25}\left(T_3 - T_4\right)\right]$$

$$= 0.3\left[\frac{1{\cdot}35}{0{\cdot}35}\left(437{\cdot}12 - 288\right) - \frac{1{\cdot}25}{0{\cdot}25}\left(303 - 219{\cdot}60\right)\right]$$

$$= 47.45\ kJ/kg$$

$$COP = \frac{R.E.}{W_{net}} = \frac{68{\cdot}39}{47{\cdot}45} = 1.44$$

$$\text{Capacity of referigerating system} = \frac{RE \times m_a\ (kg/sec)}{3.5} = \frac{68{\cdot}39 \times \frac{25}{60}}{3.5} = 8.14 \text{ tons.}$$

Example 3.9 : *5 TR refrigerating machine based on Bell coleman cycle operates between 5 bar and 20 bar. The air temperature after heat rejection to the surrounding is 37 °C and air temperature at compressor inlet is 7 °C. The η_C and η_E = 48 % respectively determine i) COP, ii) Power per TR, iii) Mass of circulation of refrigerant in kg/hr, iv) carnot COP for same entry temperature to compressor and expander. (Take $C_p = 1\ kJ/kg\ K$, $C_v = 0.718\ kJ/kg\ K$).*

Solution : Given data :

Refrigerating capacity = 5 TR = 5 × 3·52 = 17·6 kW

$p_1 = 5$ bar $\quad p_2 = 20$ bar

$T_3 = 37\,°C = 37 + 273 = 310$ K

$T_1 = 7\,°C = 7 + 273 = 280$ K

$\eta_C = \eta_E = 48\ \%$

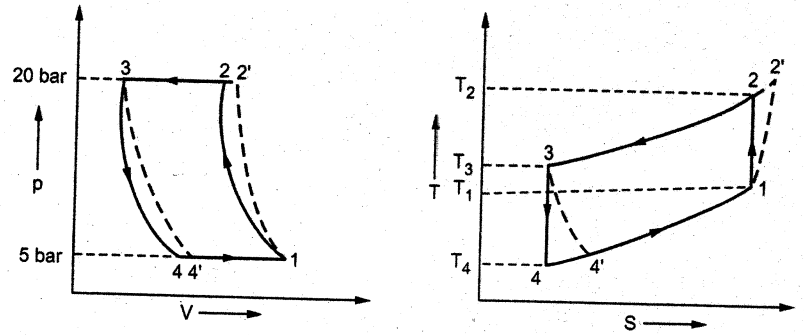


Fig. 3.22

We know process 1-2 is isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{280} = \left(\frac{20}{5}\right)^{\frac{1.4 - 1}{1.4}}$$

$$\mathbf{T_2 = 416 \cdot 82\ K}$$

Similarly process 3-4 is isentropic expansion

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{310}{T_4} = \left(\frac{20}{5}\right)^{\frac{1.4 - 1}{1.4}}$$

$$\frac{310}{T_4} = 1.488$$

$$\mathbf{T_4 = 208 \cdot 24\ K}$$

But, isentropic efficiency of compressor,

$$\eta_C = \frac{T_2 - T_1}{T_2' - T_1}$$

$$\therefore \quad 0 \cdot 48 = \frac{416 \cdot 82 - 280}{T_2' - 280}$$

$$\mathbf{T_2' = 565 \cdot 04\ K}$$

Similarly isentropic efficiency of expander

$$\eta_E = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$\therefore \quad 0 \cdot 48 = \frac{310 - T_4'}{310 - 208 \cdot 24}$$

$$\mathbf{T_4' = 261 \cdot 15\ K}$$

Net work done by the system,

$$(W)_{net} = C_P \left[(T_2' - T_1) - (T_3 - T_4')\right]$$

$$= 1\left[(565 \cdot 04 - 280) - (310 - 261 \cdot 15)\right]$$

$$= 236 \cdot 19\ kJ/kg$$

Refrigerating effect,

$$R.E. = C_p \left(T_1 - T_4'\right)$$

$$= 1\left[280 - 261 \cdot 15\right]$$

$$= 18 \cdot 85\ kJ/kg$$

$$COP = \frac{R \cdot E}{W \cdot D} = \frac{18 \cdot 85}{236 \cdot 19}$$

$$= \mathbf{0 \cdot 079}$$

Refrigerating capacity = $\dot{m}_r \times$ R. E.

$$17 \cdot 6 = \dot{m}_r \times 18 \cdot 85$$

$$\text{Power per TR} = \frac{\dot{m}_r \times \text{Work done}}{\text{Refrigerating capacity}}$$

$$= \frac{0 \cdot 93 \times 236 \cdot 19}{5}$$

$$= 44 \cdot 10 \text{ kW / TR}$$

$$(COP)_{carnot} = \frac{T_1}{T_2 - T_1}$$

$$\therefore \quad = \frac{280}{416 \cdot 82 - 280} = \mathbf{2 \cdot 046}$$

Example 3.10 : *A refrigeration system based on Bell - coleman cycle working between the pressure limit of 4 bar and 16 bar extracts 125 M J/hr. The air enters the compressor at 5 °C and enters the expander at 23 °C. The compressor and expander mechanical efficiencies are 82 % and 87 % respectively determine.*

i) Power required to run the unit.

ii) Mass circulation rate of air.

iii) COP

iv) carnot COP, Take $C_p = 1 \cdot 005$ *kJ/kg K*

$C_v = 0.718$ *kJ/kg K).*

Solution : Given data :

$p_1 = 4$ bar, $p_2 = 16$ bar

$T_1 = 5\,°C = 5 + 273 = 278$ K

$T_3 = 23\,°C = 23 + 273 = 296$ K

Heat extract = 125 MJ/hr

$$= \frac{125 \times 10^6}{3600} = 34722 \cdot 2 \text{ J/sec}$$

$$= 34 \cdot 72 \text{ kJ/sec}$$

We have given values of C_p and C_v

$$\therefore \quad \gamma = \frac{C_p}{C_r} = \frac{1 \cdot 005}{0 \cdot 718} = 1 \cdot 399 \approx 1 \cdot 4$$

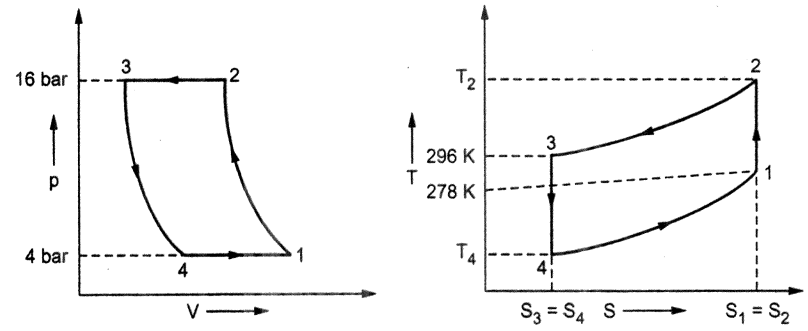


Fig. 3.23

Process 1-2 is isentropic compression,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{T_2}{278} = \left(\frac{16}{4}\right)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}}$$

$$\mathbf{T_2 = 413 \cdot 84 \text{ K}}$$

Similarly, process 3-4 is isentropic expansion,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\frac{296}{T_4} = \left(\frac{16}{4}\right)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}}$$

$$\mathbf{T_4 = 198 \cdot 84 \text{ K}}$$

Net work done,

$$(W)_{net} = C_p \left[(T_2 - T_1) - (T_3 - T_4)\right]$$

$$= 1 \cdot 005 \left[(413 \cdot 84 - 278) - (296 - 198 \cdot 84)\right] = 38 \cdot 87 \text{ kJ/kg}$$

$$\text{R.E.} = C_p (T_1 - T_4)$$

$$= 1 \cdot 005 \left(278 - 198 \cdot 84\right) = 79 \cdot 55 \text{ kJ/kg}$$

$$\text{COP} = \frac{R \cdot E}{W \cdot D} = \frac{79 \cdot 55}{38 \cdot 87} = 2 \cdot 04$$

$$\text{carnot COP} = \frac{T_1}{T_2 - T_1} = \frac{278}{413 \cdot 84 - 278} = \mathbf{2 \cdot 04}$$

To find out mass of refrigerant ($\dot{m}_r$)

$$\dot{m}_r \times \text{R. E.} = \text{Heat extracted}$$

$$\dot{m}_r \times 79 \cdot 55 = 34 \cdot 72$$

$$\dot{m}_r = 0 \cdot 436 \text{ kg/sec.}$$

Power required to run the unit,

$$P = (P)_{\text{compressor}} - (P)_{\text{Expander}}$$

$$= \left[\frac{\dot{m}_r C_P \left(T_2 - T_1\right)}{\eta_{\text{mech.comp.}}} - \frac{\dot{m}_r C_P \left(T_3 - T_4\right)}{\eta_{\text{mech.Exp.}}}\right]$$

$$= \left[\frac{0 \cdot 436 \times 1 \cdot 005 \times \left(413 \cdot 84 - 278\right)}{0 \cdot 82}\right] - \left[\frac{0 \cdot 436 \times 1 \cdot 005 \times \left(296 - 198 \cdot 84\right)}{0 \cdot 87}\right]$$

$$= 72 \cdot 58 - 48 \cdot 93 = \mathbf{23 \cdot 64 \text{ kW}}$$

Vapour Compression Cycle

3.22 Vapour Compression Cycle (VCC)

- The first Vapour Compression System (VCC) was developed in 1834 by Jacob Peskins.
- For domestic and commercial utilities point of view vapour compression system is most important.
- In vapour compression system, vapour is used as working fluid.
- Vapour compression system is most practical form of refrigeration.
- In vapour compression system, ammonia (NH_3), carbon dioxide (CO_2), sulphur dioxide (SO_2) are most commonly used refrigerants.
- Vapour compression system is used for all industrial purposes from small domestic refrigerator to big air conditioning plants.

3.23 Simple Vapour Compression System

- Fig. 3.24 shows the vapour compression system.

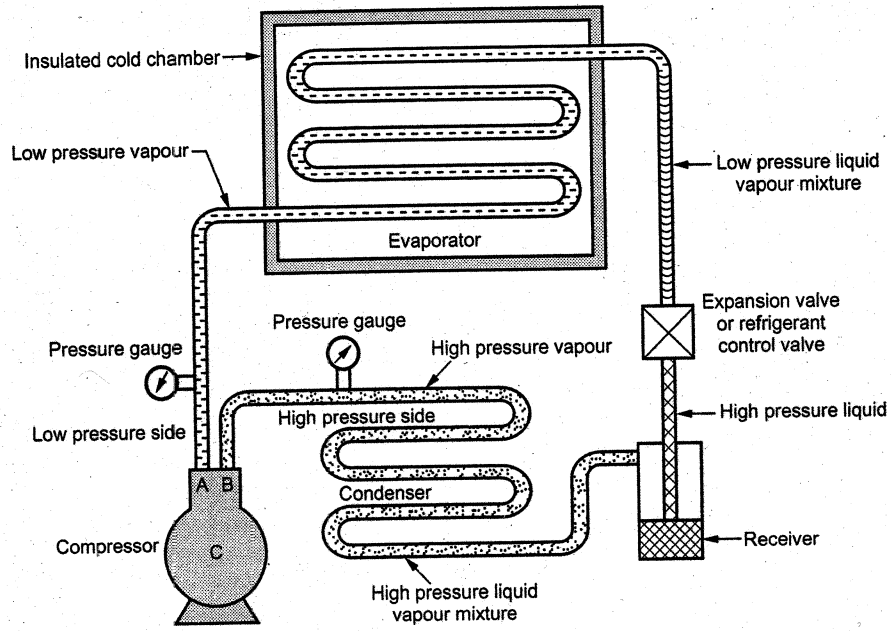


Fig. 3.24 Simple VCC

- It mainly consists of four major components
  - i) Compressor : Compression will takes place.
  - ii) Condensor : Condensation will takes place.
  - iii) Expansion device : Expansion will takes place.
  - iv) Evaporator : Evaporation will takes place.

i) Compressor :

- The low pressure low temperature vapour refrigerant from the evaporator enters into the compressor during suction stroke.
- In the compressor, compression will takes place i.e. low pressure low temperature vapour refrigerants are compressed to high pressure and high temperature.
- This compression can be considered as isentropic compression.

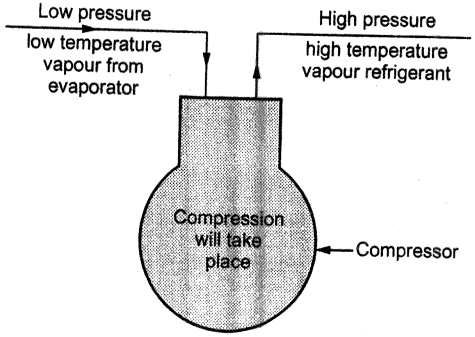


Fig. 3.25 Mechanism of compression

- At the outlet of compressor we get high pressure and high temperature vapour refrigerant.

ii) Condenser :

- High pressure, high temperature vapour refrigerant from the compressor enters into condenser where the condensation will takes place.
- In the condenser, change of phase of refrigerant will takes place. i.e. vapour refrigerants are converted into liquid refrigerant by giving latent heat to the surrounding.
- Generally condensing medium is air or water.
- During the condensation process temperature will remain constant because of latent heating.
- The liquid refrigerant from the outlet of the condenser is stored in the vessel known as receiver.

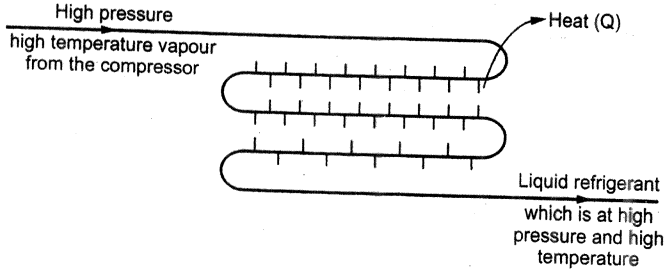


Fig. 3.26 Mechanism of condensation

iii) Expansion device/valve :

- The expansion valve is also called as throttle valve where the expansion will takes place.
- The liquid refrigerant which is at high pressure and high temperature passes through the expansion valve.
- The throttling process is pressure relieving process. During this process enthalpy will remain constant.

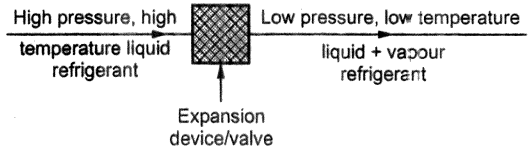


Fig. 3.27 Mechanism of expension

- Some of the liquid refrigerant evaporates as it passes through the expansionn value and hence at the outlet of expansion valve we get low pressure low temperatue vapour + liquid (mixture) refrigerants.

iv) Evaporator

- In the evaporator evaporation will takes place.
- Low pressure, low temperature liquid+vapour refrigerant enters into evaporator where it gets evaporated.
- Finaly from the outlet of the evaporator we will get low pressure, low temperature vapour refrigerants.
- In the evaporating, the liquid + vapour refrigerant absorbs latent heat of vaporisation from the medium (air, water) which is to be cooled.
- Hence we can say during evaporation heat is added and during condensation heat is rejected.

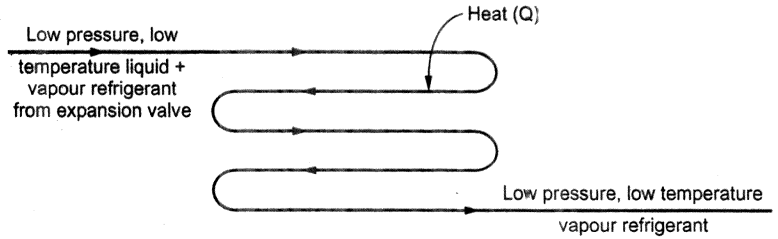


Fig. 3.28 Mechanism of evaporation

3.24 Cases of Vapour Compression System with p-h and T-S Diagram

3.24.1 Theoretical Vapour Compression System with dry Saturated Vapour after Compression

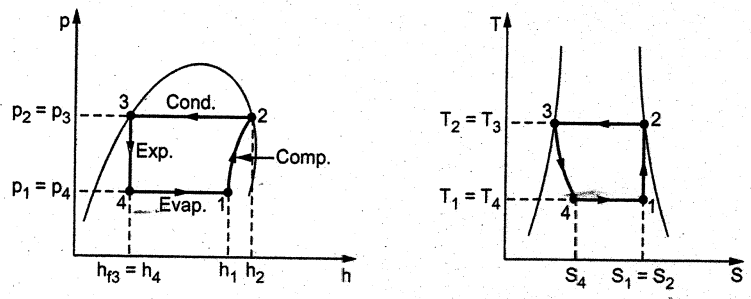


Fig. 3.29 p-h and T- S diagram for dry vapour after compression

- **Processes 1-2** is compression process.
- Initially vapour refrigerants are at low pressure (p_1) and low temperature (T_1) gets compressed isentropically to a high pressure (p_2) and high temperature (T_2).
- The compression is isentropic the entropy during compression remains constan as shown in T-S diagram i.e. ($S_1 = S_2$).
- Work done during isentropic compression per kg of refrigerant is given by

$$\boxed{W_c = h_2 - h_1}$$

Where,

h_1 - Enthalpy of refrigerrant at temperature T_1

h_2 - Enthalpy of refrigerrant at temperature T_2

- **Process 2-3** is condensing process.
- High pressure and high temperature vapour refrigerant gets condensed in condenser.
- During this process latent heat is given out to the surrounding and hence vapour refrigerant gets converted into liquid refrigerant i.e. change of phase of refrigerant takes place.
- From p-h and T-S diagram we can say during this process pressure and temperature remains constant i.e. $p_2 = p_3$ and $T_2 = T_3$.

- **Process 3-4** is an expansion process.
- Liquid refrigerant which is at pressure $p_3 = p_2$ and temperature $T_3 = T_2$ is expanded in expansion valve to low pressure ($p_4 = p_1$) and low temperature ($T_4 = T_1$).
- During throttling process no heat is added or rejected by liquid refrigerant.
- **Process 4-1** is an evaporation process.
- In evaporation liquid-vapour mixture of refrigerant is evaporated and changed into low pressure, low temperature vapour refrigerant.
- During this process heat which is absorbed by refrigerants is called as refrigerating effect. The process of evaporation continues upto point 1 which is the suetion of compressor.
- Heat absorbed or refrigerating effect can be calculated as

$$\boxed{R_E = h_1 - h_4} \text{ or } \boxed{R_E = h_1 - h_{f3}}$$

where, h_{f3} = Sensible heat at temperature T_3.

Calculation of coefficient of performance (COP)

- COP is the ratio of refrigerating effect to the work done.

$$\text{C.O.P.} = \frac{\text{Refrigerating effect (RE)}}{\text{Work done}}$$

$$\boxed{\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- The ratio of C.O.P. of vapour compression cycle to C.O.P. of Carnot cycle is known as performance index (P.I.) or refrigeration efficiency (η_R)

Procedure to solve the numerical on dry compression

- In this type the point '1' is inside the saturation curve. So it is required to find enthalpy at point 1.

$$\therefore \quad h_1 = h_{f1} + x_1 h_{fg1}$$

In some cases value of latent heat (h_{fg}) is not given then we can use the following relation.

$$\therefore \quad h_{fg1} = h_{g1} - h_{f1}$$

- The same thing is appplicable to find the entropy

$$\therefore \quad S_1 = s_{f1} + x_1 s_{fg1}$$

and $S_{f_{g1}} = s_{g1} - S_{f1}$

- COP can be found out,

$$COP = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

3.24.2 Solved Examples

Example 3.11 : *Vapour compression refrigerator works between the pressure limit of 60 bar and 25 bar. The working fluid is just dry at the end of the compression and there is no under cooling of liquid before the expansion valve Determine - i) COP of the cycle and ii) Capaciy of the refrigerator if the fluid flow is at rate of 7 kg/min.*

☞ **[GTU : Summer-18, 16]**

Pressure (bar)	Temp (K)	Enthalpy (kg/kg)		Entropy (kg/kgK)	
		Liquid	Vapour	Liquid	Vapour
60	295	151.96	293.29	0.554	1.0332
25	261	56.32	332.58	0.226	1.2464

Solution : Given data

$p_2 = p_3 = 60$ bar $\quad h_{g_1} = 322.58$ kg / kg

$S_{g_1} = 1.2464$ kg/kg K $\quad p_1 = p_4 = 25$ bar

$h_{g_2} = h_2 = 293.39$ kg / kg $\quad S_2 = S_{g2} = 1.0332$ kg/kg K

$T_2 = T_3 = 295$ K $\quad h_{f3} = h_4 = 151.96$ kg / kg

$S_{f1} = 0.226$ kg/kg K $\quad T_1 = T_4 = 261$ K

$h_{f1} = 56.32$ kg / kg

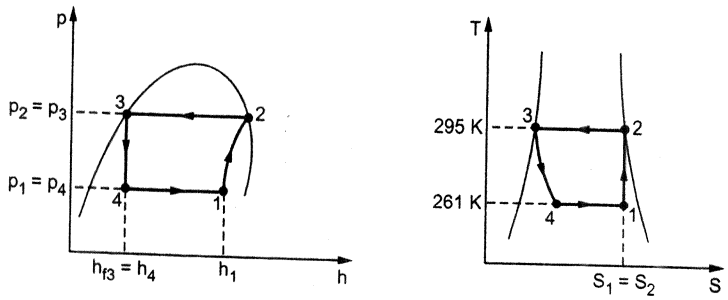


Fig. 3.30

- To find entropy at point '1'

$$S_1 = S_{f1} + x_1 S_{f_{g1}}$$

$$S_1 = S_{f1} + x_1 (S_{g_1} - S_{f1}) \quad \ldots S_{f_{g1}} = S_{g1} - S_{f1}$$

$$= 0.226 + x_1 (1.2464 - 0.226)$$

$$S_1 = 0.226 + 1.0204\, x_1 \quad \ldots(1)$$

- Entropy at point '2'

$$S_2 = S_{g2}$$

But process 1-2 is isentropic compression

$\therefore \quad S_1 = S_2$

Putting value of S_{g_2} in equation (1).

$$1.0332 = 0.226 + 1.0204\, x_1$$

$\therefore \quad \mathbf{x_1 = 0.791}$

- Now to find enthalpy at point '1'.

$$h_1 = h_{f1} + x_1 h_{f_{g1}}$$

$$h_1 = h_{f1} + x_1 (h_{g1} - h_{f1}) \quad \ldots h_{f_{g1}} = h_{g1} - h_{f1}$$

$$h_1 = 56.32 + 0.791\ (332.58 - 56.32)$$

$$\mathbf{h_1 = 266.93\ kJ/kg}$$

- COP of the cycle,

$$COP = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

$$= \frac{266.93 - 151.96}{293.29 - 266.93}$$

$$= \mathbf{4.36}$$

- Capacity of refrigerator,

$$R.E. = h_1 - h_{f3}$$

$$= 266.93 - 151.96$$

$$= \mathbf{114.97\ kg/kg}$$

We have given fluid flow rate 7 kg/min

$\therefore$ Heat extracted = m × R.E. = 7 × 114.97 = **804.79 kg/min**

$\therefore$ Capacity $= \frac{804.79}{210} =$ **3.83 TR**

Example 3.12 : *An ammonia refrigerator produces 30 tones of ice at 0 °C from water at 20 °C in a day. The temperature range of working cycle is 25 °C to –15 °C. The vapour is dry saturated at the end of compression. Assuming actual COP of 60 % of theoretical. Calculate the power required to drive the compressor. Latent heat of ice is 335 kJ/kg. Use the following properties of ammonia.*

Temperature	Enthalpy kJ/kg		Entropy of	
	Liquid	Vapour	Liquid	Vapour
25	100.04	1319.22	0.3473	4.4852
– 15 °C	– 54.58	1304.99	– 2.134	5.0585

Solution : Process 1 - 2 is isentropic compression

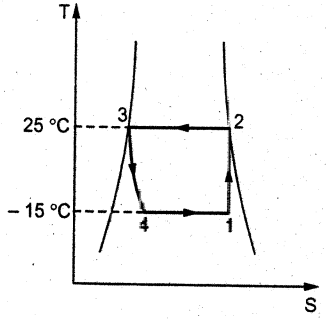


Fig. 3.31

$\therefore$ $S_1 = S_2$

$\therefore$ $S_{f1} + x_1 S_{fg1} = S_{g2}$

But, $S_{fg1} = (S_{g1} - S_{f1})$

$\therefore$ $S_{f1} + x_1(s_{g1} - S_{f1}) = S_{g2}$

$\therefore$ $-2.134 + x_1(5.0585 - 2.134) = 4.4352$

$\therefore$ $x_1 =$ **0.92**

Now, $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$

$= -54.58 + 0.92\ [(1304.99 - (-54.58)]$

$=$ **1196.43 kJ/kg**

$h_2 = h_{g2} = 1319.22$ kJ/kg

$h_3 = h_{f3} = 100.04$

Process 3 - 4 is throttling process

$\therefore$ $h_3 = h_4$

R.E. $= h_1 - h_4$

$= 1196.43 - 100.04$

$= 1096.18$ kJ/kg

Compressor work,

$W_C = h_2 - h_1$

$= 1319.22 - 1196.43$

$= 122.79$ kJ/kg

$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Work}}$

$= \frac{1096.18}{122.79} = 8.92$

Actual COP $= 0.6 \times 8.92$

$= 5.356$

Heat extracting per kg of ice,

$= C_{PW}(20 - 0) + \text{Latent heat of ice}$

$= 4.187\ (20) + 335$

$= 418.74$ kJ/kg of ice

Mass of ice/sec $= \frac{30 \times 1000}{24 \times 60 \times 60}$

$= 0.3472$ kg/sec

Actual heat extracted /sec

$$= 418.74 \times 0.7472$$

$$= 145.39 \text{ kJ/sec}$$

$$\text{Actual COP} = \frac{\text{Actual heat extracted / sec}}{\text{Actual work / sec}}$$

$$\text{Actual work/sec} = \frac{145.39}{5.348}$$

$$\text{Work/sec} = 27.18 \text{ kW}$$

$$\text{Power} = 27.18 \text{ kW}$$

3.25 Theoretical Vapour Compression System with Wet Vapour after Compression

- In case of wet compressions the point 2 is inside the saturation curve. (Refer Fig. 3.32 below)

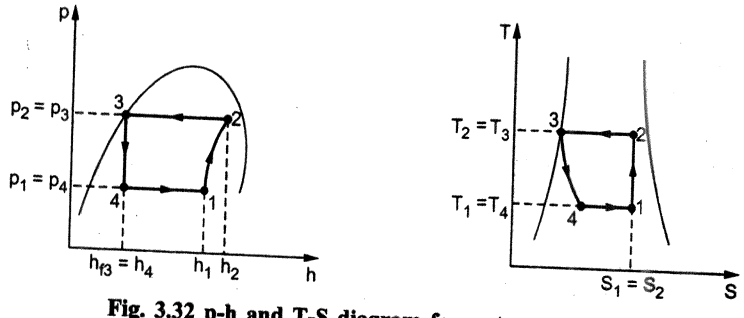


Fig. 3.32 p-h and T-S diagram for wet compression

- The wet compression is not desirable as liquid droplets in the refrigerant would enter the compressor and damage the valve and other moving parts.
- Wet compression also reduces the heat transfer rate of condenser.
- The enthalpy at point 2 i.e. h_2 can be found out with the help of dryness fraction at that point.
- COP can be found out by following formula.

$$\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

- **Procedure to solve the numerical on wet compression.**

The wet compression point '1' and point '2' are inside the saturation curve.

To find entropy at point 1 and 2, following equations are used.

$$S_1 = S_{f1} + \frac{x_1 h_{fg1}}{T_1} \quad \text{...(i)}$$

Similarly $$S_2 = S_{f2} + \frac{x_2 h_{fg2}}{T_2} \quad \text{..(ii)}$$

In some cases we have to find dryness fraction x_1 or x_2, in such time we can equate two equations (i) and (ii) as process 1-2 is isentropic compression i.e. $S_1 = S_2$.

- To find enthaly at point '1' and at point '2'

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$h_1 = h_{f1} + x_1 (h_{g1} - h_{f1})$$

Similarly $$h_2 = h_{f2} + x_2 h_{fg2}$$

$$h_2 = h_{f2} + x_2 (h_{g2} - h_{f2})$$

- To calculate COP,

$$\text{COP} = \frac{h_1 - f_{f3}}{h_2 - h_1}$$

3.25.1 Solved Examples

Example 3.13 : *Determine the theoretical COP for CO_2 machine working between the temperature range of 25 °C and –5 °C. The dryness fraction if CO_2 gas during the suction stroke is 0.6. How many tonnes of ice would machine working between the same limits and having relative COP of 45 % make in 24 hrs ? The water for the ice is supplied at 15 °C and compressor takes 9 kg/min of CO_2. Take c_{pw} = 4.18 kg/kg °C and latent heat of ice 335 kg/kg.*

Temp.	Heat kg/kg		Latent heat	Entropy (kg/kgK)	
(°C)	Liquid	Vapour	(kJ/kg)	Liquid	Vapor
25	81.17	202.5	121.34	0.251	0.644
–5	–7.53	236.8	245.2	– 0.042	0.841

Solution : Given data :

$T_1 = T_4 = -5\,°C = 268\ K$ $\quad h_{f1} = -7.53\ kJ/kg$

$T_3 = T_2 = 25\,°C = 298\ K$ $\quad h_{fg1} = 245.2\ kJ/kg$

$x_1 = 0.6$ $\quad S_{f1} = -0.042\ kJ/kg\ K$

$(COP)_{relative} = 45\ \%$ $\quad S_{f2} = 0.251\ kJ/kg\ K$

$m = 9\ kg/min$ $\quad h_{f2} = 81.17\ kJ/kg$

$C_{p_w} = 4.18\ kJ/kg\ °C$ $\quad h_{f2} = 121.34\ kJ/kg$

$(h_{fg})_{ice} = 335\ kJ/kg$

- Entropy at point '1' = Entropy at point '2'

$$S_1 = S_2$$

$\therefore$ We can write,

$$S_{f1} + \frac{x_1 h_{fg1}}{T_1} = S_{f2} + \frac{x_2 h_{fg2}}{T_2}$$

$$-0.042 + \frac{0.6 \times 245.2}{(268)} = 0.251 + \frac{x_2 \times 121.34}{(298)}$$

$$\therefore \quad x_2 = \frac{(0.507 - 0.251)}{0.407}$$

$$\mathbf{x_2 = 0.63}$$

- To find out enthalpy at point '1' and '2'.

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$= -7.53 + (0.6 \times 245.2)$$

$$\mathbf{h_1 = 139.6\ kJ/kg}$$

Similarly $\quad h_2 = h_{f2} + x_2 h_{fg2}$

$$= 81.17 + (0.63 \times 121.34)$$

$$\mathbf{h_2 = 157.6\ kJ/kg}$$

- Theoretical COP

$$COP = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

$$= \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

$$= \frac{139.6 - 81.17}{157.6 - 139.6}$$

$$= \mathbf{3.24}$$

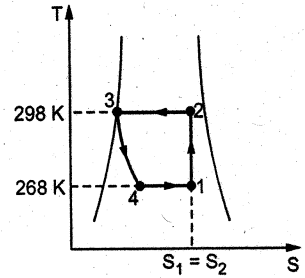


Fig. 3.33

To find out relative COP

$$(COP)_{relaive} = \frac{(COP)_{actual}}{(COP)_{theoretical}}$$

$$0.45 = \frac{(COP)_{actual}}{3.24}$$

$$\therefore \quad (COP)_{actual} = 0.45 \times 3.24$$

$$= \mathbf{1.458}$$

Heat extracted per minute $= (COP)_{actual} \times [m(h_2 - h_1)]$

$$= 1.458 \times [9(157.6 - 139.6)]$$

$$= \mathbf{236.19\ kJ/min}$$

Heat extracted per day $= 236.19 \times 60 \times 24$

$$= 340122.2\ kJ/day$$

Heat extracted per kg of water $= 1 \times C_{p_w}(15 - 0) + (h_{fg})_{ice}$

$$= 1 \times 4.18(15) + 335 = \mathbf{337.7\ kJ/kg}$$

Mass of ice per day $= \frac{340122.2}{397.7 \times 1000}$

$$= \mathbf{0.85\ tonne}$$

Example 3.14 : *A refrigerating plant works between temperature limits of – 5 °C and 25 °C. The working fluid is ammonia with dryness fraction of 0.62 at the entry to compressor. If the machine has a relative COP of 55 % calculate the amount of ice formed during a period of*
24 hrs. The ice is to be formed at 0 °C from water at
15 °C and 6.4 kg of ammonia is circulated per minute. Take specific heat of water = 4.187 kJ/kg K and latent heat of ice = 335 kJ/kg. Show process on T-S chart.

☞ **[GTU : Winter-17]**

Temperature	Enthalpy kJ/kg		Entropy kJ/kg K	
	h_f	h_g	S_k	S_g
25 °C	298.8	1166.15	1.07345	3.9121
– 5 °C	158.5	1279.85	0.6298	4.7738

Solution : $x_1 = 0.62$

Now we have to find condition of vapour at point 2

$$S_{f2} + x_2 S_{fg2} = S_{f1} + x_1 S_{fg1}$$

$$1.07842 + x_2 \times 3.9121 = 0.62985 + 0.62 \times 4.77385$$

$$\mathbf{x_2 = 0.64}$$

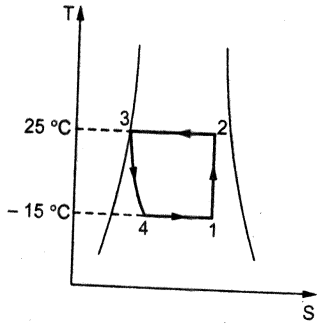


Fig. 3.34

Enthalpy at point (1) and (2) can be calculated as

$$h_1 = h_{f1} + x_1 h_{fg1}$$

$$= 158.5 + 0.62 \times 1279.85$$

$$= 952.01 \text{ kJ/kg}$$

$$h_2 = 298.8 + 0.64 \times 1166.15$$

$$= 1045.13 \text{ kJ/kg}$$

$$h_3 = 298.8 \text{ kJ/kg}$$

$h_3 = h_4$ ... throttling process enthalpy remains same

Refrigerating effect can be calculated as,

$$\text{R.E.} = h_1 - h_4$$

$$= 952.01 - 298.8$$

$$= 653.21 \text{ kJ/kg}$$

Work of compressor

$$W_C = h_2 - h_1$$

$$= 1045.13 - 952.01$$

$$= 93.126 \text{ kJ/kg}$$

$$\text{Ideal COP} = \frac{\text{R.E.}}{\text{Work done}}$$

$$= \frac{653.21}{93.12} = 7.01$$

Actual COP = $0.55 \times 7.01 = 3.85$

Heat extracted/kg of ice

$$= C_{Pw}(15 - 0) + \text{Latent heat of ice}$$

$$= 4.187\ (15) + 335$$

$$= 398.8 \text{ kJ/kg of ice}$$

Mass of refrigerant per second $= \frac{6.4}{60} = 0.106$ kg/sec

Heat extracted per second = R.E. × Mass flow rate

$$= 653.21 \times 0.106$$

$$= 69.67 \text{ kJ/sec}$$

Mass of ice produced/second $= \frac{69.67}{398.8} = 0.174$ kg/sec

Ice produced in 24 hrs. $= \frac{0.1747 \times 24 \times 3600}{1000} = 15$ tons

3.26 Superheated Vapour after Compression

- In this cycle enthalpy at point 2 can be found out with the help of degree of superheat and by using following relation. (Refer Fig. 3.35)

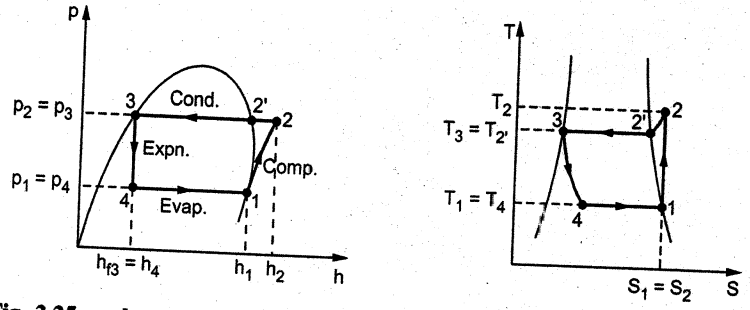


Fig. 3.35 : p-h and T-S diagram for superheated vapour after compression

$$h_2 = h_2' + C_p \times (\text{Degree of superheat})$$

$$\boxed{h_2 = h_2' + C_p \times (T_2 - T_2')}$$

- In superheating process increase in refrigerating effect is less as compared to work done in compressor.
- Hence net effect of superheating is to have low coefficient of performance.
- COP can be found out by using following relation.

$$COP = \frac{\text{Refrigerating effect}}{\text{Workdone}}$$

$$\boxed{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve the numericals on superheating**

i) In the numericals of suprheating we have to find the enthalpy at point '2'. It can be found out by using relation

$$h_2 = h_2' + C_p(T_2 - T_2')$$

ii) But in some cases, it is required to find out temperature at point (T_2). It can be found out by using relation

$$S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

In some cases we have to find out the value of C_p (when T_2 and T_2' is given in numerical)

iii) To find out the mass flow rate

$$m = \frac{Q}{h_2 - h_{f3}} \quad \text{OR} \quad m = \frac{Q}{h_1 - h_{f3}}$$

iv) Theoretical piston displacement of compressor = $m \times v_1$

v) Theoretical power of compressor

$$P = m(h_2 - h_1)\text{kW}$$

vi) Refrigerating effect

$$\text{R.E.} = h_1 - h_{f3}$$

vii) $$COP = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

3.26.1 Solved Examples

Example 3.15 : *A VCR uses methyl chloride (R-40) and operates between temp limits of – 10 °C and 45 °C at the entry of compressor the refrigerent is dry saturated and after compression it acquires a temp of 70 °C find the C.O.P. of refrigerator.*

☞ **[GTU : Summer-14, 16]**

Sat. temp.	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
In °C	Liquid	Vapour	Liquid	Vapour
– 10	45.4	460.7	0.183	1.637
45	133.0	483.6	0.485	1.587

Solution : (Refer Fig. 3.36)

$h_1 = 460.7$ kJ/kg $S_1 = S_2 = 1.637$ kJ/kg K

$h_2' = 483.6$ kJ/kg $S_2' = 1.587$ kJ/kg K

$h_{f3} = 133.0$ kJ/kg $S_{f3} = 0.485$ kJ/kg K

$h_{f1} = 45.4$ kJ/kg $S_{f1} = 0.183$ kJ/ kg K

To find out entropy at 2,

$$S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

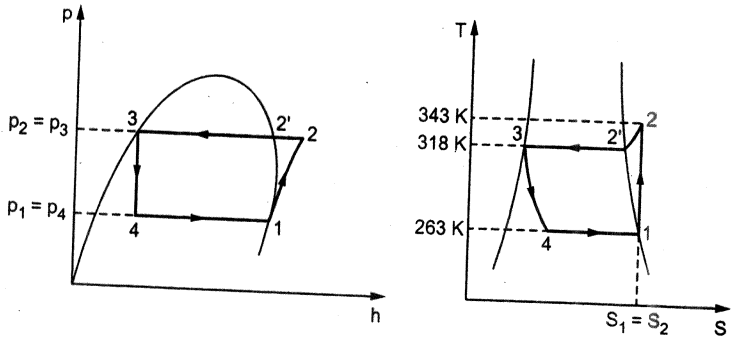


Fig. 3.36

$$1.637 = 1.587 + 2.3\, C_P \log\left(\frac{343}{318}\right)$$

$$C_P = 0.984$$

Enthalpy at point 2

$$h_2 = h_2' + C_P \times (\text{Degree of Superheat})$$

$$= h_2' + C_P\,(T_2 - T_2')$$

$$= 483 + 0.984\,(343 - 318)$$

$$= 507 \text{ kJ/kg}$$

$$\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.7 - 133}{507 - 460.7} = \mathbf{7.077}$$

Example 3.16 : *A simple F-12 heat pump for space heating operates between 15 °C and 50 °C. The heat required to be pumped is 200 MJ/hr. Determine i) Moss flow rate of refrigerant ii) Discharge temperature assuming Cp = 0.8 kJ/kg iii) Theoretical power required to run the compressor iv) COP of system.* ☞ **[GTU : Winter-19]**

Temp.	Pressure	Sp. enthalpy (kJ/kg)		Sp. entropy (kJ/kgK)	
(°C)	bar	h_f	h_g	s_f	s_g
15	4.91	50.1	193.8	0.1915	0.6902
50	12.19	84.9	206.5	0.3037	0.6797

Solution : Given data :

(Refer Fig. 3.37)

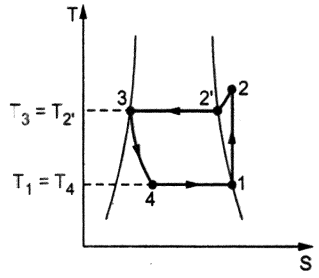


Fig. 3.37

$$T_1 = T_4 = 15\ ^\circ\text{C} = 288 \text{ K}$$

$$T_3 = T_2' = 50\ ^\circ\text{C} = 323 \text{ K}$$

$$Q = 200 \text{ MJ/hrs}$$

$$= \frac{200 \times 10^3}{3600} = 55.5 \text{ kJ/sec}$$

$$h_{g1} = h_1 = 193.8 \text{ kJ/kg}$$

$$h_2' = 206.5 \text{ kJ/kg}$$

$$h_{f3} = 84.9 \text{ kJ/kg}$$

$$S_{g1} = 0.6902 \text{ kJ/kg K } (S_{g1} = S_1)$$

$$S_{g2} = 0.6797 \text{ kJ/kg K } (S_{g2} = S_2')$$

$$C_p = 0.8 \text{ kJ/kg}$$

- i) Mass flow rate of refrigerant

$$m = \frac{Q}{h_1 - h_{f3}} = \frac{55.55}{193.8 - 84.9}$$

$$\mathbf{m = 0.5101 \text{ kg/sec}}$$

- ii) Discharge temperature,

Considering isentropic compression 1-2

$$\therefore \quad S_1 = S_2$$

$$\therefore \quad S_2 = S_2 + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$0.6902 = 0.6797 + 2.3 \times 0.8 \log\left(\frac{T_2}{323}\right)$$

$$\log\left(\frac{T_2}{323}\right) = \frac{0.6902 - 0.6797}{2.3 \times 0.8}$$

$$\log\left(\frac{T_2}{323}\right) = 0.00570$$

Taking antilog

$$\left(\frac{T_2}{323}\right) = 1.0132$$

$$\therefore \quad T_2 = 327.26 \text{ K}$$

- iii) Theoretical power required to run compressor.

$$P = m(h_2 - h_1)$$

To find value of h_2.

$$h_2 = h_2' + C_p (T_2 - T_2')$$

$$= 206.5 + 0.8 (327.26 - 323)$$

$$= 209.90 \text{ kJ/kg}$$

$$P = 0.5101(209.90 - 193.8)$$

$$\mathbf{P = 8.21 \text{ kW}}$$

- iv) $COP = \dfrac{\text{Refrigerating effect}}{\text{Power used}} = \dfrac{55.55}{8.21} = \mathbf{6.76}$

Example 3.17 : *Simple saturated VCC using ammonia has a capacity of 25 TR. Evaporator and condenser temperature are –5 °C and 40 °C respectively. Calculate mass flowrate of refrigerant, COP of the system, heat rejected in condenser. What will be the dimension bore and stroke of single acting 4 cylinder compression running at 350 rpm. L/D = 1.1, volumetric efficiency = 0.7 use the following properties sp. heat of ammonia vapour is 2.1897 kJ/kg K.*

Temp (°C)	Pressure bar	V_g m^3/kg	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg K)	s_g (kJ/kg K)
– 5	3.5571	0.346	176.9	1456.1	0.9154	5.6856
40	15.57	--	390.6	1490.4	1.6437	5.1558

Solution : Given data :

(See Fig. 3.38 on next page)

Q = 25 TR No. of cylinder = 4

= 25 × 3.52 $\quad \eta_{vol} = 0.7$

= 88 kW $\quad$ L/D = 1.1

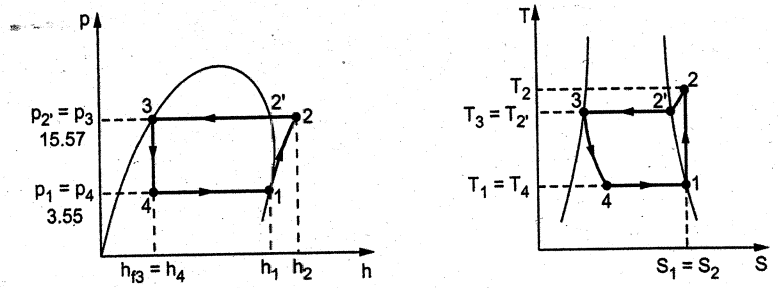


Fig. 3.38

$T_1 = T_4 = -5$ °C $\quad C_p = 2.1897$ kJ/kg

$T_3 = T_2' = 40$ °C $\quad N = 350$ rpm

$h_1 = h_{g1} = 1456.1$ kJ/kg $\quad S_1 = S_{g1} = 5.6856$ kJ/kg

$h_2' = h_{g2}' = 1490.4$ kJ/kg $\quad S_2' = S_{g2}' = 5.1558$ kJ/kg

$h_3 = h_{f3} = h_4 = 390.6$ kJ/kg

- Mass flow rate of refrigerant,

$$m = \frac{Q}{h_1 - h_{f3}}$$

$$m = \frac{88}{(1456.1 - 390.6)}$$

$$\mathbf{m = 0.0825 \text{ kg/sec}}$$

Considering isentropic compression (1 - 2)

$$S_1 = S_2$$

$$S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$5.6856 = 5.1558 + 2.3 \times 2.1897 \log\left(\frac{T_2}{313}\right)$$

$$\frac{5.6856 - 5.1558}{(2.3 \times 2.1897)} = \log\left(\frac{T_2}{313}\right)$$

$$0.1051 = \log\left(\frac{T_2}{313}\right)$$

Taking antilog,

$$1.274 = \left(\frac{T_2}{313}\right)$$

$$\therefore \quad \mathbf{T_2 = 398.78\ K}$$

The find out the enthalpy at point '2'

$$h_2 = h_2' + C_p\,(T_2 - T_2')$$

$$= 1490.4 + 2.1897\ (398.78 - 313)$$

$$\mathbf{h_2 = 1678.232\ kJ/kg}$$

- COP of system

$$COP = \frac{Q}{W} = \frac{88}{m(h_2 - h_1)}$$

$$= \frac{88}{0.0825\ (1678.23 - 145.1)}$$

$$= \mathbf{4.801}$$

- Heat rejected in the condenser,

$$Q_{condensor} = m\,(h_2 - h_3)$$

$$= 0.0825(1678.23 - 390.6)$$

$$= \mathbf{106.22\ kW}$$

- Volume flow rate

$$v = m \times v_{s_1}$$

$$= 0.0825 \times 0.346$$

$$= \mathbf{0.0285\ m^3/sec}$$

$$v = \frac{\pi}{4}D^2 \times L \times \frac{N}{60} \times \eta_{vol} \times \text{no.of cylinder}$$

$$0.0285 = \frac{\pi}{4}D^2 \times (1.10) \times \frac{350}{60} \times 0.7 \times 4$$

$$D^3 = 2.0197 \times 10^{-3}$$

$$\mathbf{D = 0.126\ m}$$

$$L = D \times 1.1$$

$$= 0.126 \times 1.1 = \mathbf{0.1386\ m}$$

Example 3.18 : *A 2 TR air conditioner based on simple saturate vapour compression cycle operates between 12 °C and 50 °C Determine i) COP ii) Power per tonn of refrigeration iii) Mass of circulation of refrigerant in kg/hrs. iv) Dryness fraction after throttling. (Solve by using property table for refrigerant R-134 a)*

Solution : Given data

Capacity = 2TR

= 2×3.52

= 7.04 kW

$T_1 = T_4 = 12\ °C = 285\ K$

$T_3 = T_2' = 50\ °C = 323\ K$

From property table (R – 134 a)

$h_1 = h_g = 405.51$ kJ/kg

$h_2' = 423.63$ kJ/kg

$h_3 = h_{f3} = h_4 = 271.59$ kJ/kg

$S_1 = 1.7215$ kJ/kg K

$S_2' = 1.7078$ kJ/kg K

$C_p = 1.218$ kJ/kg

- Process 1-2 is isentropic compression.

$$\therefore \quad S_1 = S_2$$

$$S_2 = S_2' + 2.3\ C_p \log\left(\frac{T_2}{T_2'}\right)$$

$$1.7215 = 1.7078 + 2.3 \times 1.218 \log\left(\frac{T_2}{323}\right)$$

$$\frac{1.7215 - 1.7078}{2.3 \times 1.218} = \log\left(\frac{T_2}{323}\right)$$

$$0.00489 = \log\left(\frac{T_2}{323}\right)$$

Taking antilog

$$1.01132 = \left(\frac{T_2}{323}\right)$$

$$\therefore \quad \mathbf{T_2 = 326.65\ K}$$

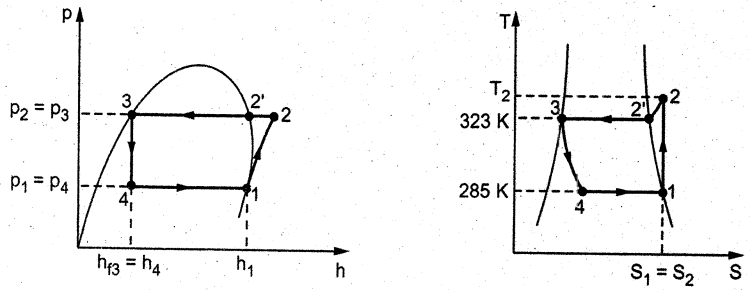


Fig. 3.39

- To find enthalpy at point '2'.

$$\therefore \quad h_2 = h_2' + C_p(T_2 - T_2')$$

$$= 423.63 + 1.218\ (326.65 - 323)$$

$$\mathbf{h_2 = 428.07\ kJ/kg}$$

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{405.51 - 271.59}{428.07 - 405.51} = 5.93$$

- Mass flow rate of refrigerant

$$\dot{m} = \frac{Q}{(h_1 - h_{f3})}$$

$$\dot{m} = \frac{7.04}{(405.51 - 271.59)}$$

$$\mathbf{\dot{m} = 0.0525\ kg/sec}$$

- To find out the power,

$$P = \dot{m}(h_2 - h_1) = 0.525\ (428.07 - 405.51)$$

$$\mathbf{P = 1.184\ kW}$$

- In problem we have given 2TR

$$\therefore \quad \text{To find per tonne of refrigration} = \frac{\text{Power}}{2TR} = \frac{1.184}{2} = 0.592 \frac{kW}{TR}.$$

- To find dryness fraction after trottling,

$$h_4 = h_{f4} + x_4(h_1 - h_{f4})$$

where, $h_{f4} = 216.384$ kJ/kg

$$271.59 = 216.384 + x_4\ (405.51 - 216.384)$$

$$\mathbf{x_4 = 0.291}$$

Example 3.19 : *A Freon-12 simple vapour compression system operating at a condenser temperature of 40 °C and evaporator temperature of 0 °C develops 15 TR using p-h chart for Freon 12, Determine.*

i) Mass flow rate of refrigerant

ii) Theoretical piston displacement of compressor and piston displacement per TR.

iii) Theoretical horse power of compressor

iv) Heat rejected in condenser.

v) Cornot COP and actual COP.

Solution :

Given data	**From the p-h chart**
$T_3 = T_2 = 40$ °C	$h_1 = 351$ kJ/kg
$T_1 = T_4 = 0$ °C	$h_2 = 370$ kJ/kg
Capacity = 15 TR	$hf_3 = h_4 = 238$ kJ/kg
$= 15 \times 3.52$	$v_{s_1} = 0.55$
$= 52.8$ kW	

- Mass flow rate of refrigerant.

$$m = \frac{Q}{h_1 - h_{f3}} \text{ or } m = \frac{Q}{h_1 - h_4}$$

$$\therefore \quad m = \frac{52.8}{(351 - 238)}$$

$$\mathbf{m = 0.467\ kg/sec}$$

Thererical piston displacement of compressor,

$$V = m \times v_{s_1} = 0.467 \times 0.55 = \mathbf{0.256\ m^3/sec}$$

- Piston displacement per TR

We have given 15 TR

$$\therefore \quad v = \frac{V}{15}$$

$$\therefore \quad v = \frac{0.256}{15} = \mathbf{0.01713\ m^3/sec/\ TR}$$

- Power consumption,

$$P = m(h_2 - h_1) = 0.467\ (370 - 351) = \mathbf{8.873\ kW}$$

- Theoretical horse power of compressor,

$$= \frac{8.873 \times 10^3}{746} = \mathbf{11.894\ HP}$$

- Horse power per TR

$$= \frac{11.894}{15} = \mathbf{0.79\ HP/TR}$$

- Heat rejected in condenser

$$= m(h_2 - h_3) = 0.467(370 - 238) = \mathbf{61.64\ kW}$$

$$(COP)_{cycle} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{351 - 238}{370 - 351} = \mathbf{5.94}$$

$$(COP)_{carnot} = \frac{T_1}{T_2 - T_1} = \frac{(0+273)}{(40+273)-(0+273)} = \mathbf{6.8}$$

Example 3.20 : *In simple vapour compression cycle following properties of refrigerant R-12 at various points*

compressor inlet : $h_1 = 183.2\ kJ/kg,\ V_1 = 0.0767\ m^3/kg$

Compressor discharge : $h_2 = 222.6\ kJ/kg,\ V_2 = 0.00164\ m^3/kg$

Condenser exit : $h_{f_3} = 84.9\ kJ/kg,\ V_3 = 0.00083\ m^3/kg$

The piston ddisplacement for compressor is 1.5 liter per stroke and η_{vol} *80 %. The speed of compressor is 1650 rpm.*

Find : i) Power rating of compressor (kW) ii) Refrigerating effect (kW)

Solution :

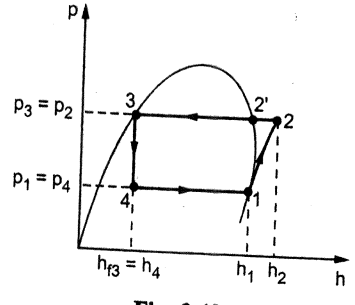


Fig. 3.40

Power rating of compressor.

Compressor discharge $= \frac{\pi}{4} D^2 \times L \times N \times \eta_{vol}$

We have given piston displacement volume = 1.5 litre

i.e. $\frac{\pi}{4} D^2 \times L = 1.5$ litre

$= 1.5 \times 1000 \times 10^{-6}\ m^3/stroke$

$= 0.0015\ m^3/rev$

Compressor discharge $= 0.0015 \times 1650 \times 0.8$

$= 1.98\ m^3/\text{min}$

- Mass flow rate of compressor,

$$m = \frac{\text{Compressor discharge}}{v_1}$$

$$= \frac{1.98}{0.0767}$$

$= 25.81$ kg/min $= 0.430$ kg/ sec

- Power rating of compressor

$$P = m(h_2 - h_1)$$

$$= 0.430\ (222.6 - 183.2)$$

$$= \mathbf{16.95\ kW}$$

Refrigerating effect $= m(h_1 - h_4)$

$$= 0.430\ (183.2 - 84.9)$$

$$= \mathbf{42.2\ kW}$$

Example 3.21 : *An ammonia refrigerator operates between – 16 °C and 50 °C. The vapour is dry-saurated at the entry of the compressor and compession is isentropic. Assuming there is no under cooling determine.*

i) The mass flow rate and power input per kW.

ii) COP of system (Refer P-h chart).

Solution : Given data :

$$T_1 = T_4 = -16\ ^\circ C = -16 + 273 = 257\ K$$

$$T_3 = T_2' = 50\ ^\circ C = 323\ K.$$

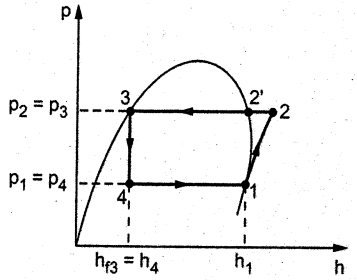


Fig. 3.41

From p-h-chart

$$h_1 = 1418\ kJ/kg$$

$$h_2 = 1785\ kJ/kg$$

$$h_{f3} = h_4 = 450\ kJ/kg$$

- To find mass flow rate per kW.

Refrigeration capacity $= m(h_1 - h_4) = m(1418 - 450)$

$\therefore$ $\mathbf{m = 1.033 \times 10^{-3}}$ kg/sec

- Power required.

$$P = m(h_2 - h_1)$$

$$= 1.033 \times 10^{-3}\ (1785 - 1418)$$

$$= \mathbf{0.397\ IW}$$

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1418 - 450}{1785 - 1418} = \mathbf{2.63}$$

3.27 Superheated Vapour before Compression

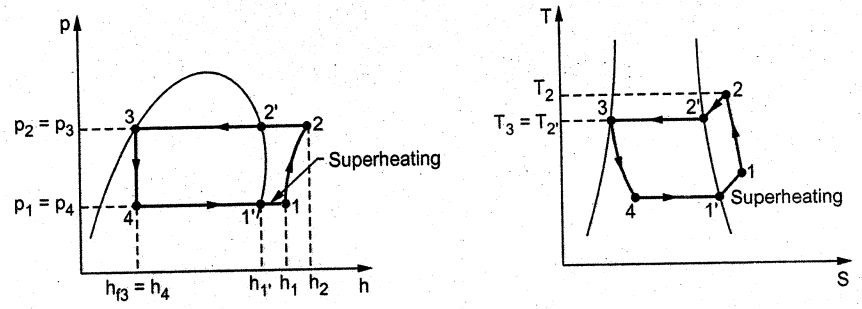


Fig. 3.42 p-h and T-S diagram for super heated vapour before compression

- In simple VCC at the entry of compressor there must be a dry saturated vapour.
- If some liquid refrigerant enters the suction line of compressor, due to wet compression. lubriating oil which is present in the compressor will be washed off and will caues more wear and tear of compressor.
- Hence the life of the compressor reduces.
- In order to increase the life of the compressor refrigerant vapour which is coming of the evaporator is allowed to stay for some more time in evaporator to ensure dry saturated vapour at evaporator exit.
- Thus we can define suprheating as, increasing the temperature of vapour refrigerant more than saturation temperature in evaporator.
- Superheating is shown in P-h and T-S diagram by process 1′ - 1

$$COP = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

$\therefore$ $$\boxed{COP = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve numericals on superheating before compression**
- In such types of numericals, it is required to find the enthalpy at point '1'

$$h_1 = h_1' + C_p(T_1 - T_1')$$

- Simillarly we can find the enthalpy at point '2'

$$h_2 = h_2' + C_p(T_2 - T_2')$$

- To find entropy a point '1'

$$S_1 = S_1' + 2.3\, C_p \log\left(\frac{T_1}{T_1'}\right)$$

- Sometimes it is required to find temperature at point '2' it can be found out by using following equation.

$$S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$$

- To find heat rejection in condenser.

$$(Q)_{reject} = m(h_2 - h_3)$$

3.27.1 Solved Examples

Example 3.22 : *A vapour compression refrigration plant works between pressure limit of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression its temperatue being 37 °C. The vapour is supperheated by 7 °C before entering to the compressor. Take C_p = 0.63 kJ/kg K. Find COP.*

Pressure (bar)	Temp (°C)	Liquid heat (kJ/kg)	Latent heat (kJ/kg)
5.3	+ 15.5	56.15	144.9
2.1	– 14.00	25.12	158.7

Solution : Given data

$p_2 = p_3 = 5.3$ bar

$p_1 = p_4 = 2.1$ bar

$T_2 = 37$ °C $= 37 + 273 = 310$ K

$T_1 - T_1' = 7$ °C

$T_1' = -14$ °C $= 259$ K

$T_2' = 15.5 + 273 = 288.5$ K

$h_{f1}' = 25.12$ kJ/kg

$h_{fg1}' = 158.7$ kJ/kg

$h_{fg2}' = 144.9$ kJ/kg

$h_{f3} = 56.15$ kJ/kg

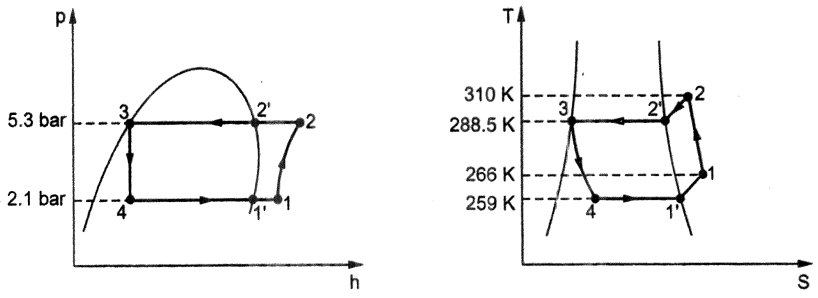


Fig. 3.43

- To find enthalpy at point '1'

$$h_1 = h_1' + C_p\,(T_1 - T_1')$$

To find the value of h_1'

$$h_1' = h_{f1}' + h_{fg1}' = 25.12 = 158.7 = 183.82 \text{ kJ/kg}$$

Now, $$h_1 = h_1' + C_p\,(T_1 - T_1') = 183.82 + 0.63\,(266 - 259)$$

$$\mathbf{h_1 = 188.23 \text{ kJ/kg}}$$

- Similarly to find out enthalpy at point '2'

$$h_2 = h_2' + C_p\,(T_2 - T_2').$$

Now to find.

$$h_2' = h_{f2}' + h_{fg2}' = 56.15 + 144.9 = 201.05 \text{ kJ/kg}$$

Now,

$$h_2 = 201.05 + 0.63\,(310 - 288.5) = \mathbf{214.59 \text{ kJ/kg}}$$

- To find COP

$$\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{188.23 - 56.15}{214.6 - 186.97} = \mathbf{4.78}$$

Example 3.23 : *A refrigerating machine using F-12 as working fluid works between temperature –18 °C and 37 °C. The enthalpy of liquid at 37 °C is 78 kJ/kg,. The enthalpy of F-12 entering and leaving the compressor are 200 kJ/kg and 238 kJ/kg respectively. The rate of circulation of refregerant is 5 kg/min and efficiency of compressor is 0.90. Determine.*
i) Capacity of plant in TR ii) Power required to run plant

Solution :

Given data :

$T_1' = T_4 = -18\ °C = 255\ K$ $\quad h_3 = 78\ kJ/kg$

$T_2' = T_3 = 37\ °C = 310\ K$ $\quad h_1 = 200\ kJ/kg$

$m = 5\ kg/min = 0.083\ kg/sec$ $\quad h_2 = 280\ kJ/kg$

$\eta_c = 0.90$

- To find capacity of plant in TR

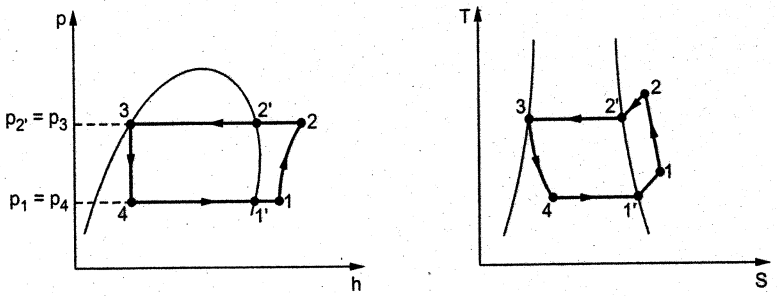


Fig. 3.44

$$= \frac{m(h_1 - h_4)}{3.5} = \frac{0.083(200-78)}{3.5} \quad \ldots (h_3 = h_4)$$

$$= \mathbf{2.893\ TR}$$

- Power required to run plant

$$P = \frac{m(h_2 - h_1)}{\eta_c} = \frac{0.083(238-200)}{0.90} = \mathbf{3.5\ kW}$$

3.28 Undercooling or Subcooling of Refrigerant

- The process of cooling the refrigerant below the condensing temperature for a given pressure is known as undercooling or subcooling.
- The process 3'-3 represents the undercooling.
- The effect of subcooling is to increase the refrigerating effect and therefore COP of the system increases.

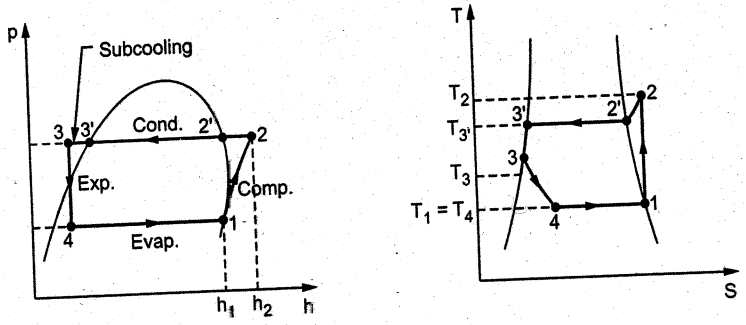


Fig. 3.45 p-h and T-S diagram for subcooling and under cooling

- In subcooling process refrigerating effect can be found out by using following relation.

$$\text{R.E.} = h_1 - h_{f3} \quad \text{or} \quad \text{R.E.} = h_1 = h_4$$

- In above case, value of h_{f3} can be found out as

$$h_{f3} = h'_{f3} - Cp \times (\text{Degree of subcool})$$

$$h_{f3} = h'_{f3} - C_p \times (T_3' - T_3)$$

$$\text{COP} = \text{Refrigerating effect / Work done}$$

$\therefore$ $$\boxed{\text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1}}$$

- **Procedure to solve numericals on subcooling**
- To find the enthalpy at point '2'

$$h_2 = h_2' + C_{p_v}(T_2 - T_2')$$

- To find enthalpy at point '3'

$$h_3 = h'_{f3} - C_{p_l}(T_3' - T_3) \quad \ldots (h_3 = h_{f3})$$

- To find the entropy at point '2'

$$S_2 = S_2' + 2.3\, C_{p_v} \log\left(\frac{T_2}{T_2'}\right)$$

- To find work during the compression

$$w = m(h_2 - h_1)$$

- To find refrigerating effect

$$\textbf{R.E.} = \mathbf{m(h_1 - h_4)}$$

- To find COP of system

$$COP = \frac{R.E.}{W.D.} = \frac{(h_1 - h_4)}{(h_2 - h_1)}$$

3.28.1 Solved Examples

Example 3.24 : *A vapour compression refrigeration plant operates between evaporator and condenser temperature at – 15° C and 40° C respectively. The refrigerant is dry and saturated at the suction discharge temperature of refrigerant is 98° C. The bore and stroke of compressor are 85 mm each. It runs at 750 rpm with η_{vol} 82 %. The liquid enters the expansion value at 32° C. Calculate, i) COP ii) Mass of flow of refrigerant iii) Capacity. C_{Pl} = 1.62 and C_{Pv} = 0.96 kJ/kg K.*

Temperature °C	V_g m^3/kg	h_f (kJ/kg)	h_g (kJ/kg)	S_f (kJ/kg K)	S_g (kJ/kg K)
– 15	0.24	43.4	458.7	0.18	1.742
40	0.043	131	468.6	0.48	1567

Solution : Given data :

$T_1 = T_4 = -15$ °C = 258 K

$T_3' = T_2' = 40°$ C = 313 K

$T_2 = 98°$ C = 371 K

$T_3 = 32°$ C = 305 K

L = 85 mm

$h_1 = h_{g1}$ = 458.7 kJ/kg

$h_2' = h_{g2}'$ = 468.6 kJ/kg

h_{F3}' = 131.0 kJ/kg

d = 85 mm

N = 750 rpm

η_{vol} = 82 % C_{Pl} = 1.62 and C_{Pv} = 0.96 kJ/kg K

- To find enthalpy at point '2'.

$$h_2 = h_{g2}' + C_{Pv}(T_2 - T_2')$$

$$= 468.6 + 0.96(371 - 313)$$

$$\mathbf{h_2 = 524.28\ kJ/kg}$$

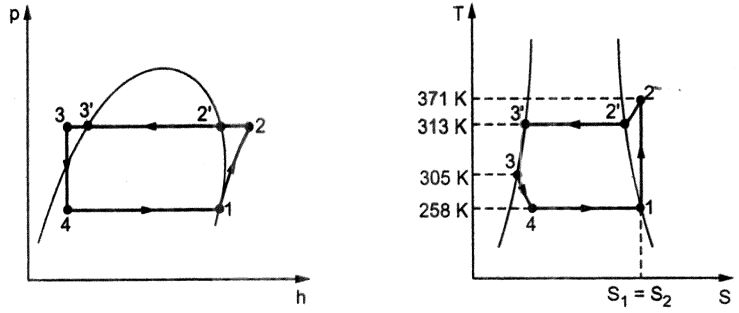


Fig. 3.46

- To find enthalpy at point '3'.

$$h_3 = h_{f3}' - C_{Pl}(T_3' - T_3) = 131 - 1.62(313 - 305)$$

$$\mathbf{h_3 = 118.04\ kJ/kg}$$

- COP of the system

$$COP = \frac{R.E.}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1} \qquad \ldots h_4 = h_3$$

$$= \frac{458.7 - 118.04}{524.28 - 458.7} = \mathbf{5.194}$$

- Volume flow rate of refrigerant

$$V = \frac{\pi}{4} D^2 L \times \frac{N}{60} \times \eta_{vol} = \frac{\pi}{4}(0.085)^2 \times 0.085 \times \frac{750}{60} \times 0.82$$

$$= \mathbf{4.943 \times 10^{-3}\ m^3/sec}$$

$$m = \frac{V}{V_{g1}} = \frac{4.943 \times 10^{-3}}{0.24} = 0.020\ \text{kg/sec}$$

Capacity = $m(h_1 - h_4)$

$= 0.020(458.7 - 118.04) = 7.153$ kW $= \mathbf{2.03\ TR}$

Example 3.25 : *A vapour compression system operates between – 10° C and 45° C. The refrigerant is dry-saturated at the entry of the compressor and attains 102° C after compression. The temperature of liquid refrigerant at the entry of throttle valve is 35° C. Take C_{Pl} = 1.62 kJ/kg K and C_{Pv} = 1.06 kJ/kg K. Determine COP of system.*

[GTU : Summer-18]

Temperature °C	h_f (kJ/kg)	h_g (kJ/kg)	S_f (kJ/kg K)	S_g (kJ/kg K)
– 10	45.4	460.7	0.183	1.762
45	133	483.6	0.485	1.587

Solution : Given data :

$T_1 = T_4 = -10\,°C = 263\ K$ $\quad h_1 = h_{g1} = 460.7\ kJ/kg$

$T_3' = T_2' = 45°\,C = 318\ K$ $\quad S_1 = 1.762\ kJ/kg\,K$

$T_3 = 308\ K$ $\quad h_{f3}' = 133\ kJ/kg$

$C_{Pl} = 1.62\ kJ/kg\,K$ $\quad S_{g2} = 1.587\ kJ/kg\,K$

$T_2 = 102\ °C = 102 - 273 = 375\ K$

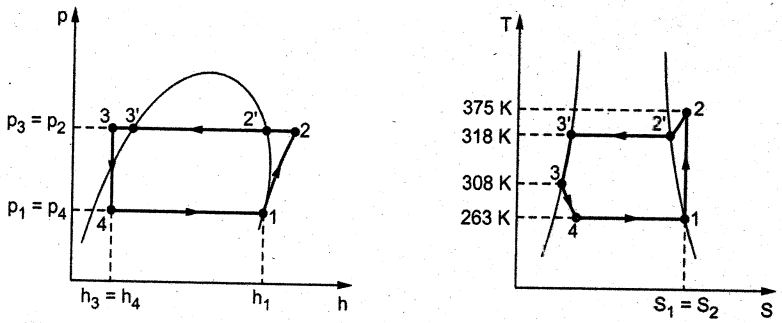


Fig. 3.47

- The process 1-2 is isentropic compression

$\therefore \quad S_1 = S_2$

In this numerical the value of C_{Pv} is not given

- To find the value of C_{Pv}

$$S_2 = S_2' + 2.3 \times C_{Pv} \times \log\left(\frac{T_2}{T_2'}\right)$$

But $\quad S_1 = S_2$

$$\therefore \quad 1.762 = 1.587 + 2.3 \times C_{Pv} \times \log\left(\frac{375}{318}\right)$$

$$C_{Pv} = \frac{1.762 - 1.587}{0.165} = \mathbf{1.06\ kJ/kg\,K}$$

- To find enthalpy at point '2'.

$$h_2 = h_2' + C_{Pv}\,(T_2 - T_2') = 483.6 + 1.06\,(375 - 318) = \mathbf{538\ kJ/kg}$$

- To find the value at h_3

$$h_3 = h_{f3}' - C_{Pl}\,(T_3' - T_3) = 133 - 1.62\,(318 - 308) = \mathbf{149.2\ kJ/kg}$$

- The process 3-4 is throttling process

$\therefore \quad h_3 = h_4 = 149.2\ kJ/kg$

- COP of the system

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{460.7 - 149.2}{538 - 460.7} = \mathbf{4.03}$$

Example 3.26 : *A food storage locker requires a refrigeration capacity of 60 kW. It works between a condenser temperature of 35° C and evaporator temperature of – 10° C. The refrigerant is ammonia. It is subcooled by 7° C before entering the expansion valve by dry saturated vapour leaving the evaporator. Assuming single cylinder single acting compressor operating at 1000 rpm with stroke equal to 1.2 times the bore determine Assume specific volume 0.417477 m^3/kg.*

i) Power required ii) The cylinder dimensions ☞ **[GTU : Winter-17]**

Temperature °C	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Sp. heat (kJ/kg K)	
	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
– 10	154.056	1450.22	0.8296	5.7550	–	2.492
35	366.072	1488.57	1.5660	5.2086	4.556	2.903

Solution : Given data :

Capacity = 60 kW $\quad h_1 = 1450.22\ kJ/kg$

$T_3' = T_2' = 35°\,C = 308\ K$ $\quad S_1 = 5.7550\ kJ/kg\,K$

$T_1 = T_4 = -10°\,C = 263\ K$ $\quad S_2' = 5.2086\ kJ/kg\,K$

$h_2' = 14.88.57\ kJ/kg$

$T_3 = 35°\,C - 7°\,C = 28°\,C = 301\ K$ $\quad h_{f3}' = 366.072\ kJ/kg$

$N = 1000\ rpm$ $\quad L = 1.2 \times D$

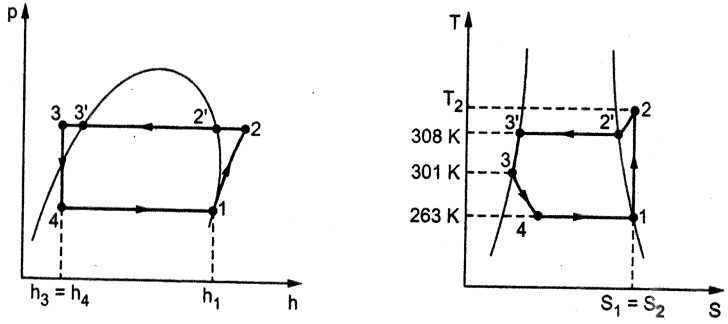


Fig. 3.48

- The process 1-2 is isentropic process,

$$\therefore \quad S_1 = S_2$$

$$S_2 = S_2' + 2.3 \times C_{Pv} \log\left(\frac{T_2}{T_2'}\right)$$

$$\therefore \quad 5.755 = 5.2086 + 2.3 \times 2.903 \log\left(\frac{T_2}{308}\right)$$

$$\frac{5.755 - 5.2086}{2.3 \times 2.903} = \log\left(\frac{T_2}{308}\right)$$

$$0.08183 = \log\left(\frac{T_2}{308}\right)$$

Taking antilog

$$1.2073 = \left(\frac{T_2}{308}\right)$$

$$\mathbf{T_2 = 371.86\ K}$$

- To find enthalpy at point '2'.

$$h_2 = h_2' + C_{Pv}\,(T_2 - T_2') = 1488.57 + 2.903\,(371.86 - 308)$$

$$= \mathbf{1673.96\ kJ/kg}$$

To find enthalpy at point 3

$$h_3 = h_{f3}' - C_P\,(T_3' - T_3) = 366.07 - 4.556\,(308 - 301)$$

$$= \mathbf{334.178\ kJ/kg}$$

- Mass of refrigerant flowing

$$m = \frac{Q}{h_1 - h_3} = \frac{Q}{h_1 - h_4} = \frac{60}{1450.22 - 334.178}$$

$$= 0.0537\ \text{kg/sec}$$

- Power required,

$$P = m\,(h_2 - h_1) = 0.0537\,(1673.96 - 1450.22) = \mathbf{12.02\ kW}$$

- Cylinder dimension

$$m = \frac{\pi}{4} D^2 \times L \times \frac{N}{60} \times \text{(Specific) volume}$$

$$0.0537 = \frac{\pi}{4} D^2 \times 1.2\,D \times \frac{1000}{60} \times 0.417477$$

$$D^3 = \frac{0.04517 \times 4 \times 60}{\pi \times 1.2 \times 1000 \times 0.417477}$$

$$\therefore \quad \mathbf{D = 0.19\ m}$$

$$L = 1.2 \times D = 1.2 \times 0.19$$

$$\mathbf{L = 0.228\ m}$$

Example 3.27 : *Standard ammonia at 2.5 bar enters a 160 mm × 150 mm (bore × stroke) twin cylinder, single acting compressor whose η_v is 79 % and speed is 250 rpm. The head pressure is 12 bar. The subcooled liquid ammonia at 22 °C enters the expansion valve find i) Ammonia circulated in kg/min ii) The refrigeration in TR iii) COP of cycle.*

Assume (C_{Pl} = 4.6006 and C_{Pv} = 2.703 kJ / kg.K). ☞ **[GTU : Summer-15]**

Pressure (bar)	Temperature (°C)	Specific volume (m^3 / kg)	Specific enthalpy kJ/kg		Specific entropy kJ / kg K	
			L	V	L	V
2.5	– 15	0.5098	112.4	142.58	0.4572	5.5497
12	30	0.1107	323.08	1468.87	1.203	4.9842

Solution : $p_1 = p_4 = 2.5$ bar, D = 160 mm = 0.16 m, L = 0.15 m

No. of cylinder = 2, $\eta_v = 0.79$, N = 250 rpm, $p_2 = p_3 = 12$ bar, $T_3 = 22$ °C,

$T_1 = T_4 = 258$ K, $T_2' = T_3' = 30$ °C, $v_1 = 0.5098$, $v_2' = 0.1107$, $h_{f1} = 112.4$,

$h_{f3}' = 323.08$, $h_1 = 1426.58$, $h_2' = 1468.87$, $S_{f1} = 0.4572$, $S_{f3} = 1.2037$,

$S_1 = S_2 = 5.5497$, $S_2' = 4.9842$, $C_{Pl} = 4.6006$, $C_{Pv} = 2.703$ kJ /kg

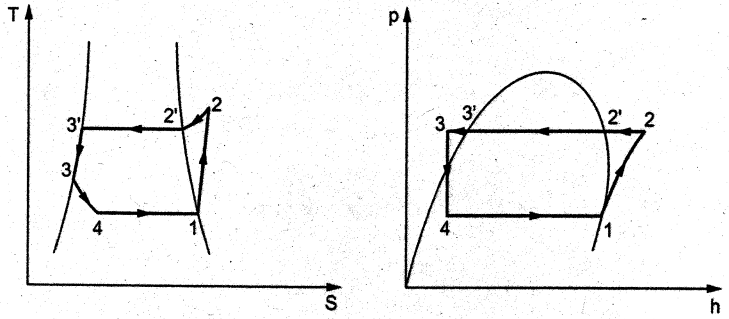


Fig. 3.49

Ammonia circulated (m_R)

Piston displacement / min = Area × Stroke × RPM × no. of cylinder

$$= \pi/4\, D^2\, LN \times 2 = \pi/4 \times (0.16)^2 \times (0.15) \times 250 \times 2$$

$$= 1.508\ m^3/min \qquad \ldots (1)$$

We also know, piston displacement / min

$$= m_R \times V_1 \times 1/\eta_v = m_R \times 0.5098 \times 1/0.79 = 0.6453\, m_R \quad \ldots (2)$$

equate (1) and (2)

$$m_R = 1.508 / 0.6453 = 2.34\ kg/min$$

Refrigeration in TR

$$h_{f3} = hf_3' - C_{Pl}(T_3' - T_3) = 286.23\ kJ/kg$$

Total RE $= m_R(h_1 - h_{f3}) = 2668.4$ kJ/min

Capacity $= 2668.4 / 210 = 12.7$ TR

To find COP

$$S_2 = S_2' + 2.3\, C_{Pv} \log\left(\frac{T_2}{T_2'}\right)$$

$$5.5497 = 4.9842 + 2.3 \times 2763 \log \frac{T_2}{T_2'}$$

$$\frac{T_2}{T_2'} = 1.227$$

$$T_2 = T_2' \times 1.227 = 303 \times 1.227 = 371.78$$

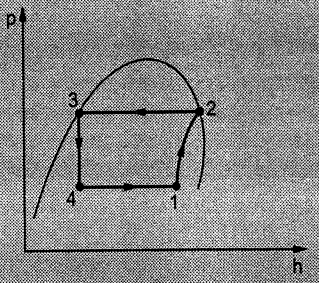
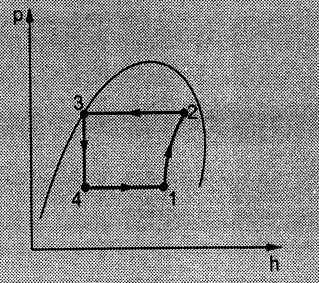
$$= 371.78 - 273 = \mathbf{98.78\ °C}$$

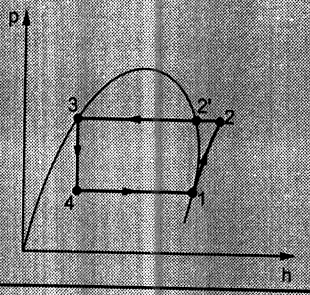
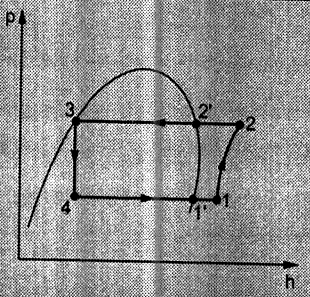
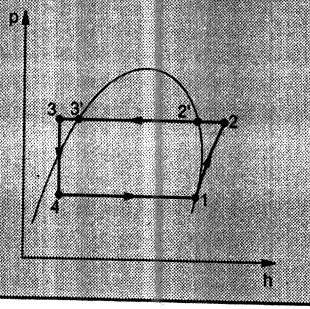
Enthalpy at (2)

$$h_2 = h_2' + C_{Pv}(T_2 - T_2') = 1658.9\ kJ/kg$$

$$COP = \frac{h_1 - h_{f3}}{h_2 - h_1} = 4.91$$

Table 3.15 - Review of all the processes with p- h representation

Dry compression	• $h_1 = h_{f1} + x_1 h_{fg1}$ $h_1 = h_{f1} + x_1 (h_{g1} - h_{f1})$ • $S_1 = S_{f1} + x_1 S_{fg1}$ $S_1 = S_{f1} + x_1 (S_{g1} - S_{f1})$	
Wet compression	• $h_1 = h_{f1} + x_1 h_{fg1}$ $h_1 = h_{f1} + x_1 (h_{g1} - h_{f1})$ • $h_2 = h_{f2} + x_2 h_{fg2}$ $h_2 = h_{f2} + x_2 (h_{g2} - h_{f2})$ • $S_1 = S_{f1} + \frac{x_1 h_{fg1}}{T_1}$ • $S_2 = S_{f2} + \frac{x_2 h_{fg2}}{T_2}$	

Superheating after compression or simple saturated VCC	• $h_2 = h_2' + C_p (T_2 - T_2')$ $S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$	
Superheating before compression	• $h_1 = h_1' + C_p (T_1 - T_1')$ • $h_2 = h_2' + C_p (T_2 - T_2')$ • $S_1 = S_1' + 2.3\, C_p \log\left(\frac{T_1}{T_1'}\right)$ • $S_2 = S_2' + 2.3\, C_p \log\left(\frac{T_2}{T_2'}\right)$	
Subcooling	• $h_2 = h_2' + C_{Pv} (T_2 - T_2')$ $h_3 = h_{f3}' + C_{Pl} (T_3' - T_3)$ $S_2 = S_2' + 2.3\, C_{Pv} \log\left(\frac{T_2}{T_2'}\right)$	

Vapour Absorption Cycle

3.29 Vapour Absorption System (VAS)

- The principle of vapour absorption system (VAS) was first introduced by Michael Faraday in 1824.
- Vapour absorption refrigeration system is one of the oldest methods of producing refrigerating effect.
- In vapour absorption system, compressor is replaced by an absorber, pump, generator and pressure reducing valve.

- The vapour absorption system require at least two fluids.
- One fluid acts as refrigerant while other will acts as an absorber.
- The vapour absorption system can be built in the capacities above 1000 TR.

3.30 Working of Simple Vapour Absorption System

- It consists of following parts
  - i) Absorber ii) Pump
  - iii) Generator iv) Pressure reducing valve
- These four components replaces the compressor which is present in VCC.
- Remaining components are
  - v) Condenser vi) Receiver
  - vii) Expansion valve and viii) Evaporator
- In this system, low pressure ammonia vapours leave the evaporator.
- These vapours enter into absorber.
- These ammonia vapours are absorbed by the water and forms a solution known as aqua ammonia.
- Water has strong affinity of ammonia vapours.
- Some form of cooling arrangement is made in absorber to remove the heat of solution.
- The strong solution which is formed in absorber is pumped to generator by liquid pump.
- In the generator strong solution of ammonia is heated by external source.
- Due to heating process ammonia vapours are formed and the vapours move to the condenser leaving behind the hot weak ammonia solution in generator.
- This weak ammonia solution flows back to absorber through pressure reducing valve.
- High pressure ammonia vapours are condensed in condenser where the change of phase of ammonia vapour takes place.
- This liquid ammonia is passed to expansion valve through the receiver and then it moves to evaporator.
- In this way simple vapour absorption cycle completes.

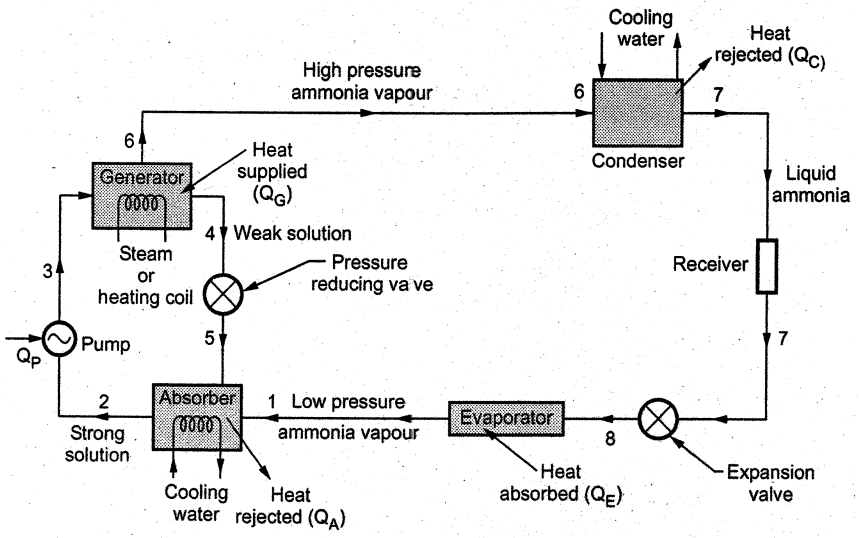


Fig. 3.50 : Simple vapour absorption system

3.31 Practical Vapour Absorption System

☞ **[GTU : Summer-16, 18, Winter-17]**

- Simple vapour absorption system is not very economical and in actual practice COP of system is very low.
- Therefore to make it practical some modifications are made.
- Following components are added to improve the performance of cycle.

  i) Analyzer ii) Rectifier iii) Heat exchanger
- Analyzer is provided to remove unwanted water particles associated with ammonia vapour before going to condenser.
- If these unwanted water particles are not removed they will enter in the expansion valve where they freeze and choke the pipe line.
- Analyzer consists of series of trays mounted above the generator.
- In some cases, vapour leaving the analyzer still contains some amount of water particles.
- Rectifier is cooling coil provided to remove moisture so that almost all water particles will be removed leaving only dehydrated ammonia vapour to enter into condenser.

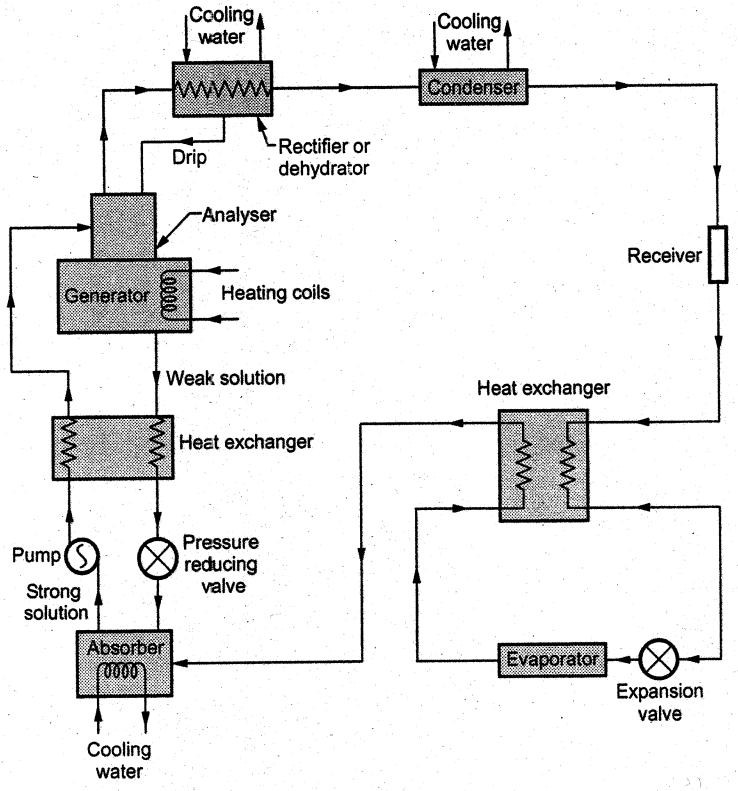


Fig. 3.51 : Practical vapour absorption system

- Heat exchanger is provided between pump and generator.
- Its function is to cool the weak hot solution returning from generator to absorber.
- The heat removed from weak solution raises the temperature of strong solution leaving the pump and going to analyser and generator.
- The heat exchanger which is provided between condenser and evaporator is also called as liquid subcooler.
- COP can be determined by following formula.

$$COP = \frac{\text{Heat absorbed in evaporator}}{\text{Work done by pump} + \text{Heat supplied in generator}}$$

3.32 Desirable Properties

- The desirable Properties of solvent and those of refrigerant solvent combination are given below.

3.32.1 Desirable Properties of (Absorber) Solvent :

- Absorber should have high boiling point.
- It should have low viscosity.
- The absorber should have great affinity to absorb the refrigerant.
- Ideal absorbent should remain in liquid state.
- The solvent should have low specific heat for better heat transfer.
- Heat liberated during absorption of refrigerant should be as small as possible.

3.32.2 Desirable Properties of Refrigerant - Solvent Combination

- The combination of refrigerant absorbent should be chemically stable.
- The mixture should have low specific heat and low viscosity.
- The mixture should be non corrosive.
- The mixture should have high freezing point.
- The refrigerant should have high latent heat to have low mass flow rates.

3.32.3 Characteristics of Ammonia

- Water has strong affinity to absorb ammonia vapour.
- The amount of ammonia vapour that water can absorb increases with increase in pressure and decrease with increase in temperature.
- During the absorption of ammonia vapour in water large amount of heat is liberated.
- It is colourless gas.
- Its boiling point at atmospheric pressure is -33.3° C.

3.33 Performance Evaluation of Vapour Absorption System

- Q_G - Heat given to the refrigerant in Generator
- Q_C - Heat discharged to atmosphere
- Q_E - Heat absorbed by refrigerant in evaporator

- Q_P - Heat added to refrigerant due to pumpwork.

Neglecting the heat due to pumpwork.

- According to I law of Thermodynamics,

We can write,

$$Q_C = Q_G + Q_E \quad \ldots(i)$$

Now,

- T_G - Temperature at which heat is given to generator
- T_C - Temperature at which heat is discharged to atmosphere
- T_E - Temperature at which heat is added to evaporator

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_C}{T_C} \quad \ldots(ii)$$

But from equation (i) we can write,

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_G + Q_E}{T_C}$$

Rearranging above equation,

$$\frac{Q_G}{T_G} - \frac{Q_G}{T_C} = \frac{Q_E}{T_C} - \frac{Q_E}{T_E}$$

$$Q_G\left(\frac{1}{T_G} - \frac{1}{T_C}\right) = Q_E\left(\frac{1}{T_C} - \frac{1}{T_E}\right)$$

$$Q_G\left(\frac{T_C - T_G}{T_G \times T_C}\right) = Q_E\left(\frac{T_E - T_C}{T_C \times T_E}\right)$$

$$\therefore \quad Q_G = Q_E\left[\frac{T_E - T_C}{T_C \times T_E}\right]\left[\frac{T_G \times T_C}{T_C - T_G}\right]$$

$$Q_G = Q_E\left[\frac{T_C - T_E}{T_C \times T_E}\right]\left[\frac{T_G \times T_C}{T_G - T_C}\right]$$

$$Q_G = Q_E\left[\frac{T_C - T_E}{T_E}\right]\left[\frac{T_G}{T_G - T_C}\right] \quad \ldots(iii)$$

- Maximum COP of the system is given by,

$$(COP)_{max} = \frac{Q_E}{Q_G}$$

$$= \frac{Q_E}{Q_E\left[\frac{T_C - T_E}{T_E}\right]\left[\frac{T_G}{T_G - T_C}\right]}$$

$$\therefore \quad \boxed{(COP)_{max} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]}$$

- COP of Carnot Refrigerator $= \frac{T_E}{T_C - T_E}$
- Efficiency of Carnot Engine $= \frac{T_G - T_C}{T_G}$
- Maximum COP can be written as,

$$\therefore \quad \boxed{(COP)_{max} = (COP)_{carnot} \times \eta_{carnot}}$$

3.33.1 Solved Examples on VAS

Example 3.28 : *In a vapour absorption system, heating, cooling and refrigeration temperature are 115° C, 30° C and – 20° C respectively. Find the C.O.P. of the system. In case of heating temperature is increased to 200° C and refrigeration temperature and cooling temperature remaining same, find the new COP.*

Solution : Given Data : (I - Case)

$$T_G = 115\,°C = 115 + 273 = 388\ K$$

$$T_C = 30° C = 30 + 273 = 303\ K$$

$$T_E = -20° C = -20 + 273 = 253\ K$$

We have,

$$COP = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{253}{303-253}\right]\left[\frac{388-303}{388}\right] = 1.10$$

Given Data : (II - Case)

$$T_{G1} = 200° C = 200 + 273 = 473\ K$$

$$T_{E1} = -20° C = -20 + 273 = 253\ K$$

$$T_{C1} = 30° C = 30 + 273 = 303\ K$$

$$COP = \left[\frac{T_{E1}}{T_{C1} - T_{E1}}\right]\left[\frac{T_{G1} - T_{C1}}{T_{G1}}\right]$$

$$= \left[\frac{253}{303-253}\right]\left[\frac{473-303}{473}\right] = 1.81$$

Example 3.29 : *In an absorption system, heating, cooling and refrigeration takes place at temperature of 115° C, 30° C and – 20° C respectively. Find Theoretical COP of the system. If the generator temperature increased to 200° C and eveporator temperature decreased to – 40° C. Find % change in COP of system.*

☞ **[GTU : Summer-14, Winter-16]**

Solution : Given Data : (I - Case)

$$T_G = 115\,°C = 115 + 273 = 388\ K$$

$$T_C = 30° C = 30 + 273 = 303\ K$$

$$T_E = -20° C = -20 + 273 = 253\ K$$

We have,

$$COP = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{253}{303-253}\right]\left[\frac{388-303}{388}\right] = 1.1081$$

Given Data : (II - Case)

$$T_{G1} = 200° C = 200 + 273 = 473\ K$$

$$T_{C1} = 30° C = 30 + 273 = 303\ K$$

$$T_{E1} = -40° C = -40 + 273 = 233\ K$$

$$(COP)_1 = \left[\frac{T_{E1}}{T_{C1} - T_{E1}}\right]\left[\frac{T_{G1} - T_{C1}}{T_{G1}}\right]$$

$$(COP)_1 = \left[\frac{233}{303-233}\right]\left[\frac{473-303}{473}\right]$$

$$= 1.1963$$

$$\% \text{ change in COP} = \frac{COP_1 - COP}{COP} \times 100$$

$$= \frac{1.1963 - 1.1081}{1.1081} \times 100$$

$$= 7.96\ \% \text{ (increased)}$$

Example 3.30 : *In a vapour absorption refrigeration system, the refrigeration temperature is −15°. The generator is operated by solar heat where temperature reached is 110° C. The temperature of heat sink is 55° C. What is the maximum possible COP ?*

Solution : Given Data :

$$T_G = 110\ °C = 110 + 273 = 383\ K$$

$$T_C = 55° C = 55 + 273 = 328\ K$$

$$T_E = -15° C = -15 + 273 = 258\ K$$

$$(COP)_{max} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{258}{328-258}\right]\left[\frac{383-328}{383}\right]$$

$$(COP)_{max} = \mathbf{0.5292}$$

Example 3.31 : *In an aqua - ammonia absorption system, heat is supplied to the generator by condensing the steam at 20 kPa and 0.95 dry. The evaporator is maintained at −5° C. The cooling water available for condenser at 30° C. Find ideal COP. If actual COP of the system is 80 % of ideal COP. Find mass of the steam needed in kg/hr to produce refrigeration load of 60 TR.*

Solution : Given Data :

$$p = 200\ kPa = 2\ bar$$

$$x = 0.95$$

$$T_E = -5° C = -5 + 273 = 268\ K$$

$$T_C = 30° C = 30 + 273 = 303\ K$$

$$[COP]_{actual} = 80\ \%\ [COP]_{ideal}$$

$$R.E. = 60\ TR$$

From steam table, at p = 2 bar

$$T_{sat} = T_G = 120° C = 120 + 273 = 393\ K$$

$$h_{fg} = 2200\ kJ/kg$$

$$(COP)_{ideal} = \left[\frac{T_E}{T_C - T_E}\right]\left[\frac{T_G - T_C}{T_G}\right]$$

$$= \left[\frac{268}{303-268}\right]\left[\frac{393-303}{393}\right]$$

$$= \mathbf{1.7535}$$

$$\text{Actual COP} = 0.8 \times (COP)_{ideal} = 0.8 \times 1.7535$$

$$= \mathbf{1.4028}$$

$$\text{Actual COP} = \frac{\text{Refrigerating load}}{\text{Actual heat supplied}}$$

$$\text{Actual heat supplied} = \frac{60 \times 3.516}{1.4028} = 150.38\ kJ/sec$$

$$\text{Heat supplied per kg} = x \times h_{fg} = 0.95 \times 2200$$

$$= 2090\ kJ/kg$$

$$\text{Mass of steam needed} = \frac{150.38 \times 3600}{2090}$$

$$= \mathbf{259.02\ kg/hrs}$$

3.34 Lithium - Bromide Absorption Refrigeration System

- This refrigeration system is used to chill water.
- In this system water is used as refrigerant and Li - Br salt used as absorbent.
- Because of low vapour pressure Li - Br solution has a strong affinity for water vapour.
- It consists of absorber, generator, heat exchanger, condenser and evaporator.

- In the evaporator, refrigerant water evaporates by absorbing the latent heat from water which is to be chilled.
- The water vapours are drawn in absorber.
- In the absorber weak solution of Li - Br absorbs the water vapour and gets converted into strong solution.
- In the absorber cooling coil is placed, to increase the affinity for water vapour.
- The strong solution of Li - Br is pumped to a generator through heat exhanger.
- The function of heat exchanger is to exchange the heat from weak solution to strong solution.
- In the generator strong solution is heated by means of heating coil to release the water vapour.
- As the water vapours are releases, the strong solution is converted into weak solution.
- This weak solution is passed to the absorber through heat exchanger.
- The water vapours which are generated in the generator are passed to condenser, where it is condensed by condensing water supplied externally.
- This condensed liquid refrigerant finally passed to the evaporator where evaporation will takes place.

3.34.1 Practical Li - Br Absorption Refrigeration System OR Two Shell type Li - Br Absorption System

- It consists of following major components.
  - i) Generator
  - ii) Heat exchanger
  - iii) Absorber
  - iv) Solution Pump
  - v) Evaporator pump
  - vi) Condenser
  - vii) Evaporator
- The weak Li - Br solution transfered to generator through heat exchanger with the help of solution pump.
- In the heat exchanger, weak solution from the absorber is heated by strong solution flowing from generator.
- It reduces heat requirement in the generator and cooling load in absorber.
- The water vapour refrigerant is condensed in condenser by circulation of water.

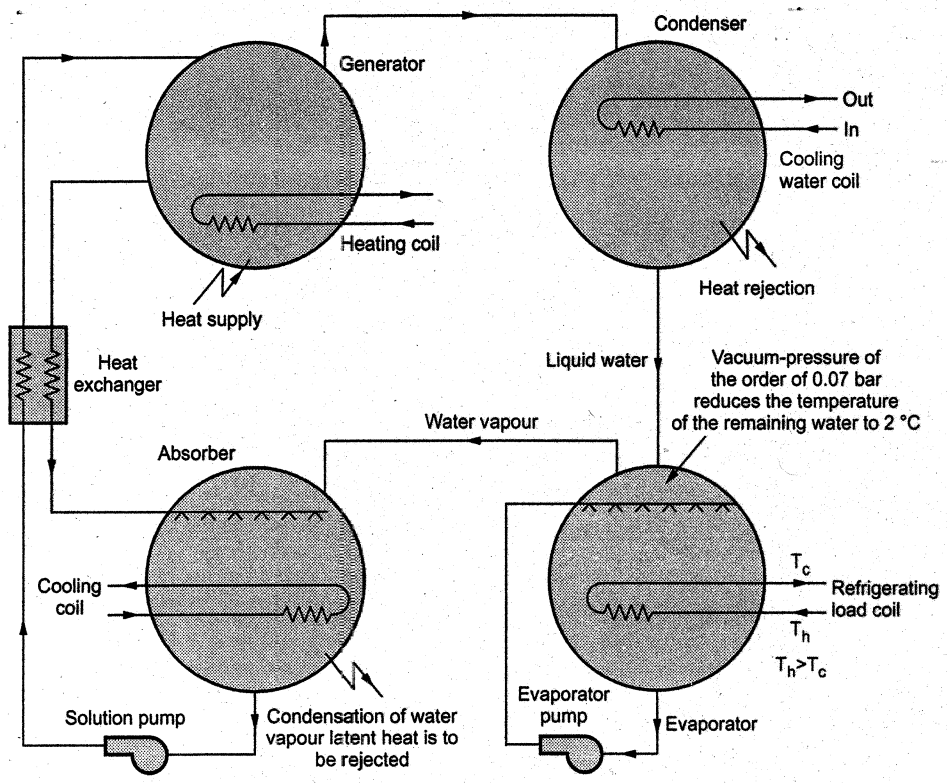


Fig. 3.52 Li - Br - Absorption Refrigeration System

- The condensed water vapour refrigerant flows down towards evaporator. During which the refrigerants pressure is reduced upto evaporator pressure by pressure reducing valve.
- In the evaporator cooled water absorbs the heat from hot water.
- During this process hot water gives away the heat and converted into chilled water.

3.34.2 Advantages of the System

- Operation and maintenance cost is very low.
- The pressure inside the system is below atmospheric.
- These plants have been built up to 1 lakh TR capacity.
- The work energy requirements are negligible.

3.34.3 Disadvantages of System

- Lithium - Bromide is corrosive.
- All joints must be made leakproof to prevent the leakages.
- Once the system stops working, the salt solution may solidify and it causes replacement of pipe.

3.35 Three Fluid System (Electrolux Refrigerator)

☞ **[GTU : Summer-16, Winter-14]**

- In this system, three fluids are used hence it is called as three fluid system.
- The three fluids used in this system are ammonia (NH_3), Hydrogen (H_2) and Water (H_2O).
- The main purpose of this system is to eliminate the pump hence system will become noiseless.

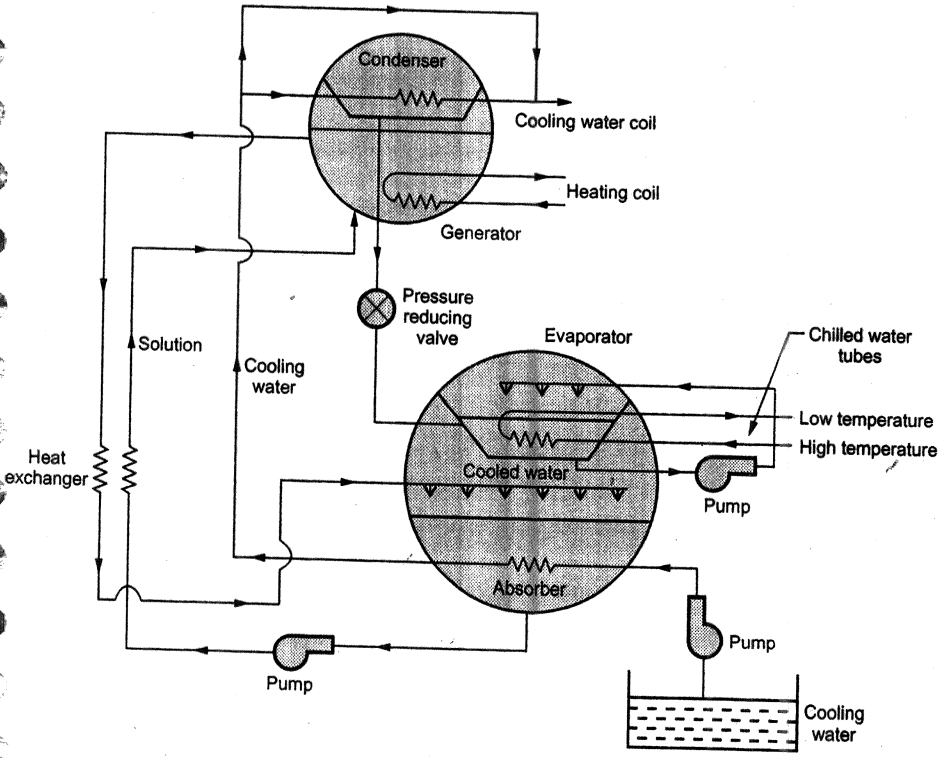


Fig. 3.60 Practical VAS (Li - Br)

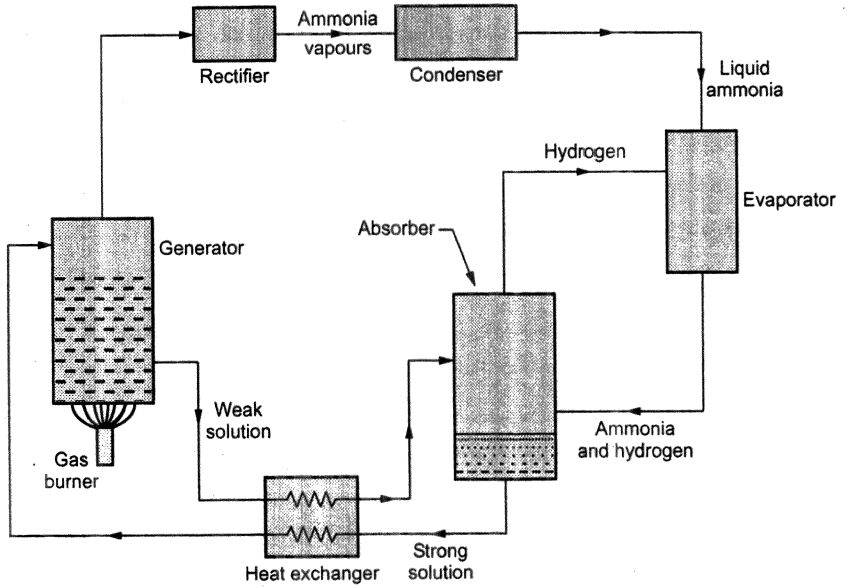


Fig. 3.54 Three Fluid System

- **Ammonia** (NH_3) used as **refrigerant** because it possesses most desirable properties.
- The **hydrogen** (H_2) is the lightest gas used to **increase the rate of evaporation** of liquid ammonia passing through evaporator.
- The **water** is used as **solvent** because it has the ability to absorb ammonia.
- The strong ammonia solution from absorber enters into the generator through heat exchanger.
- In the generator ammonia solution is heated by external source.
- During heating process, ammonia vapours are formed, and these ammonia vapours are removed from solution and passed to condenser.
- The rectifier is fitted between the generator and condenser to remove the water vapour. If these water vapours are not removed it will enter into the evaporator and causes freezing and choking of machine.
- The hot weak solution which is in the generator is passed to absorber through heat exchanger.
- In the heat exchanger heat exchange will takes place between weak solution and strong solution.

- Ammonial vapours in condenser gets condensed by using external cooling source.
- The liquid refrigerant from the condenser flows into the evaporator where it meets hydrogen (H_2) gas.
- Finally the mixture of ammonia vapour and hydrogen is passed to absorber, where ammonia (NH_3) is absorbed in water and hydrogen flows back to evaporator.
- COP of the system can be find out,

$$COP = \frac{\text{Heat absorbed in evaporator}}{\text{Heat supplied to generator}}$$

3.36 Applications of Vapour Absorption System

- VAS usually comes in higher capacities (more than 50 TR) but now China has invented lesser capacity VAS (2 TR), so it found its applications in industries, malls and theaters.
- VAS has to be applied for bigger tonnage plants
- VAS acts as standby plants for VCC.

3.37 Comparison of VCC and Vapour Absorption System

☞ **[GTU : Summer-14, 17, Winter-16]**

Sr.No.	Vapour Compression System	Vapour Absorption System
1.	This system is provided with compressor.	Compressor is replaced by absorber, generator, pump and pressure reducing valve.
2.	More wear and tear due to more no. of parts in compressors.	No wear and tear.
3.	Due to presence of compressor system is noisy.	System is quite.
4.	Cooling effect is instant.	System takes some time to produce cooling effect.
5.	Poor performance at partial load.	Better performance at partial load.
6.	It requires high grade energy.	It can be used with low grade energy.

7.	More chances of refrigerant leakage.	Less chances of refrigerant leakage.
8.	COP of the system is high.	COP of system is low.
9.	More maintenance is required	Less maintenance is required.
10.	Charging of refrigerant is simple.	Charging of refrigerant is difficult.

Compound Compression

3.38 Vapour Compression System

- Vapour compression system is used in cooling and heating applications i.e. summer air - conditioning system, winter air - conditioning system, ice formation.
- Vapour compression system mainly includes

a) Simple vapour compression system

b) Multi stage vapour compression system

a) Simple Vapour Compression System :

- The simple vapour compression system works between two pressure limits which are as follows :

  i) Low pressure in evaporator

  ii) High pressure in condenser

- In actual applications, pressure lift from evaporator to condensor is high, hence it require multi stage in compression system.

b) Multi stage compression system :

- Multi stage compression system is used for larger applications where pressure lift is high.
- In this system, pressure limits is more than two.
- This system is mainly used for reducing compressor work and increasing refrigeration effect.
- Commonly used multi pressure system are :

  1) Flash gas removal 2) Intercooler

3.39 Different Methods of Multi Stage Compression System

- The different method of multi-stage compression system are as follows :
  1) Single stage system with multi evaporators having same temperature.
  2) Single stage system with multi evaporators having different temperature.
  3) Individual compressor with multi expansion valve and evaporator.
  4) Single stage compressor with multiple expansion valve.
  5) Multi stage compression with multi evaporator system with intercooling.
  6) Multi evaporator with individual compressors and individual expansion valves.
  7) Multi stage system with flash tank and intercooling
  8) Cascade refrigeration system

3.39.1 Single Stage System with Multi Evaporators having Same Temperature

- In this system, more than two evaporators are used and each evaporator is maintained with same temperature.
- In this system, single compressor is used.
- The schematic diagram of this system is as shown in the Fig. 3.62.
- In the above schematic diagram, three evaporators are used with same temperature to increase refrigeration effect.
- The C.O.P of above system can be calculated as follows :

The refrigeration effect from process 4 - 1 can be written as,

$$m_1 = \frac{(R.E)_1}{(h_1 - h_4)}; \quad m_2 = \frac{(R.E)_2}{(h_1 - h_4)};$$

$$m_3 = \frac{(R.E)_3}{(h_1 - h_4)}$$

- The total mass in the system can be written as

$$m = m_1 + m_2 + m_3$$

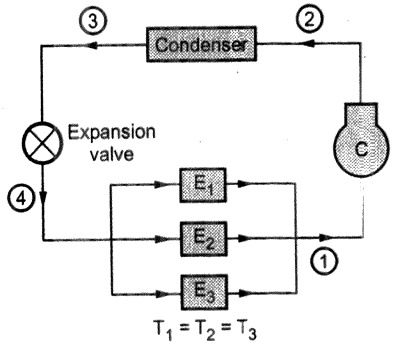
where m_1 = Mass flow rate of refrigerant in evaporator 1

m_2 = Mass flow rate of refrigerant in evaporator 2

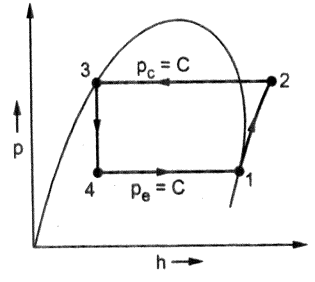
m_3 = Mass flow rate of refrigerant in evaporator 3

- The compressor work of the system can be written as,

$$W_c = m(h_1 - h_2)$$



(a)



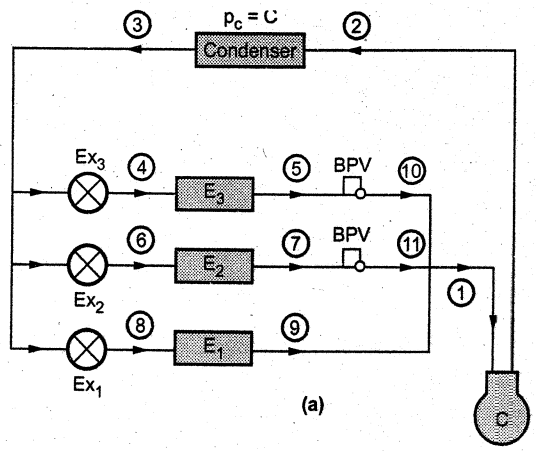
(b)

Fig. 3.55

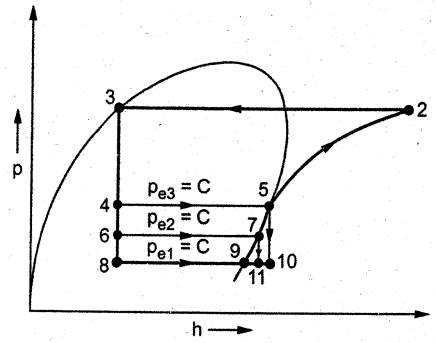
$$\therefore \quad \boxed{C.O.P = \frac{(h_2 - h_1)}{(h_1 - h_4)}}$$

3.39.2 Single Stage System with Multi Evaporators having Different Temperatures

- In this system, more than two evaporators are used and each evaporator has different temperature.
- In this system, single compressor is used.
- The schematic diagram of this system is as shown in Fig. 3.56.
- In the above schematic diagram, 3 evaporators are used with different temperature to increase refrigeration effect.
- To reduce the back pressure individual back pressure valves (B.P.V.)



(a)



(b)

Fig. 3.56

- The C.O.P. of above system can be calculated as follows :

The refrigeration effect from different processes with different masses is as follows :

$$m_1 = \frac{(R.E)_1}{(h_9 - h_8)};$$

$$m_2 = \frac{(R.E)_2}{(h_7 - h_6)};$$

$$m_3 = \frac{(R.E)_3}{(h_5 - h_4)}$$

The total mass in the system can be written as

$$m = m_1 + m_2 + m_3$$

Calculation of Mass :

By energy balance :

Enthalpy before mixing = Enthalpy after mixing

$$m \cdot h_1 = m_1 h_9 + m_2 h_{11} + m_3 h_{10}$$

where $h_{11} = h_7$ and $h_{10} = h_5$

$\therefore$ $$m \cdot h_1 = m_1 h_9 + m_2 h_7 + m_3 h_5$$

The compressor work of the system can be calculated as $m (h_2 - h_1)$

$\therefore$ $$\text{C.O.P} = \frac{(R.E)_1 + (R.E)_2 + (R.E)_3}{m (h_2 - h_1)}$$

Example 3.32 : *A data pertaining to refrigeration system is as follows : Two separate evaporators E_1 (STR) and E_2 (10TR) at temperatures 5 °C and 15 °C respectively. Common condenser is at a temperature 38 °C. The liquid from the condenser is saturated. After evaporator E_2, pressure reducing valve bring the pressure to the suction pressure of the compressor. Single compressor and individual expansion valves are used in the system. Use refrigerant data provided. Make the neat flow diagram and a cycle diagram on p-h plane, Find;*
i) Power required to run the system ii) COP of the system

Refrigerant Data								
T	p	v_f	v_g	h_f	h_g	h_{f0}	s_f	s_g
°C	bar	m^3/kg		kJ/kg			kJ/kgK	
5	3.63	0.000725	0.0475	40.69	189.65	148.96	0.1586	0.6943
15	4.91	0.000743	0.0355	50.1	193.79	143.69	0.1915	0.6902
38	9.14	0.000794	0.0191	72.56	202.51	129.95	0.2654	0.683

R134-a Data								
T	p	v_g	h_f	h_g	s_f	S_g	(C_p)	(C_p)
°C	bar	m^3/kg	kJ/kg	kJ/kg	kJ/kgK	kJ/kgK	kJ/kgK	kJ/kgK
–10	2.0052	0.09963	186.78	-	0.9509	1.7337	-	0.842

–6	2.3418	0.8591	192.03	395.15	0.9707	1.731	-	0.858
2	3.145	0.0647	202.68	399.84	-	1.7263	1.341	0.892
12	4.4289	0.04636	-	405.51	1.0579	1.7215	1.374	0.939
24	6.4566	0.03189	233.05	411.93	-	1.7169	-	1.006
30	7.7008	0.02667	241.65	414.94	1.1432	1.7149	-	1.044
50	13.177	-	271.59	423.63	1.2373	1.7078	1.569	1.218

Solution :

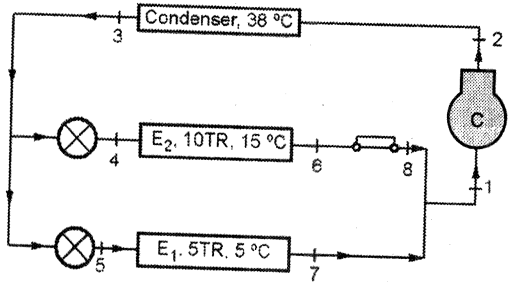


Fig. 3.57

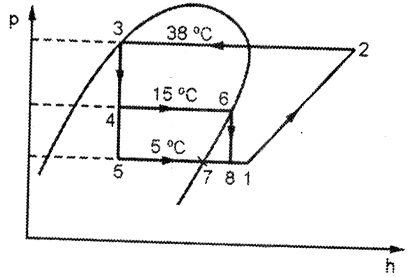


Fig. 3.58

$h_7 = h_{g7} = 189.65$ kJ/kg

$h_6 = h_8 = h_{g6} = 193.79$ kJ/kg

$h_5 = h_4 = h_3 = h_{f3} = 72.56$ kJ/kg

$Q_{e1} = 5TR = 5 \times 3.517 = 17.585$ kW

$Q_{e2} = 1OTR = 10 \times 3.517 = 35.17$ kW

The flow rate through each evaporator is given as,

$$\dot{m}_I = \frac{Q_{e1}}{h_7 - h_5} = \frac{17.585}{189.65 - 72.56}$$

$$= 0.1525 \text{ kg/s}$$

$$\dot{m}_{II} = \frac{Q_{e2}}{h_6 - h_4} = \frac{35.17}{193.79 - 72.56}$$

$$= 0.2901 \text{ kg/s}$$

From energy balance

$$\dot{m}_I \cdot h_7 + \dot{m}_{II} \cdot h_6 = \dot{m}_c \cdot h_1$$

$$\therefore 0.1525 \times 189.65 + 0.29 \times 193.79 = (0.15 + 0.29)\, h_1$$

$$\therefore \quad h_1 = 193.456 \text{ kJ/kg}$$

We know that for point 1

$$\therefore \quad h_{sup} = h_g + C_p (T_{sup} - T_{sat})$$

$$\therefore \quad 193.456 = 189.65 + 1.4\,(T_{sup} - 278)$$

$$\therefore \quad T_{sup} = 280.7185 \text{ K}$$

Also, for point 1

$$S_{sup} = S_g + C_p \, ln \left[\frac{T_{sup}}{T_{sat}}\right]$$

$$S_{sup} = 0.6943 + 1.4 \, ln \left[\frac{280.7185}{278}\right]$$

$$= 0.7079 \text{ kJ/kgK}$$

For isentropic process 1-2

$$S_1 = S_2 = S_g + C_p ln \left[\frac{T_{sup}}{T_{sat}}\right]$$

$$\therefore \quad 0.7079 = 0.6830 + 1.4 \, ln \left[\frac{T_{sup}}{311}\right]$$

$$T_{sup} = 316.58 \text{ K}$$

Now for point 2,

$$h_2 = h_g + C_p(T_{sup} - T_{sat})$$

$$h_2 = 202.51 + 1.4[316.58 - 311]$$

$$= 210.322 \text{ kJ/kg}$$

Power required is given by,

$$P = \dot{m}_C \times (h_2 - h_1)$$

$$= (0.15 + 0.29)\ (210.322 - 193.456)$$

$$\mathbf{P = 7.421\ kW}$$ **...Ans.**

COP of the machine is given as,

$$COP = \frac{(Q_{e1} + Q_{e2}) \times 3.517}{\text{Power required}} = \frac{(5+10) \times 3.517}{7.421}$$

$$\mathbf{COP = 7.108}$$ **...Ans.**

Example 3.33 : *A single compressor using R-12 as refrigerant has three evaporators capacity 10TR, 20TR, 30TR. All the evaporators operates at –10 °C and the vapour leaving the evaporators are dry and saturated. The condenser temperature is 40 °C. The liquid refrigerant leaving the condenser is subcooled to 30 °C, assuming isentropic compression, find :*

(1) The mass of refrigerant flowing through each evaporator.

(2) The power required to drive the compressor.

(3) The COP of the system.

Solution : Given data :

Refrigerant - 12, $R.E_1 = 10 \times 3.52 = 35.2$ kJ/sec, $R.E_2 = 20 \times 3.52 = 70.4$ kJ/sec

$R.E_3 = 30 \times 3.52 = 105.6$ kJ/sec, $T_E = -10°C$,

$T_C = 40°C = T_3'$, $T_3 = 40°C$

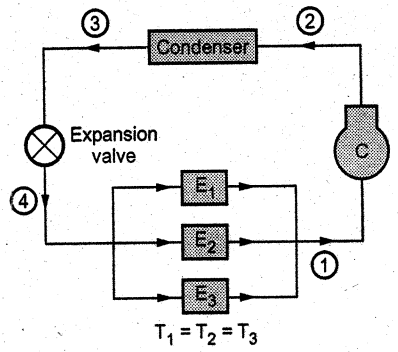
To find:

1) $\dot{m}$, 2) P , 3) C.O.P

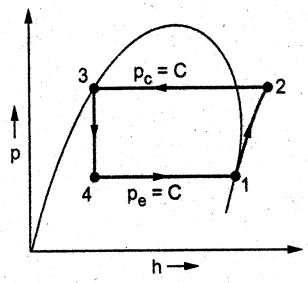
1) Mass of refrigerant flowing through each evaporator ($\dot{m}$)

For Evaporator 1 :

$$(R.E)_1 = \dot{m}_1(h_1 - h_4)$$



(a)



(b)

Fig. 3.59

$$\therefore \quad \dot{m}_1 = \frac{(R.E)_1}{h_1 - h_4}$$

From R - 12 Refrigeration Chart,

$$h_1 = 410 \text{ kJ/kg}, \quad h_2 = 438 \text{ kJ/kg}$$

$$h_3 = 235 \text{ kJ/kg} = h_4$$

$$\therefore \quad \dot{m}_1 = \frac{35.2}{(410-235)} = \mathbf{0.2011\ kg/sec}$$

For Evaporator 2 :

$$R.E_2 = \dot{m}_2(h_1 - h_4)$$

$$\therefore \quad \dot{m}_2 = \frac{R.E_2}{h_1 - h_4}$$

$$\dot{m}_2 = \frac{70.4}{(410-235)}$$

$$\dot{m}_2 = \mathbf{0.4022\ kg/sec}$$

For Evaporator - 3

$$R.E_3 = \dot{m}_3 (h_1 - h_4)$$

$$\therefore \quad \dot{m}_3 = \frac{R.E_3}{h_1 - h_4} = \frac{105.6}{(410-235)}$$

$$\dot{m}_3 = \mathbf{0.6034\ kg/sec}$$

Total mass flowing through evaporator is

$$\dot{m} = \dot{m}_1 + \dot{m}_2 + \dot{m}_3$$

$$= 0.2011 + 0.4022 + 0.6034$$

$$\therefore \quad \dot{m} = \mathbf{1.2067\ kg/sec}$$

2) Power required to drive the compressor (P)

$$P = \dot{m} \times W_c$$

$$W_c = (h_2 - h_1) = (438 - 410) = 28\ kJ/kg$$

$$\therefore \quad P = \mathbf{1.2067\ (28) = 33.78\ kW}$$

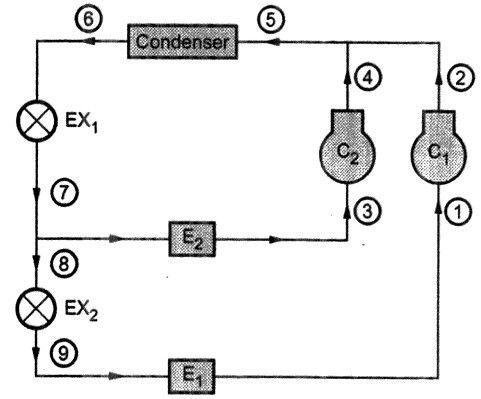
3) C.O.P of System

$$C.O.P = \frac{R.E_1 + R.E_2 + R.E_3}{P}$$

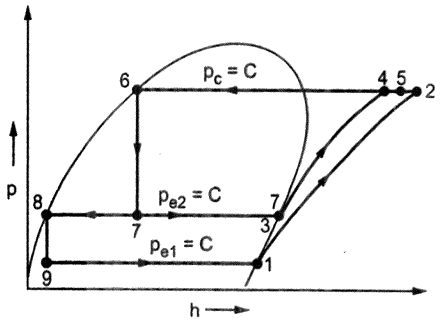
$$= \frac{35.2 + 70.4 + 105.6}{33.78} = 6.25$$

3.39.3 Individual Compressor with Multi Expansion Valve and Multi Evaporators

- In this system, multiple evaporators, multiple expansion valves and seperator compressor for each evaporator is used.
- The schematic diagram is as shown in the Fig. 3.67.
- In the above schematic diagram, two evaporators, two compressors and two expansion valves are used to inprove refrigeration effect and reduce compressor work.



(a)



(b)

Fig. 3.60

- The C.O.P. of the above system can be calculated as follows :
- Refrigeration effect can be written as

For Evaporator 1

$$(R.E)_1 = m_1 (h_1 - h_9)$$

For Evaporator 2

$$(R.E)_2 = m_2 (h_3 - h_7)$$

- Total refrigeration effect can be written as,

$$(R.E.) = (R.E)_1 + (R.E)_2$$

- Total mass can be calculated as,

$$m = m_1 + m_2$$

- Compressor work of the system can be written as,

For Compressor 1

$$W_{C1} = m_1(h_2 - h_1)$$

For Compressor 2

$$W_{C2} = m_2(h_4 - h_3)$$

- Total compressor work is

$$W_C = W_{C1} + W_{C2}$$

- Hence C.O.P is

$$\text{C.O.P.} = \frac{\text{R.E.}}{W_C} = \frac{(\text{R.E})_1 + (\text{R.E})_2}{(W_{C1} + W_{C2})}$$

$$\text{COP} = \frac{m_1(h_1 - h_9) + m_2(h_3 - h_7)}{m_1(h_2 - h_1) + m_2(h_4 - h_3)}$$

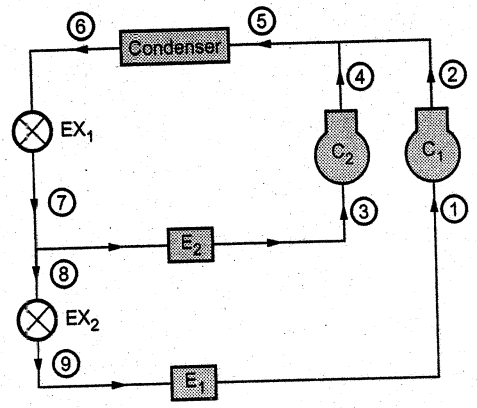
Example 3.34 : *A R - 12 refrigeration system as shown in Fig. 5.7 works with two evaporators, individual expansion valves, individual compressors and with single condenser. Draw the cycle on (p - h) diagram and calculate :*
i) Mass of refrigerant circulated to each evaporator.
ii) Compressor power for each compressor and total power.
iii) COP of the system.
Assume that refrigerant leaving the each evaporator is dry saturated and the liquid refrigerant leaving the condenser is subcooled to 30° C. Compression is isentropic in each compressor.

Solution : Given data :

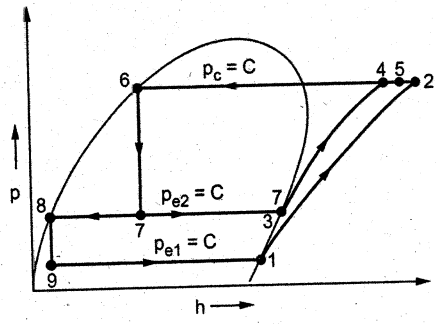
Refrigerant = R - 12, $R.E_1 = 60 \times 3.52 = 211.2$ kW, $R.E_2 = 50 \times 3.52 = 176$ kW
$T_{E1} = -20\,°C$, $T_{E2} = 0°C$, $T_C = 40\,°C = T_3'$, $T_3 = 30°C$

To find:

1) Mass of refrigerant circulated to each evaporator ($\dot{m}$) ,
2) Compressor power for each compressor and total power
3) C.O.P of System



(a)



(b)

Fig. 3.61

1) Mass of refrigerant flowing through each evaporator ($\dot{m}$)

For Evaporator 1 :

$$R.E_1 = \dot{m}_1(h_1 - h_7)$$

$$\therefore \quad \dot{m}_1 = \frac{R.E_1}{h_1 - h_7}$$

From R - 12 Refrigeration Chart,

$$h_1 = 344 \text{ kJ/kg}, \quad h_2 = 367 \text{ kJ/kg}$$

$$h_3 = 352 \text{ kJ/kg}, \quad h_4 = 366 \text{ kJ/kg},$$

$$h_5 = 229 \text{ kJ/kg} = h_6 = h_7$$

$$\therefore \quad \dot{m}_1 = \frac{211.2}{(344-229)} = \mathbf{1.836\ kg/sec}$$

For Evaporator 2 :

$$R.E_2 = \dot{m}_2 (h_3 - h_6)$$

$$\therefore \quad \dot{m}_2 = \frac{R.E_2}{(h_3 - h_6)}$$

$$\dot{m}_2 = \frac{176}{(352-229)} = \mathbf{1.430\ kJ/sec}$$

$$\text{Total mass } \dot{m} = \dot{m}_1 + \dot{m}_2 = 1.836 + 1.430$$

$$= \mathbf{3.266\ kg/sec}$$

2) Compressor Power for each compressor and total power

For Compressor 1 :

$$P_1 = \dot{m}_1 (h_2 - h_1) = 1.836\,(367 - 344)$$

$$= \mathbf{42.22\ kW}$$

For Compressor 2 :

$$P_2 = \dot{m}_2 (h_4 - h_3) = 1.430\,(366 - 352)$$

$$= \mathbf{20.02\ kW}$$

$$\text{Total compressor work } P = P_1 + P_2$$

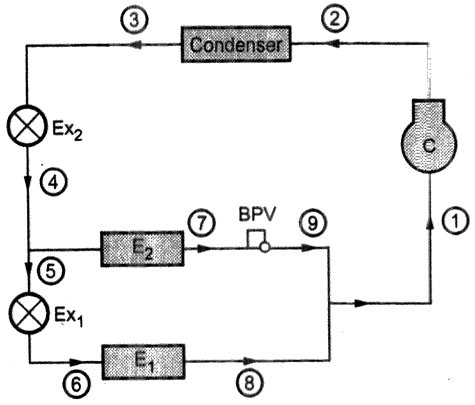
$$= 42.22 + 20.02 = \mathbf{62.24\ kW}$$

3) COP of System :

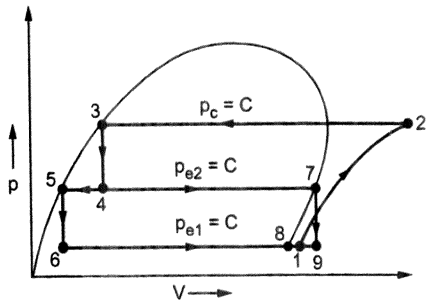
$$COP = \frac{R.E_1 + R.E_2}{P} = \frac{211.2+176}{60.24} = \mathbf{6.42}$$

3.39.4 Single Stage Compressor with Multiple Expansion Valve

- In this system, single compressor multiple expansion valve and seperate evaporators are used.
- Schematic diagram of this system is as shown in the Fig. 3.69.
- In the above diagram two evaporators, two expansion valves and single compressor is used.



(a)



(b)

Fig. 3.62

- COP of the system can be written as

$$COP = \frac{\text{Refrigeration effect}}{\text{Work done}}$$

$$COP = \frac{(R.E)}{(W_C)}$$

$$(R.E.) = (R.E)_1 + (R.E)_2$$

$$(R.E)_1 = m_1 (h_8 - h_6)$$

$$(R.E)_2 = m_2 (h_7 - h_4)$$

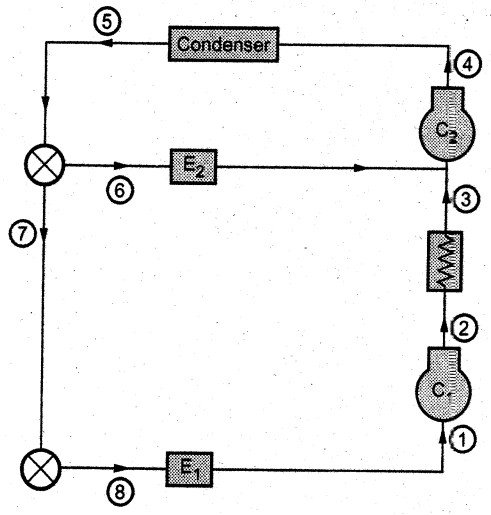
Compressor Work

$$W_C = W_{C1} + W_{C2}$$

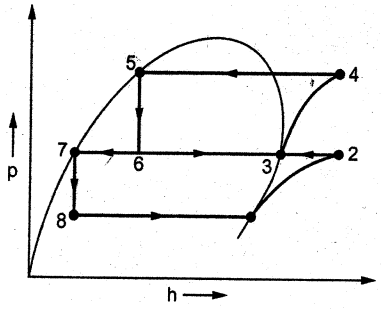
$$W_{C1} = W_{C2} = m(h_2 - h_1)$$

$$\therefore \quad \boxed{COP = \frac{m_1(h_8 - h_6) + m_2(h_7 - h_4)}{m(h_2 - h_1)}}$$

3.39.5 Multi Stage Compression with Multi Evaporator System with Intercooling



(a)



(b)

Fig. 3.63

- In this system, multi stage compressor is used to reduce power required.
- In multistage compressor, intercooling is necessary for improving efficiency of compressor.
- Hence intercooling is used between the compressor.
- Schematic diagram of this system is shown in Fig. 5.9.
- In the above system, two evaporators and one multistage compressor is used with individual expansion valves.
- The COP of the system can be written as,

$$COP = \frac{(R.E)_1 + (R.E)_2}{(W_{C1} + W_{C2})}$$

$$\boxed{COP = \frac{m_1(h_1 - h_8) + m_2(h_3 - h_6)}{m_1(h_2 - h_1) + m_2(h_4 - h_3)}}$$

3.39.6 Multi Evaporator with Individual Compressors and Individual Expansion Valves

- In this system, more than two evaporators, more than two expansion valves and individual compressor for each evaporator is used.
- This system is mainly used to improve refrigeration effect and to reduce compressor capacity.
- The schematic diagram of this system is shown in Fig. 3.64.
- In the above system, 3 evaporators, 3 expansion valves and 3 separate compressor are used.
- The C.O.P of this system can be calculated as,

$$COP = \frac{(R.E)_1 + (R.E)_2 + (R.E)_3}{W_{C1} + W_{C2} + W_{C3}}$$

$$(R.E)_1 = m_1(h_1 - h_{12}) ;$$

$$(R.E)_2 = m_2(h_3 - h_{10});$$

$$(R.E)_3 = m_3(h_5 - h_8);$$

Total refrigeration effect

$$(R.E.) = (R.E)_1 + (R.E)_2 + (R.E)_3$$

$$(R.E.) = m_1(h_1 - h_2) + m_2(h_3 - h_{10}) + m_3(h_5 - h_8)$$

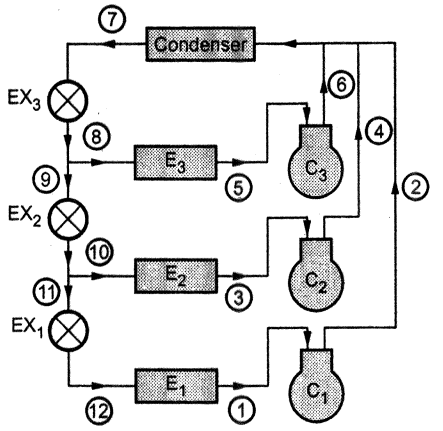


Fig. 3.64

$$W_{C1} = m_1(h_2 - h_1);$$

$$W_{C2} = m_2(h_4 - h_3);$$

$$W_{C3} = m_3(h_6 - h_5);$$

Total compressor work,

$$W_C = W_{C1} + W_{C2} + W_{C3}$$

$$W_C = m_1(h_2 - h_1) + m_2(h_4 - h_3) + m_3(h_6 - h_5)$$

$$COP = \frac{m_1(h_1 - h_2) + m_2(h_3 - h_{10}) + m_3(h_5 - h_8)}{m_1(h_2 - h_1) + m_2(h_4 - h_3) + m_3(h_6 - h_5)}$$

3.39.7 Multi Stage System with Flash Gas Removal Intercooling

- In this system, flash tank is used for removal of gases coming from an expansion valve.
- If the gases is removed, the refrigeration effect in the evaporator is increased.
- From this system, the C.O.P. is better compared to other system.
- The schematic diagram of the system is as shown in the Fig. 3.65.

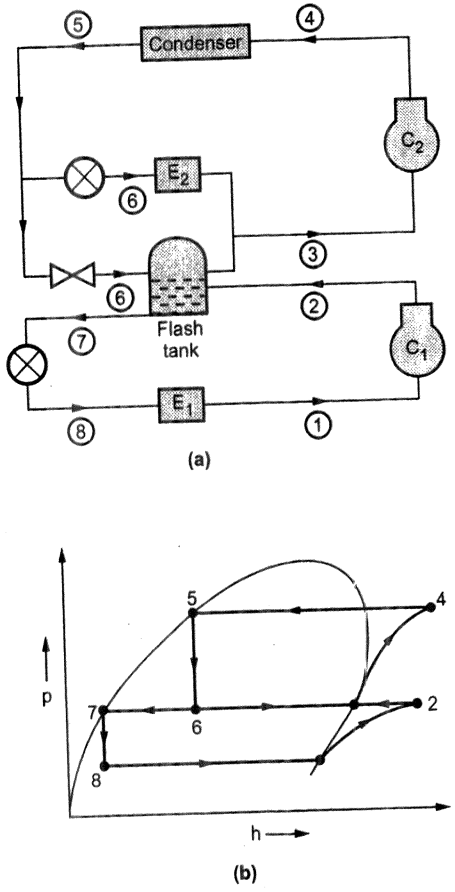


Fig. 3.65

- In the above system, flash gas removal is used between the evaporators and one multi - stage compressor is used.

- Individual expansion valve is used for each evaporator.
- In this system, the refrigerant coming out from the compressor is super heated and before compression is dry saturated.
- Therefore the C.O.P of this system is maximum. The C.O.P can be written as,

$$COP = \frac{(R.E.)}{(W_C)}$$

$$COP = \frac{(R.E)_1 + (R.E)_2}{W_{C1} + W_{C2}}$$

$$\boxed{COP = \frac{m_1(h_1 - h_8) + m_2(h_3 - h_6)}{m_1(h_2 - h_1) + m_2(h_4 - h_3)}}$$

Review Questions

1. *Explain reverse carnot cycle.*
2. *Explain difference between heat engine, heat pump and refrigeration.*
3. *Explain Bell-coleman cycle.*
4. *Explain milk processing - commercial application.*
5. *Explain with heat sketch ice plant.*
6. *Explain air conditioning system used in multiplex.*
7. *Explain the properties of ideal refrigerant.*
8. *Give the classification of refrigerant.*
9. *Explain thermodynamic properties of refrigerant.*
10. *What is ozone depletion and ODP ?*
11. *Explain global warming.*
12. *Write a short note on selection of refrigerant.*
13. *What is LCCP and RRR ?*
14. *Explain with neat sketch simple VCC.*
15. *Define dry compression, wet compression, superheating and subcooling.*
16. *Define COP, EER, IPLV, NPLV.*
17. *Explain effect of operating parameters on performance of VCC.*
18. *Explain methods of improving the COP.*
19. *Explain actual VCC how it differs from theoretical VCC..*
20. *What is vapour absorption refrigeration system ?*

21. *Explain with neat sketch practical vapour absorption system.*
22. *State the advantages of vapour absorption system.*
23. *Distinguish between VCC and VAS.*
24. *Explain the desirable properties of absorbent used in VAS.*
25. *Explain Li - Br vapour absorption system.*
26. *Explain three fluid refrigeration system.*
27. *Explain simple vapour compression cycle and multistage vapour compression cycle.*
28. *Give the advantages of multistage compression with flash gas removal and intercooling.*
29. *With the help of p - h diagram, explain multipressure compression system with individual compressor and multiple expansion valve.*
30. *Explain multievaporator with different temperature refrigeration system with help of p - h diagram.*
31. *With the help of neat sketch explain cascade refrigeration system.*

Refrigerant and Refrigeration Cycles ends ...

Notes

CHAPTER - 4

FUEL AIR AND ACTUAL CYCLES

Syllabus : Assumptions for fuel-air cycles, reasons for variation of specific heats of gases, change of internal energy and enthalpy during a process with variable specific heats, isentropic expansion with variable specific heats, effect of variable specific heats on Otto, Diesel and Dual cycle, dissociation, comparison of air standard and fuel air cycles, effect of operating variables, comparison of air standard and actual cycles, effect of time loss, heat loss and exhaust loss in petrol and diesel engines.

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4.2	IC and EC Engine	4 - 3
4.3	I.C. Engine and Its Components	4 - 3
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4.1 Heat Engine

- Heat Engine is a **mechanical device**, which converts heat energy into mechanical energy.

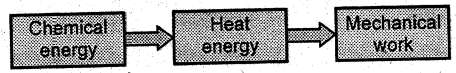


Fig. 4.1

- **Examples :** Gas turbine power plant, Steam power plant, Internal combustion engine, etc.
- **Heat engines** are classified into two types, etc.
  1) Internal combustion (I.C.) engine
  2) External combustion (E.C.) engine

4.2 IC and EC Engine

☞ **[GTU : Winter-16, Summer-18]**

- Any machine which derives heat energy from the combustion of fuel (chemical energy of fuel) and convert part of this energy into mechanical work is called as **heat engine.**
- These heat engines may be externally combustion type or internally combustion type.
- In case of external combustion (E.C) engines, the combustion of fuel takes place outside the cylinder.
  For example : Steam engines, steam and gas turbines, air engines, etc.
- In case of internal combustion engines, the combustion of fuel in the presence of air takes place inside the cylinder and produced gases act on piston to develop the power.
  For example : Petrol engines, diesel engines, gas engines, etc.
- Internal combustion (I.C.) engines are commonly used in vehicles, locomotives and in various industrial applications.

4.3 I.C. Engine and Its Components

☞ **[GTU : Summer-16,19]**

- Fig. 4.2 shows reciprocating I.C. engine and its components. (See Fig. 4.2 on next page). The main components of this 4-stroke petrol engine are as follows :

1. Cylinder	2. Piston
3. Crank and crankshaft	4. Connecting rod
5. Spark plug	6. Inlet and exhaust valve
7. Crank pin	8. Gudgeon pin
9. Cylinder head	10. Piston rings
11. Suction and exhaust manifold	12. Carburettor

1. **Cylinder :** It is the main body of an I.C. engine in which piston reciprocates to develop power. It has to withstand very high pressures (upto 75 bar) and temperatures (upto 2400 C), because the the combustion takes place in the cylinder. It may be air cooled or water cooled. It is generally made of cast iron or alloy steel.

2. **Piston :** It is used to compress the charge (air-fuel mixture) during compression stroke and to transmit the gas force to the connecting rod and then crank during power stroke.

3. **Crank and crankshaft :** Crank is the integral part of the crankshaft and crankshaft is the backbone of the engine, crankshaft is supported in main bearings and it has a heavy wheel called as flywheel to control the fluctuation of torque.

4. **Connecting rod :** It is a rod of circular or rectangular cross section. Its small end is connected to the piston and big end is connected to the crank. It converts reciprocating motion of pistion into rotary motion of crankshaft.

5. **Spark plug :** It is used to initiate the spark for the combustion of air-fuel mixture in petrol engines.

6. **Inlet and exhaust valve :** Inlet valve controls the admission of the charge during suction stroke and exhaust valve is used for the removal of exhaust gases after doing work on the pistion.

7. **Crank pin and Gudgeon pin :** Crank pin is used to connect big end of connecting rod to the crank and gudgeon pin is used to connect small end of connecting rod to the piston.

8. **Cylinder head :** It is used for closing the one end of the cylinder. It houses inlet and exhaust valves through which charge is taken inside the cylinder and burned gases are exhausted to the atmosphere from the cylinder.

9. **Piston rings :** Piston rings are housed in the circumferential grooves provided on the outer surface of the piston. It gives tight fit between the pistion and cylinder hence prevents the leakage of high pressure gases.

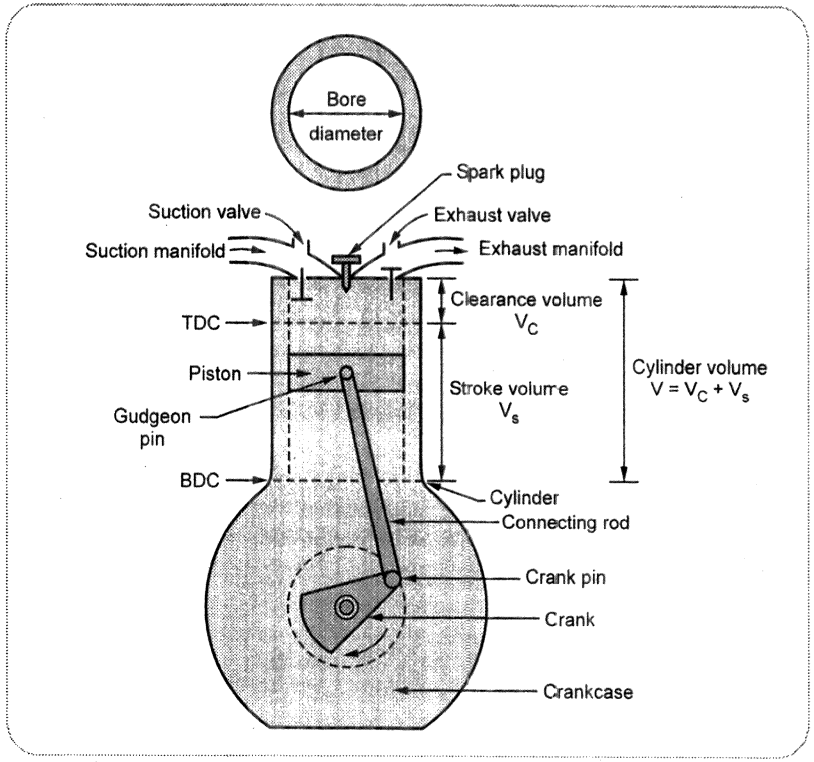


Fig. 4.2 : Four stroke I.C. engine (petrol) and its components

10. Suction and exhaust manifold : Suction manifold is the passage which carries the charge from carburettor to the engine where as exhaust manifold is the passage which carries the exhaust gases from the exhaust valve to the atmosphere.

11. Carburettor : It is used to supply the uniform air-fuel mixture to the cylinder of petrol engine through the intake manifold. It is controlled by a throttle valve.

4.4 Classification of I.C. Engine

☞ **[GTU : Summer-14 Winter-16]**

I.C. engines are classified as follows :

❑ 1. According to the cycle of operation

a) Four stroke engines : In these engines cycle is completed in two revolution of crank.

b) Two stroke engines : In these engines cycle is completed in one revolution of crank.

❑ 2. According to the nature of thermodynamic cycle

a) Constant volume or Otto cycle.

b) Constant pressure or diesel cycle.

c) Partly Otto and partly diesel cycle or dual cycle.

❑ 3. According to the method of ignition

a) **Spark Ignition (S.I.) engines :** In spark ignition or **petrol engines**, air-fuel mixture supplied by carburettor is ignited by producing spark.

b) **Compression Ignition (C.I.) engines :** In compression ignition or **diesel engines** air is compressed to higher pressure and in fine atomised form fuel is injected in combustion chamber. Hence air-fuel mixture is ignited without producing spark.

❑ 4. According to the number of cylinders

a) Single cylinder engines

b) Multi-cylinder engines

❑ 5. According to the type of cooling system

a) Air cooled engines : For cooling purpose air is used.

b) Water cooled engines : Water jackets are provided around the cylinder to cool the engine.

❑ 6. According to the speed of engine

a) Low speed engines

b) Medium speed engines

c) High speed engines

❑ 7. According to the lubrication method

a) Wet sump lubrication

b) Dry sump lubrication

c) Pressure lubrication

❑ 8. According to the fuel used

a) Petrol engines

b) Diesel engines

c) Gas engines

❑ 9. According to the field of operation

a) Automotive engines

b) Stationary engines

c) Locomotive engines

d) Marine engines

❑ 10. According to the arrangement of cylinder (Refer Fig. 4.3)

a) Vertical engines : In these engines, cylinder is in vertical position.

b) Horizontal engines : In these engines, cylinder is in horizontal position.

c) V-engines : In these engines, two cylinders are arranged at an angle connected to a common crank.

d) Radial engines : In these engines 4 to 6 cylinders are placed radially and equally spaced and connected to a common crank.

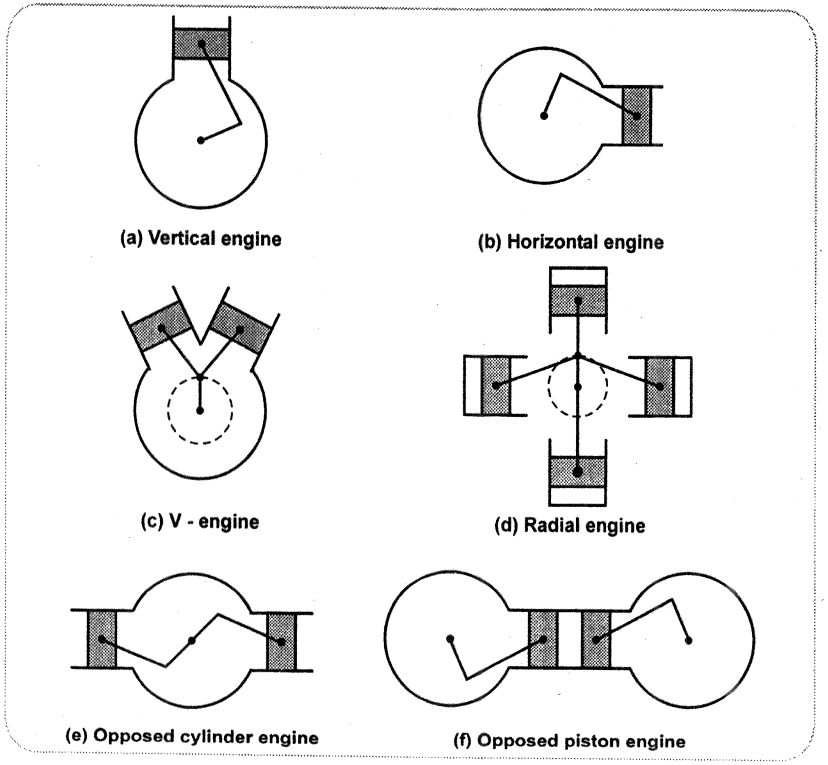


Fig. 4.3 : Arrangement of cylinder in I.C. engine

e) **Opposed cylinder engines :** In these engines, cylinders are arranged on opposite side of a common crank.

f) **Opposed piston engines :** In these engines, single cylinder houses two pistons, each of which drives separate crank.

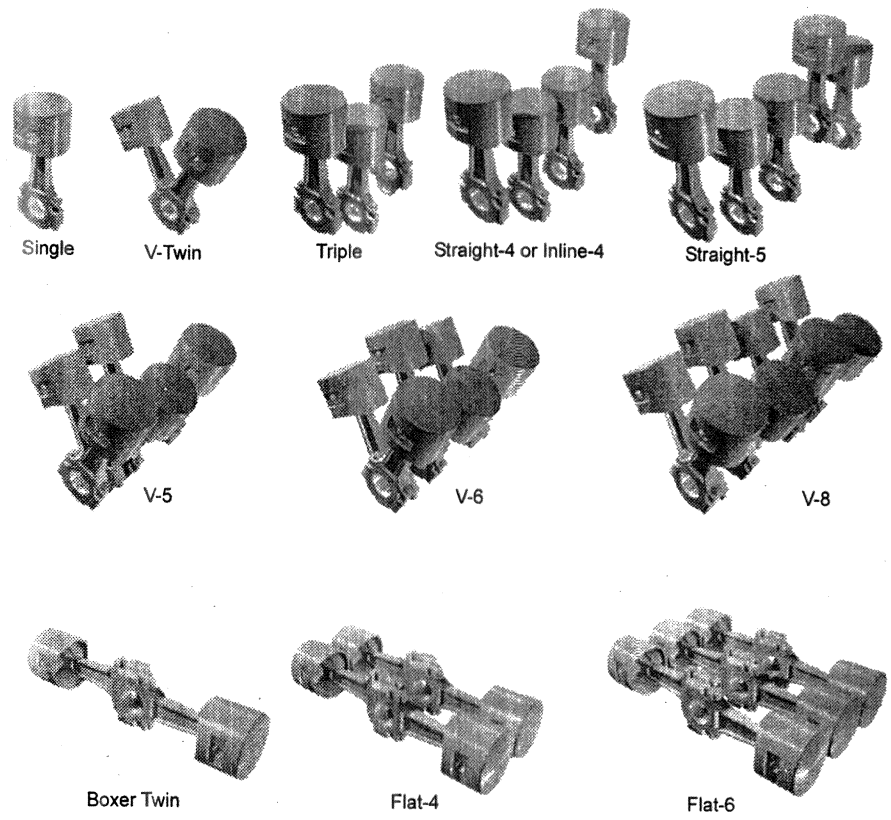


Fig. 4.3(a) : Arrangement of cylinders in I.C. engine

4.5 Terminology used in I.C. Engine ☞ [GTU : Summer-16,18, Winter-17]

In addition to the above components of I.C. engine, certain standard terminology used in I.C. engine is as follows :

1. Bore
2. Stroke
3. Top Dead Centre (T.D.C.)
4. Bottom Dead Centre (B.D.C.)
5. Clearance volume
6. Swept volume
7. Compression ratio

1. **Bore :** It is the inside diameter of the cylinder.
2. **Stroke :** It is the maximum distance travelled by the piston in the cylinder in one direction. It is generally twice the radius of crank.
3. **Top Dead Centre (T.D.C.) :** It is the extreme position of the piston at the top of the cylinder. For horizontal engines it is called as inner dead centre (I.D.C.).
4. **Bottom Dead Centre (B.D.C.) :** It is the extreme position of the piston at the bottom of the cylinder. For horizontal engines it is called as outer dead centre (O.D.C.).
5. **Clearance volume :** It is the volume contained in the cylinder above the top of the piston when it is at T.D.C. It is denoted by .
6. **Swept volume :** It is the volume swept by the piston in moving between TDC and BDC. It is denoted by .
7. **Compression ratio :** It is the ratio of the volume when the piston is at BDC to the volume when the piston is at TDC. It is denoted by r.

4.6 Four Stroke Petrol Engine

- In a four stroke cycle engine, the cycle is completed in two revolutions of crankshaft and four strokes are performed.

1. Suction stroke	2. Compression stroke
3. Expansion/Power stroke	4. Exhaust stroke

- Each stroke consists of 180 of crankshaft rotation, hence the cycle consists of 720 of crankshaft rotation for one cycle.
- Four stroke cycle engine consists of following strokes (Refer Fig. 4.4) :

1. Suction stroke

- This stroke starts when the piston is at TDC and about to move downwards.
- During this stroke inlet valve remains open and exhaust valve remains closed.
- Due to low pressure created by downward moving piston, the charge (air-fuel mixture) is drawn into the cylinder.
- At the end of this stroke the inlet valve closes.

2. Compression stroke

- During this stroke, the compression of fresh sucked charge takes place by the return stroke (BDC to TDC) of piston.
- During this stroke, both inlet and exhaust valve remains closed.
- Just before completion of compression stroke a charge is ignited by a spark plug and combustion takes place.

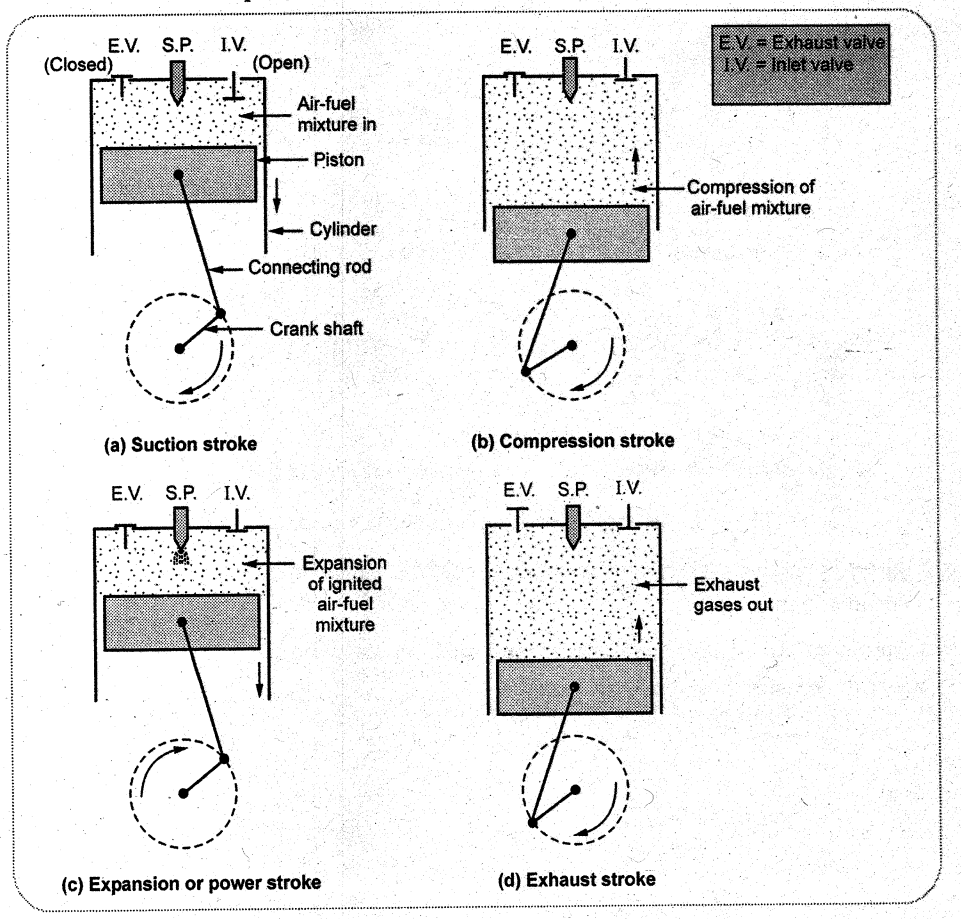


Fig. 4.4 : Four stroke petrol (S.I.) engine

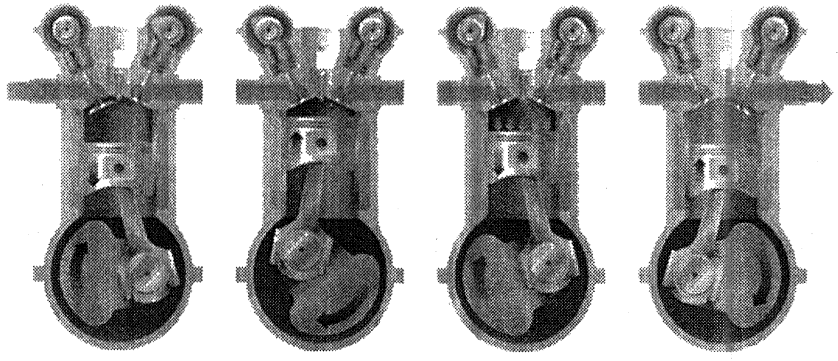


Fig. 4.4(a) : Four stroke petrol (S.I.) engine

❑ 3. Expansion or power stroke

- The products of combustion exerts pressure on the piston hence forcing it to move downwards i.e. TDC to BDC.
- During this stroke, both inlet and exhaust valve remains closed.
- Power is developed during expansion of gases hence this stroke is also called as **power stroke**.
- During power stroke, both temperature and pressure decreases.

❑ 4. Exhaust stroke

- At the end of power stroke the exhaust valve opens but the inlet valve remains closed.
- During this stroke piston moves from BDC to TDC and burnt gases inside the cylinder are exhausted.
- At the end of this stroke exhaust valve closes but some residual gases remain in the cylinder.
- In this way, again the inlet valve opens and the new cycle starts.

Note : Each cycle of four stroke engine completes the above strokes in two crank rotations. One revolution of crankshaft occurs during the suction and compression strokes and the other revolution during the expansion and exhaust stroke.

📖 4.7 Four Stroke Diesel Engine

- The working of four stroke diesel engine is similar to four stroke petrol engine, except the following two major points :

- In C.I. engines the air is only drawn during the suction stroke instead of air-fuel mixture drawn in S.I. engines.
- In C.I. engines the fuel is injected by using injector into the cylinder and the high pressure and temperature mixture is burnt. Hence there is no need of spark plug.

- Diesel engine cycle consists of following four strokes (Refer Fig. 4.5) :

1. Suction stroke	2. Compression stroke
3. Expansion/Power stroke	4. Exhaust stroke

❑ 1. Suction stroke

- During this stroke air is drawn into the cylinder and piston moves from TDC to BDC.
- The exhaust valve remains closed during this stroke.

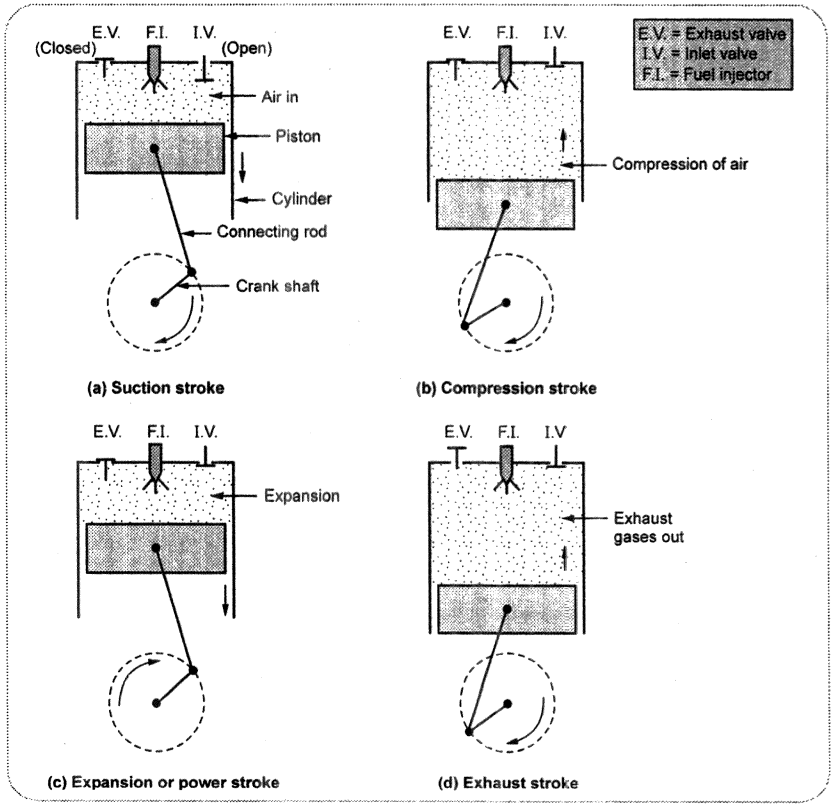


Fig. 4.5 : Four stroke diesel (C.I.) engine

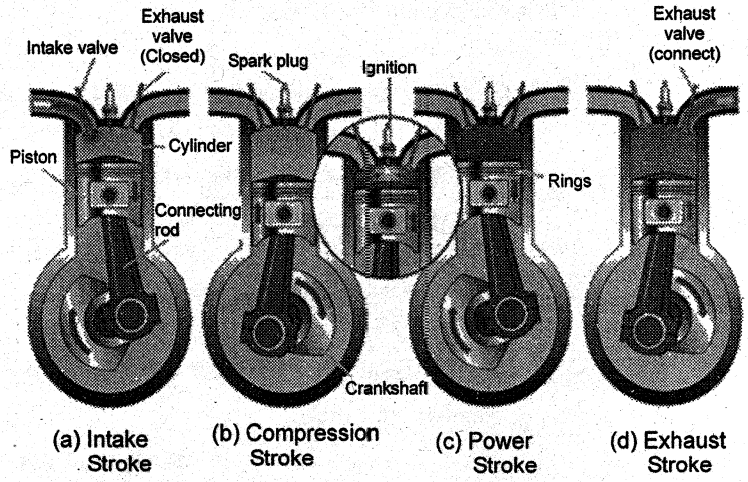


Fig. 4.5(a) : Four stroke diesel (C.I.) engine

❑ 2. Compression stroke

- During this stroke the piston moves from BDC to TDC and both the valves remain closed.
- The air sucked during the suction stroke is compressed hence there is rise in the pressure and temperature of the air.
- Before the end of this stroke, a fine spray of fuel is injected into the compressed air which is at high temperature and hence combustion takes place.

❑ 3. Expansion or power stroke

- During this stroke both the valves remain closed and the piston moves from TDC to BDC.
- The heat energy released by the combustion of fuel results in rise in pressure of the gases which drives the piston downwards. Hence, power is developed.

❑ 4. Exhaust stroke

- In this stroke exhaust valve is opened and inlet valve remains closed and the piston moves from BDC to TDC.
- The upward movement of the piston forces the burnt gases out of the cylinder through the exhaust valve.

🕮 4.8 Comparison between Petrol Engine and Diesel Engine

☞ **[GTU : Summer-18, Winter-16]**

The comparison between four stroke petrol and four stroke diesel engines is as follows :

Sr. No.	Petrol (S.I.) engines	Diesel (C.I.) engines
1.	Petrol engine works on Otto cycle.	Diesel engine works on Diesel cycle.
2.	In these engines, air-fuel mixture is sucked during suction stroke.	In these engines, only air is sucked during suction stroke.
3.	Fuel supply is controlled by throttle valve in the carburettor.	Fuel supply is controlled by fuel pump.
4.	For ignition, spark plug is required.	Spark plug is not required.
5.	Compression ratio is low (about 6 to 12).	Compression ratio is high (about 14 to 22).
6.	Due to low compression ratio, efficiency of these engines is low.	Due to high compression ratio, efficiency of these engines is high.
7.	Due to low compression ratio, starting of these engines is easy.	Due to high compression ratio, starting of these engines is difficult.
8.	Petrol engine is light in weight.	Diesel engine is heavier due to high pressure.
9.	Due to light weight these engines can rotate at high speed.	Due to heavy weight these engines cannot rotate at high speed.
10.	The operation of these engines is silent.	The operation of these engines is noisy.
11.	Running cost is more since it uses costilier fuel.	Running cost is low since it uses cheaper fuel.
12.	Initial cost is low.	Initial cost is high.
13.	These engines are used in light duty vehicles like motor cycles, scooters, cars, etc.	These engines are used in heavy duty vehicles like buses, trucks, etc.

🕮 4.9 Two Stroke Petrol Engine

- In four stroke engines, there is one power stroke in one cycle i.e. in two revolutions of crankshaft.
- In two stroke engines the suction and exhaust strokes are eliminated. Also there are no inlet and exhaust valves, instead of that these engines have inlet and exhaust ports.

- A piston of these engines is given a specific crown shape which helps to prevent the loss of fresh incoming charge which can be escaped alongwith the gases through exhaust port.
- A two stroke petrol engine consists of following strokes (Refer Fig. 4.6) :

1. First stroke 2. Second stroke

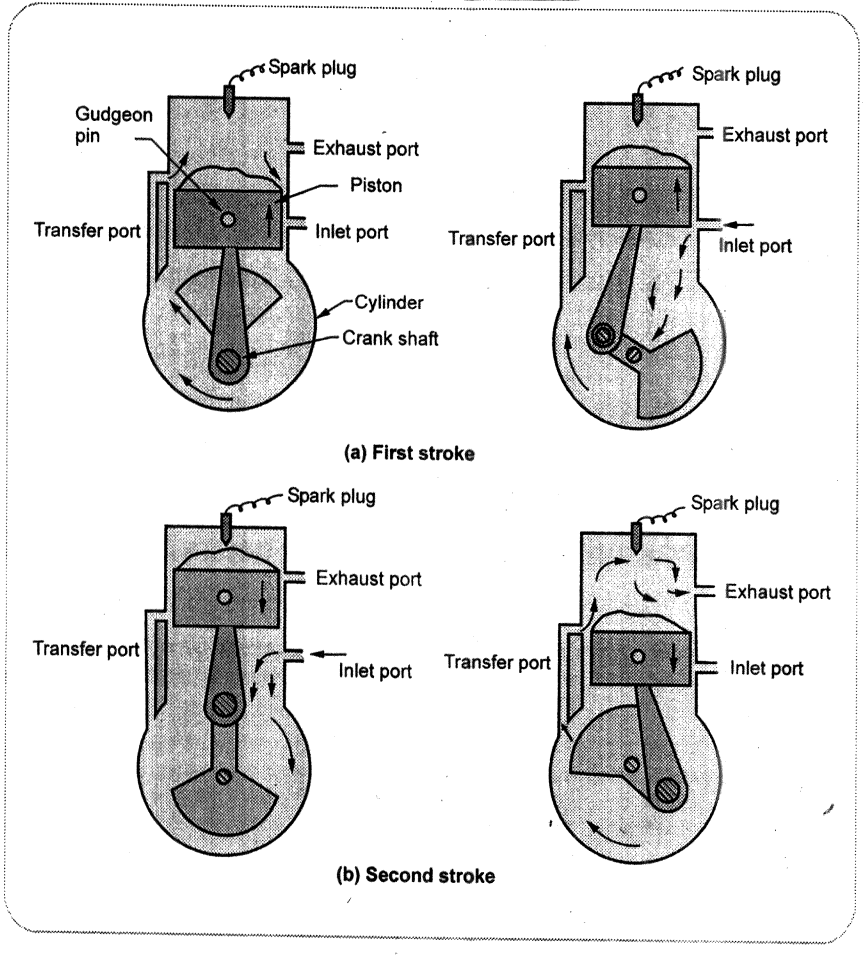


Fig. 4.6 : Two stroke petrol (S.I.) engine

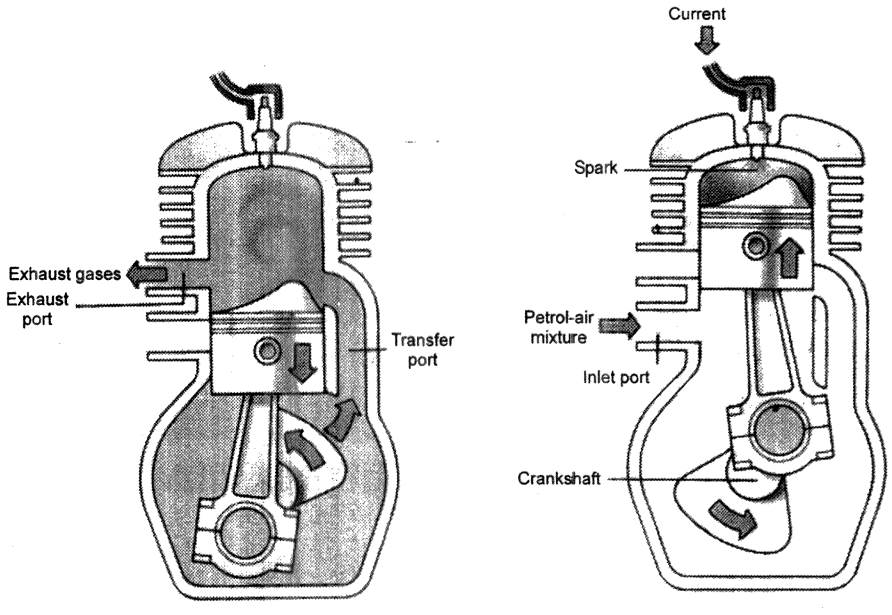


Fig. 4.6(a) : Two stroke petrol (S.I.) engine

❑ 1. First stroke

- Initially the piston is at BDC. The arrangement of the ports is such that the piston performs two operations simultaneously.
- When the piston starts rising from BDC it closes the transfer port and exhaust port and the already existing charge is compressed.
- At that time, vacuum is created in the crankcase which is gas tight.
- As soon as the inlet port is uncovered the fresh air is sucked in the crankcase and the charging is continued untill the crankcase is filled.
- At the end of this first stroke piston reaches TDC.

❑ 2. Second stroke

- In this stroke, piston moves from TDC to BDC.
- Before the completion of compression stroke, the compressed charge is ignited using spark plug and the gas pressure is exerted on the crown of the piston.
- This pressure forces the piston in downward direction which produces some useful work.

- The downward movement of piston closes the inlet port and compresses the charge already sucked in the crankcase. This fresh charge is transferred on the top side of the piston through transfer port.
- At that time, exhaust port opens and burnt gases are escaped.

Note : The difference between two stroke petrol engine and diesel engine is similar to the difference between four stroke petrol engine and diesel engine.

4.9.1 Two Stroke C.I. Engine

- The Fig. 4.7 shows the schematic diagram of two stroke C.I. engine.
- In case of two stroke C.I. engine, only air is inducted in the crank case as compared to air fuel mixture in two stroke S.I. engine.
- The fuel is injected by the fuel injector after the air is compressed in the clearance volume.

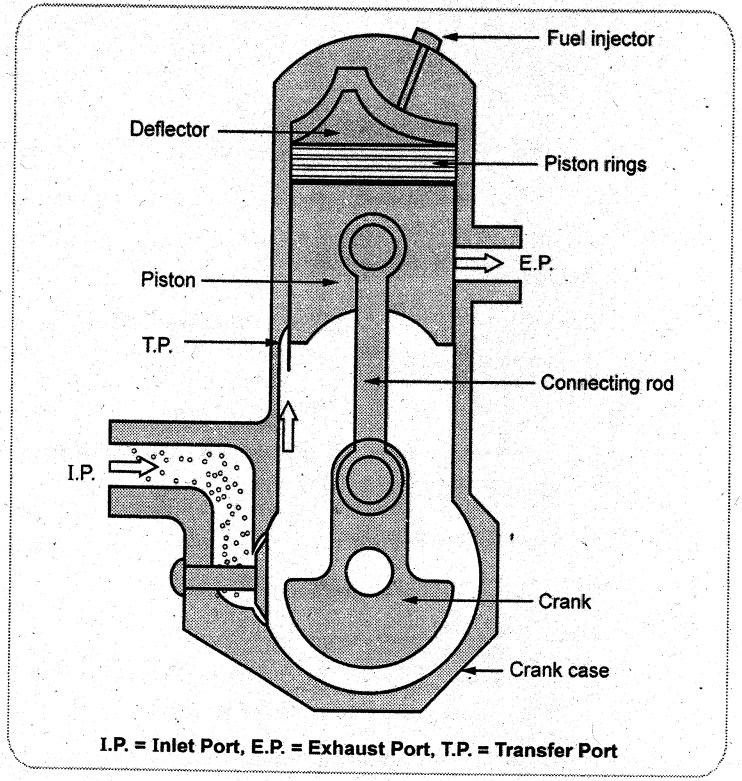


Fig. 4.7 : Two stroke C.I. engine

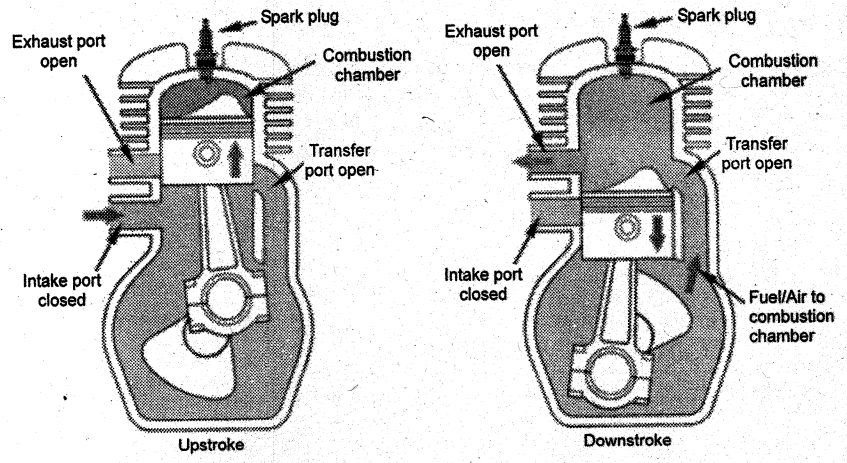


Fig. 4.7(a) : Two stroke diesel (C.I.) engine

- The detailed explanation is given below.

1. Combustion :

- Assuming the piston is at TDC with air under high pressure and high temperature after a compression stroke, the fuel injector injects the fuel under high pressure.
- Due to this, the atomised fuel auto ignites due to high temperature of air present in cylinder. The combustion products under high pressure push the piston towards BDC.

2. Power stroke :

- Due to very high pressure created by the combustion process, is forced the piston down towards BDC in the power stroke.
- At about 70° before BDC, the exhaust port opens and blow-down occurs.

3. Transfer and scavenging :

- At about 45° before BDC, the transfer port on the side of the cylinder is uncovered and fresh air enters in the cylinder under pressure.
- As the piston moves towards TDC, it covers the transfer port and then the exhaust port.

4. Compression stroke :

- With all ports closed, the piston travels towards TDC and compresses the air present above it to a higher pressure and temperature.

- Near the end of the compression stroke, the fuel injector injects proper amount of fuel into the cylinder (20° before TDC).
- While compression of air present above the piston is taking place, the fresh air is inducted into the crank case.

4.10 Comparison between Two Stroke and Four Stroke Engine

☞ **[GTU : Summer-16,17]**

Sr. No.	Two stroke engine	Four stroke engine
1.	The cycle is completed in one revolution of crankshaft.	The cycle is completed in two revolutions of crankshaft.
2.	Power stroke is obtained in every revolution of crankshaft.	Power stroke is obtained in every two revolutions of crankshaft.
3.	Due to one power stroke in one revolution, power produced for same size of engine is more.	Due to one power stroke for two revolutions, power produced for same size of engine is small.
4.	In these engines ports are present.	Instead of ports these engines contains valves.
5.	Due to simplicity and light weight, its initial cost is low.	Due to complication and heavy weight, its initial cost is high.
6.	The piston is having crown shape or dome shape.	The crown of the piston is flat.
7.	Low thermal efficiency.	High thermal efficiency.
8.	It consumes more amount of lubricating oil.	It consumes less amount of lubricating oil.
9.	During the operation it produces more noise.	It produces less noise during the operation.
10.	These type of engines are used in scooters, mopeds, etc.	These type of engines are used in cars, buses, trucks, etc.

4.11 Valve Timing Diagram (VTD)

- The valve timing diagram shows opening and closing of inlet and exhaust valve.
- The valve timing diagrams are usefull for designing the cam shaft.
- The valve timing diagram study improves volumetric efficiency.

4.11.1 Valve Timing Diagram for Four Stroke SI Engine

☞ **[GTU : Summer-18, Winter-17]**

- The inlet and exhaust valves are opened at TDC and BDC respectively (Theoretically).
- In actual engine, the inlet valve is opened 10 before TDC and if closes at 15 after BDC for low speed engine.
- High speed engines the inlet valve opens at same of low speed engine but closes at 50 to 55 after BDC.

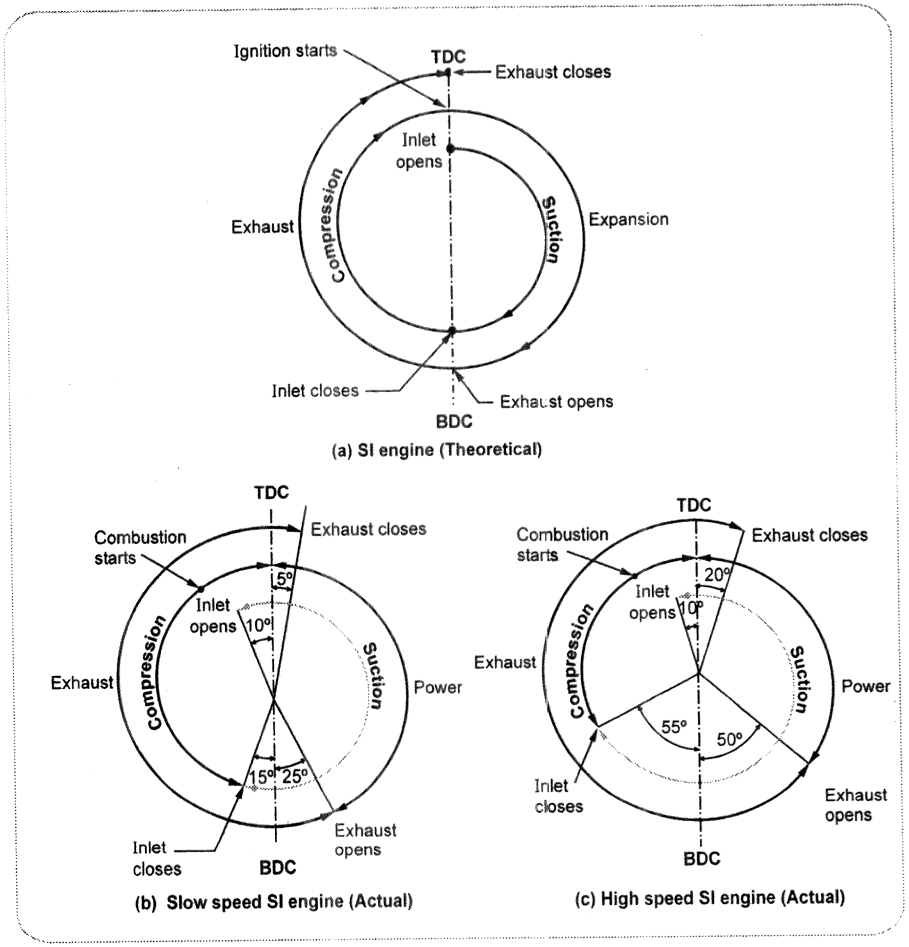


Fig. 4.8 : Valve timing diagrams for 4 stroke S.I. engine

- The valve timing diagrams are drawn to ensure maximum amount of air-fuel mixture to increase volumetric efficiency.
- The exhaust valve opens 25 C before BDC to reduce piston work for slow speed engine.
- Spark plug sparks 15 C before TDC to allow the time delay and helps for beginning of combustion.
- The exhaust valve is kept open for 20 after TDC.
- The inlet and exhaust valve opens at same time is called **overlap.**
- The following diagrams and table shows different valve timing diagram.

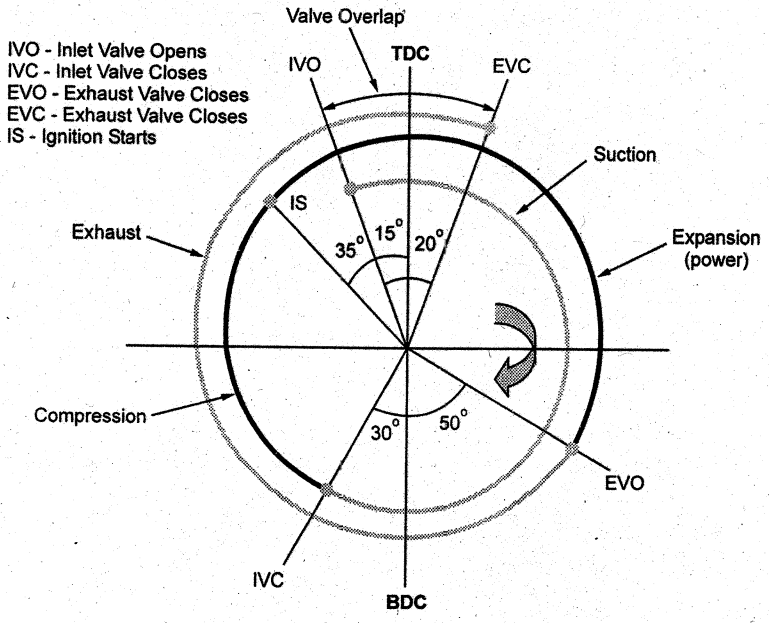


Fig. 4.8(a) : Valve Timing Diagram of 4 Stroke Petrol S.I. Engine

	Engine type	Valve open	Valve close	Time of stroke
Intake Stroke	Low Speed	10 before TDC	15 after BDC	205
	High Speed	10 before TDC	55 after BDC	245
Exhaust Stroke	Low Speed	25 before BDC	5 after TDC	210
	High Speed	50 before BDC	20 after TDC	250

Table 4.1 : Valve timing angle for 4 stroke S.I. engine

4.11.2 Valve Timing Diagram for Four Stroke CI Engine

- Inlet and exhaust valves are opens at TDC and BDC for theoretical engine.
- For actual engine inlet valve is opened at 25 before TDC and closes 30 after BDC.
- The exhaust valve is opened at 40° before BDC and closes 15 after TDC.
- The following figure and table shows valve timing diagram for CI engine.

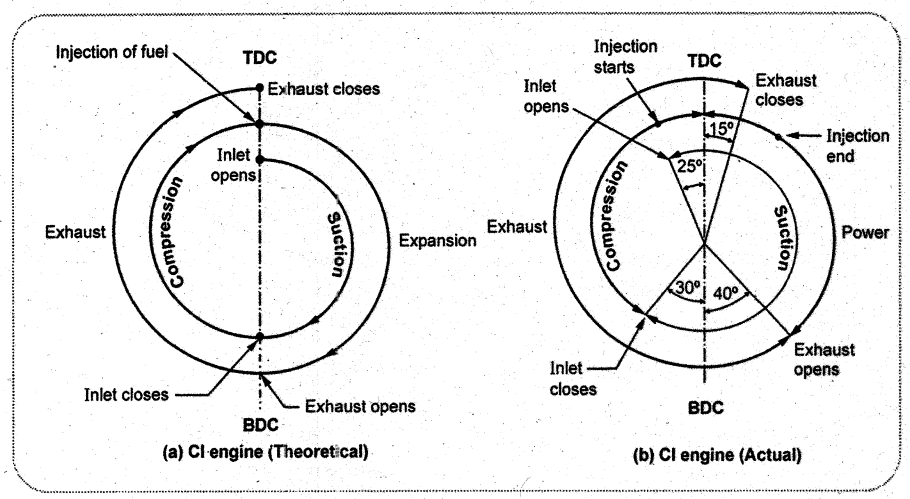


Fig. 4.9 : Valve timing diagrams for 4 stroke C.I. engine

Stroke	Valve open	Valve close	Time of stroke
Intake	25 before TDC	30 after BDC	235
Exhaust	40 before BDC	15 after TDC	235

Table 4.2 : Valve timing angles for 4 stroke C.I. engine

4.11.3 Valve Timing Diagram for Two Stroke Engine [GTU : Summer-16]

a) Two stroke C.I. engine

Two following figure shows valve timing diagram for two stroke C.I. engine.

The Fig 4.10 shows self-explanatory valve timing diagram for two stroke CI engines.

- Exhaust Port Opens : 70° before BDC.
- Exhaust Port Closes : 70° after BDC.
- Transfer Port Opens : 45° before BDC.
- Transfer Port Closes : 45° after BDC.

- Fuel injection Starts : 20° before TDC.

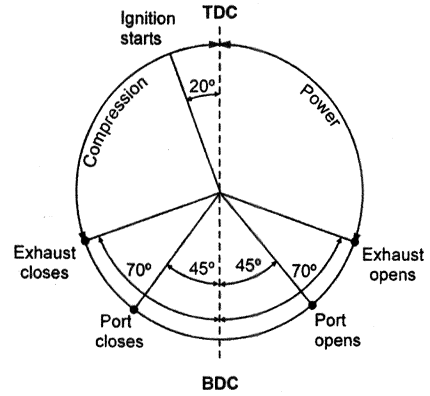


Fig. 4.10 : Valve timing diagram for 2 stroke C.I. engine

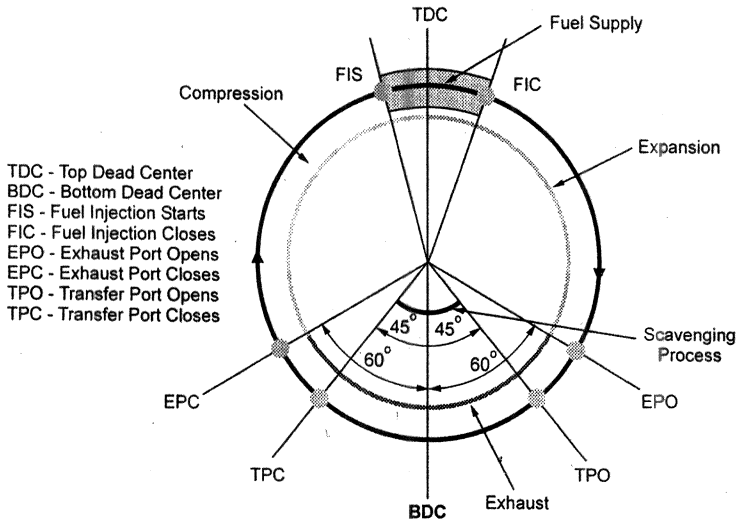


Fig. 4.10(a) : Valve timing diagram of 2 stroke diesel engine

❑ b) Two stroke petrol engine

The following figure shows valve timing diagram for two stroke S.I. engine.

The Fig 4.11 shows self-explanatory valve timing diagram for two stroke S.I. engines.

- Exhaust Port Opens : 45° before BDC.
- Exhaust Port Closes : 45° after BDC.
- Transfer Port Opens : 35 before BDC.
- Transfer Port Closes : 35° after BDC.
- Spark plug Ignites : 20° before TDC

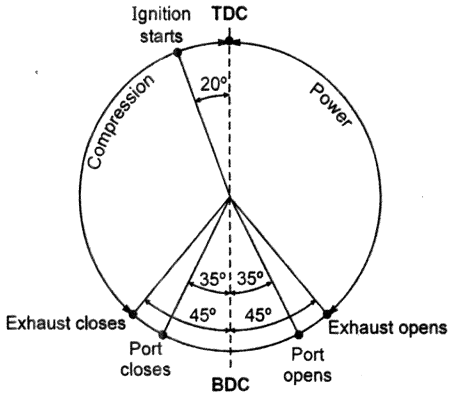


Fig. 3.11 : Valve timing diagram for 2 stroke S.I. engine

4.12 Intake and Exhaust Systems

[GTU : Summer-14]

- The purpose of the exhaust and inlet processes is to remove the burned gases at the end of the power stroke and admit fresh charge for the next cycle.
- Indicated power of an ICE at a given speed is proportional to the mass flow rate of air.
- Inducting the maximum air mass and retaining the mass within the cylinder is the primary goal of the gas exchange processes in engines.
- Engine gas exchange processes are characterized by volumetric efficiency and it depends on the design of engine subsystems such as manifolds, valves, and ports, as well as engine operating conditions.
- Supercharging and turbo-charging are used to increase air flow into engine cylinder, and hence power density.
- In SI engine, the intake system typically consists of an air filter, a carburetor and throttle or fuel injector and throttle or throttle with individual fuel injectors in each intake port and intake manifold.
- During the induction process, pressure losses occur as the mixture passes through or by each of these components.
- The pressure drop depends on engine speed, the flow resistance of the elements in the system, the cross-sectional area through which the fresh charge moves, and the charge density.
- In a CI engine intake system, the carburetor or EFI system and the throttle plate are absent.

- The exhaust system typically consists of an exhaust manifold, exhaust pipe, often a catalytic converter for emission control, and a muffler or silencer.
- One complete four stroke cycle requires two complete rotation of the crank-shaft and therefore one complete rotation of the camshaft The opening and closing of the valves are controlled by the camshaft.

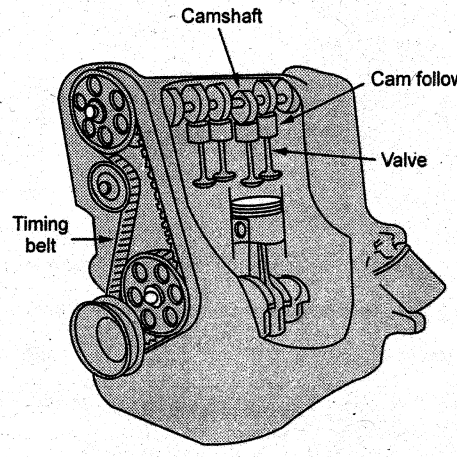


Fig. 4.12 : Intake and exhaust system

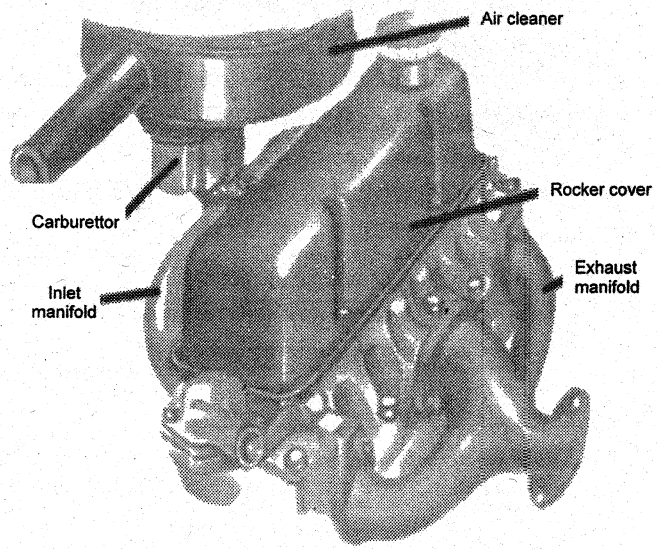


Fig. 4.12(a)

4.13 Applications of IC Engine

The IC engines are used in following applications :

- Road vehicles
- Locomotives
- Cinemas
- Several industrial application
- Air crafts
- Pumping sets
- Hospitals

4.14 IC Engine Components and its Material

☞ **[GTU : Summer-17, Winter-19]**

Component	Materials	Production method
Cylinder block	Alloy cast iron, Aluminimum alloy	Casting
Cylinder head	Alloy cas iron, Aluminium alloy	Casting
Cylinder liner	Alloy cast iron, Nickel alloy steel, Aluminium alloy	Casting
Engine gasket	Soft copper, Asbestos sheet, Synthetic rubber cork, Treated paper	Rolling
Piston	Alluminium alloy, Semi-steel, Cast iron	Casting
Piston rings	Alloy cast iron	Casting
Gudgeon pin	Alloy steel, Nickel or Vanadium steel for high speed engines	Forging, extrusion
Connecting rod	High grade nickel alloys, Plain carbon steel, Duralumin	Forging
Crankshaft	Alloy cast iron, Forged sheel, Plain carbon steel	Forging casting
Crankcase	Grey cast iron, Aluminium alloy, Aluminium-copper-zinc alloy	Casting
Oil sump	Pressed steel, Cast iron, Aluminium	Casting, stamping
Camshaft	Alloy steel, Alloy cast iron, Forged steel	Forging
Engine valves	Austenitc steel, Precipitation hardening steel, Low alloy steels (Cr-Ni steel, Cr-Ni-Mo steel)	Forging
Manifolds	Alloy cast iron, Alloy steel, Aluminimum alloy	Casting
Flywheel	Cast iron	Casting

4.15 Introduction to Air Standard Cycle

In the subject Thermodynamics, we have already studied otto cycle, diesel cycle and dual cycle (Air standard cycles).

- Those cycles are theoretical cycles because we have neglected the losses that occur during the processes.

But, practically actual cycles are different than the theoretical cycles.

- Here, we are going to discuss the difference between theoretical and actual cycle for S.I. engine and C.I. engine.

4.16 Theoretical and Actual (p-V) Diagrams for 4-Stroke Petrol Engine

[GTU : Summer-17]

The theoretical and actual (p-V) diagrams for petrol engine working on otto cycle are shown in Fig. 4.13.

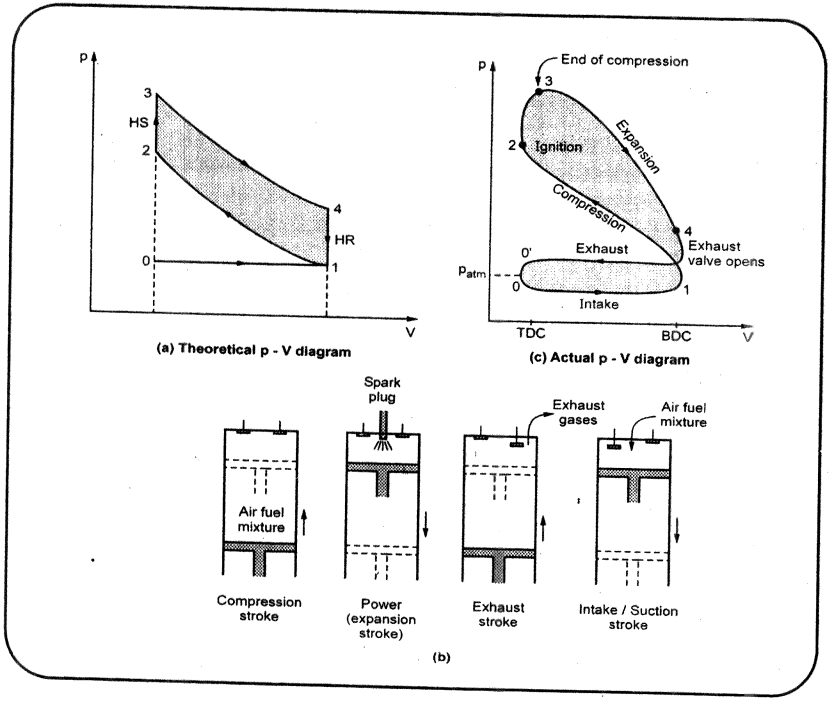
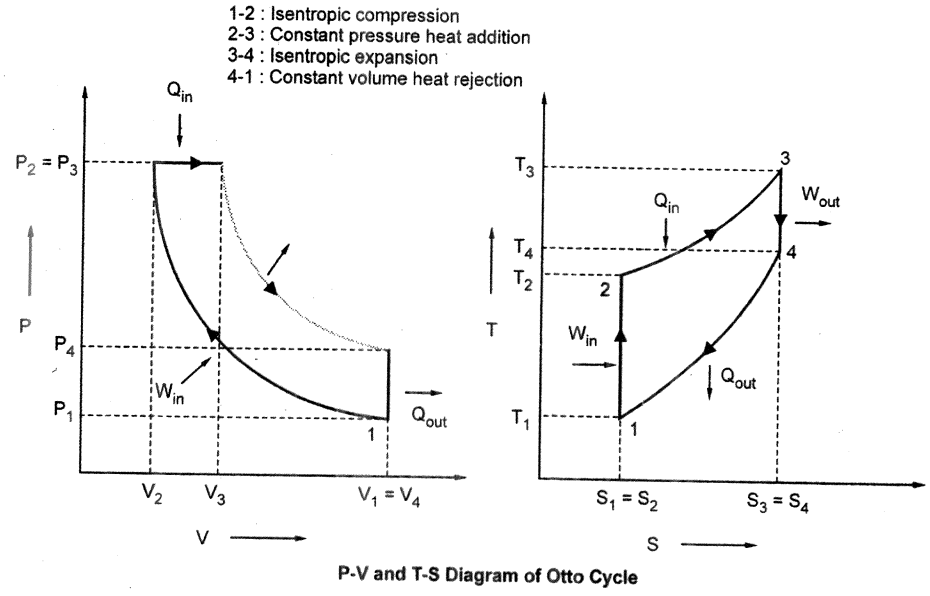


Fig. 4.13 : Theoretical and actual p-V diagram for otto cycle (SI engine)



P-V and T-S Diagram of Otto Cycle

Fig. 4.13(a) : Theoretical and actual p-V diagram of otto cycle (S.I.) engine

4.16.1 Assumptions Made in Theoretical Otto Cycle

- Suction and exhaust stroke takes place at atmospheric pressure.
- Suction stroke (0-1), compression stroke (1-2), expansion stroke (3-4) and exhaust stroke (4-0) takes place during 180° of crank rotation.
- Heat addition process (2-3) and heat rejection process (4-1) are at constant volume and takes place instantly.
- Air behaves like a perfect gas.
- Compression and expansion processes are reversible adiabatic.

4.16.2 Actual Cycle

Actual cycle differs from theoretical cycle in the following factors :

- During the suction stroke, mixture of air and fuel is admitted instead of only air. This process is possible only when the pressure inside the cylinder is below atmospheric pressure.
- Due to heat transfer between system and surroundings, the compression process and expansion process are polytropic.
- Heat addition process is carried out by providing a spark to mixture few degrees before T.D.C.

- As the gases has to be expelled to the atmosphere, the exhaust stroke is above atmospheric pressure.
- The processes are not ideal.
- Exhaust at point 4 is not instantaneous since there is a time gap in opening the exhaust valve.

4.17 Theoretical and Actual (p-V) Diagram for 4-Stroke Diesel Engines

The theoretical and actual (p-V) diagram for a diesel engine working on diesel cycle is as shown in Fig. 4.14.

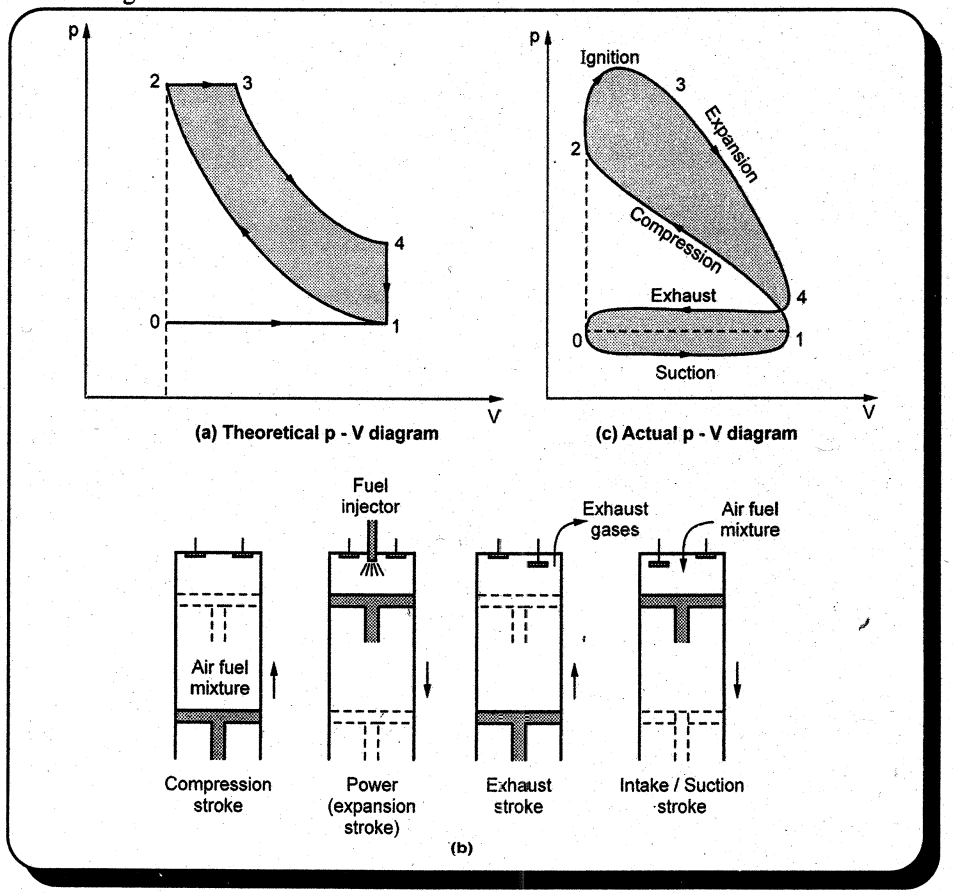


Fig. 4.14 : Theoretical and actual p-V diagram for diesel cycle (CI engine)

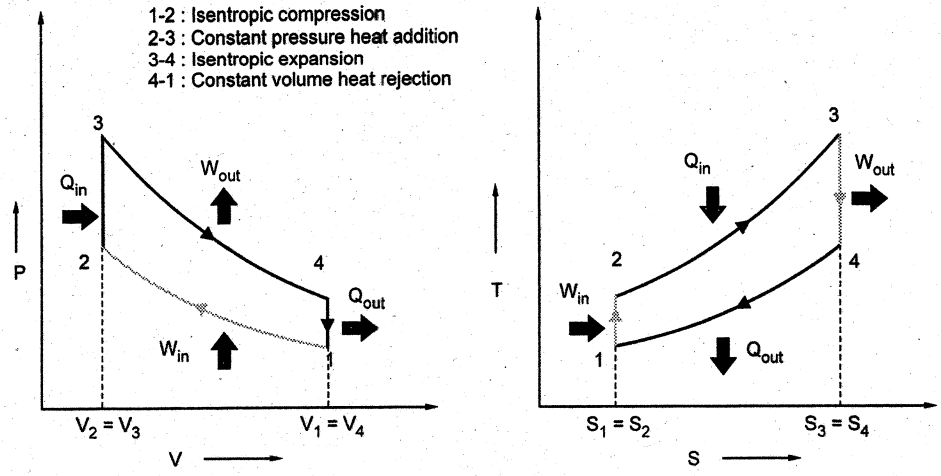


Fig. 4.14 (a) : Theoretical and actual p-V diagram of diesel cycle (C.I.) engine

4.17.1 Assumptions Made in Theoretical Diesel Cycle

- Suction and exhaust stroke takes place at atmospheric pressure.
- Suction stroke (0-1), compression stroke (1-2), expansion stroke (3-4) and exhaust stroke (4-0) takes place during 180° of crank rotation.
- Heat addition process (2-3) is at constant pressure and heat rejection process (4-1) is at constant volume and takes place instantly.
- Air behaves like a perfect gas.
- Compression and expansion processes are reversible adiabatic.

4.17.2 Actual Cycle

- Actual cycle differs from theoretical cycle in following aspects :
- During suction stroke, air is admitted at a pressure below atmospheric pressure.
- Due to heat transfer between system and surroundings, the compression and expansion processes are polytropic.
- Heat addition process is due to self ignition of high temperature gases caused by the high compression ratio. Here, combustion process is not at constant pressure due to continuous fuel injection.
- Exhaust stroke is only possible when the pressure inside the cylinder is above atmospheric pressure as the gases has to be expelled to atmosphere.

- Exhaust at point 4 is not instantaneous due to time gap in opening the exhaust valve.
- Processes are not ideal.
- The mixture does not behave like a perfect gas.

4.18 Theoretical and Actual (p-V) Diagram for Dual Cycle (Semi-Diesel Cycle)

- The theoretical and actual (p-V) diagram for dual cycle are shown in Fig. 4.15.

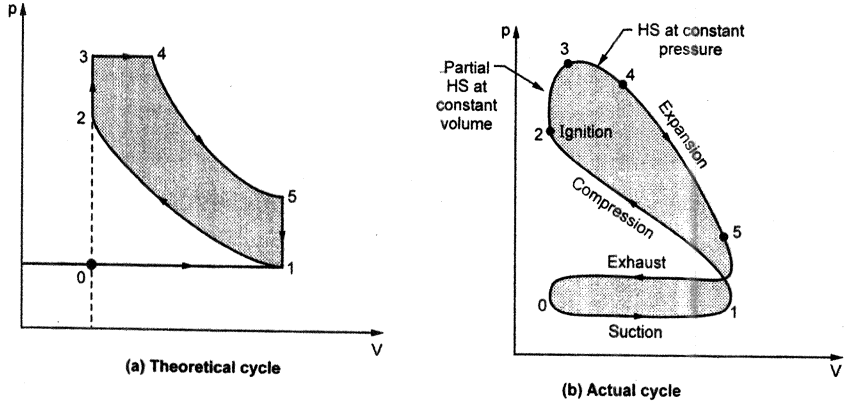


Fig. 4.15 : Theoretical and actual p-V diagram for dual cycle

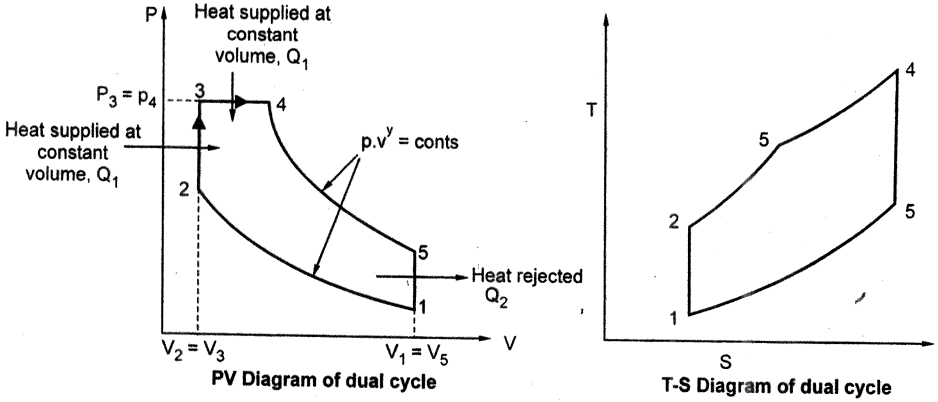


Fig. 4.15 (c) : Theoretical and actual p-V diagram of dual cycle

4.18.1 Assumptions Made in Theoretical Cycle

- Air behaves like a perfect gas.
- The suction and exhaust strokes are at atmospheric pressure.

- Suction (0-1), compression (1-2), expansion (4-5), exhaust (5-0) strokes takes place during 180° of crank rotation.
- Compression and expansion processes are reversible adiabatic.
- Heat addition process (2-4) takes place at constant volume and constant pressure whereas heat rejection takes place at constant volume.
- All friction losses are neglected.

4.18.2 Actual Cycle

Actual cycle differs from theoretical cycle in the following aspects :

- During suction stroke, the mixture of air and fuel is admitted instead of only air. The suction stroke is carried out below atmospheric pressure.
- Exhaust stroke is carried out above atmospheric pressure as the gases have to be expelled to the atmosphere.
- Due to heat transfer between system and surroundings, compression and expansion processes are polytropic.
- Processes are not ideal.
- Exhaust at point 5 is not instantaneous since there is time lag in opening and closing of exhaust valve.

4.19 Fuel-Air Cycles

- In case of ideal cycles, we assumed that the working substance is air which behaves like a perfect gas and the specific heat of air remains constant throughout the process.
- In actual cycles, the working substance is a mixture of air and fuel alongwith the residual gases of previous stroke.
- With increase in temperature, the specific heat of gases increases, except in case of monoatomic gases as the value of γ (ratio of specific heat at constant pressure to specific heat at constant volume) varies during various processes of the actual cycle.
- The products of combustion tends to dissociate at high temperatures as they are unstable at temperature above 1200 °C.
- Due to the above mentioned factors, the performance of actual cycle is not similar to that of ideal cycle.
- The actual-cycle which takes into account the molecular structure, variation of specific heats as well as air-fuel mixture approximating to actual engine working substance, are called as fuel-air cycles.

4.19.1 Assumptions in Fuel-Air Cycles

- The compression and expansion processes are adiabatic.
- The air-fuel mixture is homogeneous in case of otto cycle.
- During the cycle, the suction and exhaust process takes place at atmospheric pressure.
- For good combustion, the mixture is in chemical equilibrium.
- The change in kinetic energy is neglected.

4.20 Effect of Various Factors on Analysis of Fuel-Air Cycles

[GTU : Summer-17,18, Winter-16]

The effect of various factors on analysis of fuel-air cycles are discussed below :

4.20.1 Variation of Specific Heats

- In theoretical cycle, it is assumed that the specific heat of working fluid remains constant. But in case of actual cycle, the specific heat of working fluid increases with increase in temperature, except in case of monoatomic gases.
- The specific heat increases drastically above 1200 °C. Specific heat at constant pressure (C_p) and specific heat at constant volume (C_v) are functions of temperature (T) and are given as,

$$C_p = a + bT + CT^2$$

$$C_v = a_1 + bT + CT^2$$

Where, a, b, c and a_1 are constants and the ratio of specific heat (C_p and C_v) is,

$$\frac{C_p}{C_v} = \gamma$$

As, $C_p - C_v = R$ ...(Characteristic gas constant)

$\therefore \quad a - a_1 = R$

- The variation of specific heat in the temperature range of 27 °C to 1227 °C is almost a straight line. Hence, the above equations may be written as :

$$C_p = a + bT \qquad C_v = a_1 + bT$$

- During the compression stroke, the value of specific heat (γ) decreases with increase in temperature. So, the actual gas temperature is less than ideal gas temperature.
- Cycle (1-2-3-4) represents ideal otto cycle and cycle represents the actual otto cycle.
- The work developed in actual cycle is less than the work developed in theoretical (ideal) cycle.

- The difference between theoretical work and actual work is called as **loss of work due to the variation in specific heat.**

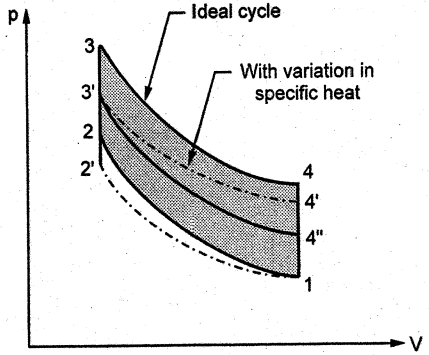


Fig. 4.16 : Otto cycle with variation in specific heat

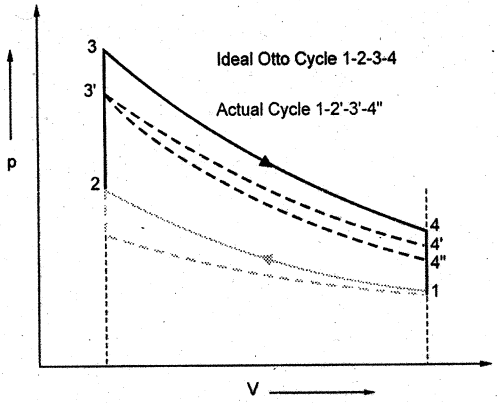


Fig. 4.16 (a) : Otto cycle with variation in specific heat

4.20.2 Dissociation Losses

[GTU : Sumer-14, Winter-16]

- During the dissociation process, fuel absorbs heat from the combustion space representing endothermic reaction. Therefore, the temperature of combustion goes on reducing till chemical equilibrium is achieved. At this temperature, the chemical reaction can proceed in either direction. The equation becomes,

$$CO_2 + O_2 \rightleftharpoons 2CO_2$$

- The effect of dissociation is to reduce the maximum temperature and pressure expected during combustion process.
- When the temperature falls at the late stage of expansion process, a part of dissociated products may recombine to release the heat energy which cannot be utilized effectively for producing work.
- Reduced maximum pressure and temperature are achieved during the combustion process. Due to dissociation, there is loss of work and efficiency. These losses are termed as dissociation losses.
- Fig. 4.17 represents graph of temperature verses air-fuel ratio. It can be seen that without dissociation the temperature achieved is more compared to the temperature obtained when dissociation takes place.

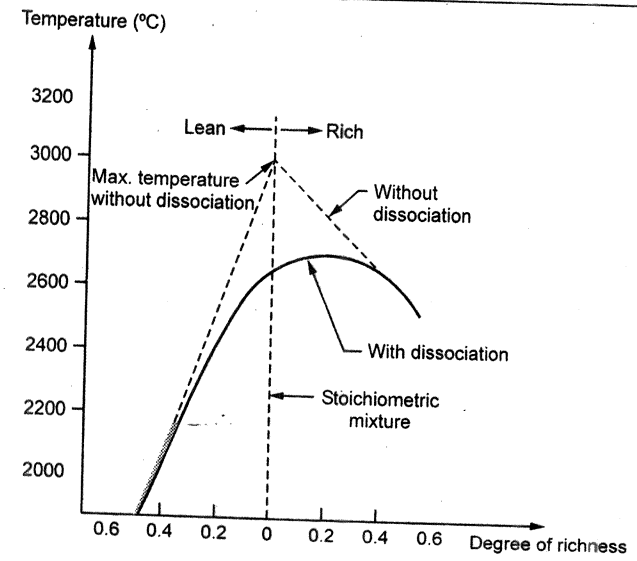


Fig. 4.17 : Effect of dissociation on temperature

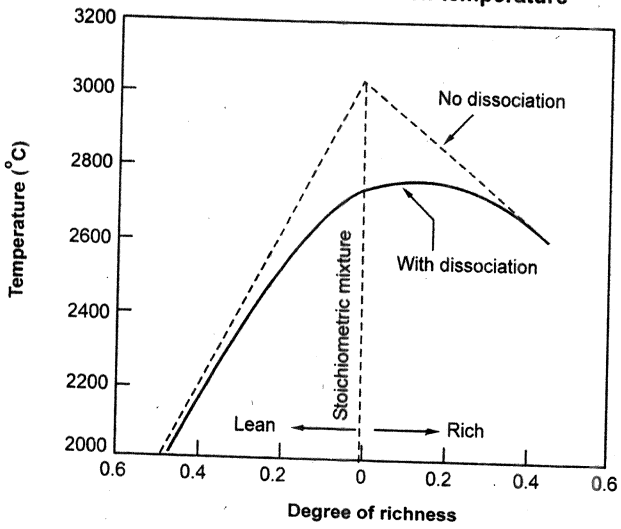


Fig. 4.17 (a) : Effect of dissociation on temperature

- Fig. 4.18 represents graph of brake power verses air-fuel ratio. It shows that maximum power is obtained with stoichiometric mixture when there is no dissociation. The shaded area shows the loss of power due to dissociation.

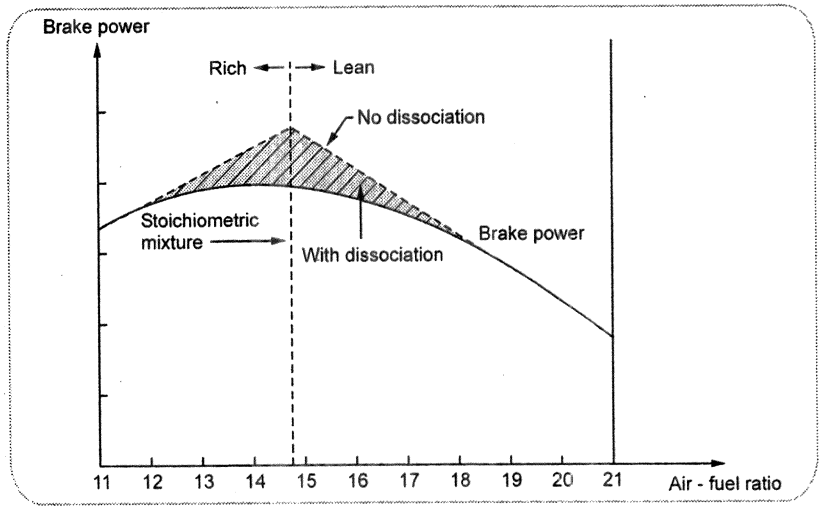


Fig. 4.18 : Brake power vs air - fuel ratio (for SI engine)

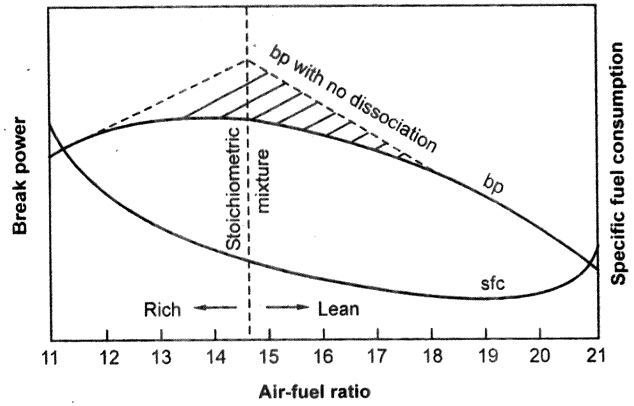


Fig. 4.18 (a) : Brake power vs air - fuel ratio for SI engine

4.20.3 Compression and Air-Fuel Ratio

- Fig. 4.19 shows the effect of air-fuel ratios on the thermal efficiency with the variation of compression ratio.
- It can be seen that as the compression ratio increases, the thermal efficiency of air fuel cycle increases.
- It could be observed that the fuel-air cycles with lean mixtures are closer to the theoretical cycle compared to rich mixtures.

- The main reason is that the lean mixtures tends to reach their chemical equilibrium at the beginning of expansion stroke.

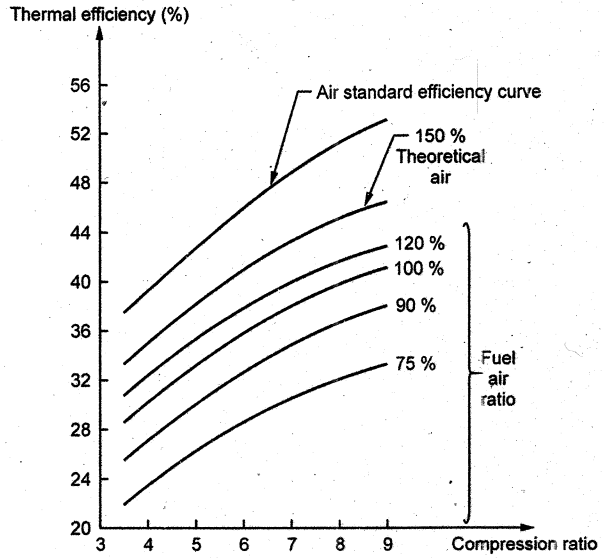


Fig. 4.19 : Effect of compression and air-fuel ratio on thermal efficiency

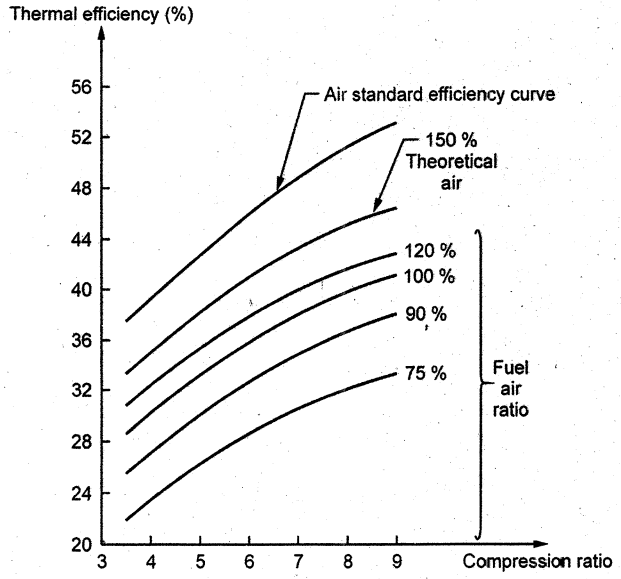


Fig. 4.19 (a) : Effect of compression and air-fuel ratio on thermal efficiency

4.20.4 Molecular Change

- It is assumed that the number of moles of the working substance before and after combustion remains constant in case of ideal cycles. But, in case of fuel-air cycles, the number of moles present before and after combustion would be different. For example :

i) $2H_2O + O_2 \rightarrow 2H_2O$
2 moles 1 mole 2 moles

ii) $C + O_2 \rightarrow CO_2$
1 mole 1 mole 1 mole

- The total number of moles before and after combustion are different.
- The deflection / change in number of moles is known as molecular expansion or contraction.

4.21 Actual Cycles

- Fig. 4.20 shows the actual and ideal cycle for SI engine. The actual cycle is different from ideal cycle in following aspects :
- During the suction stroke, mixture of fuel and air is admitted inside the cylinder.
- Heat is supplied to the cycle due to combustion of fuel.
- The temperature, pressure and chemical composition of the fresh air-fuel mixture changes due to burnt gases leftover in the cylinder.
- During heat addition, suction and exhaust stroke, there are some losses (discussed later) present in the cylinder. These losses reduce the efficiency of the engine.

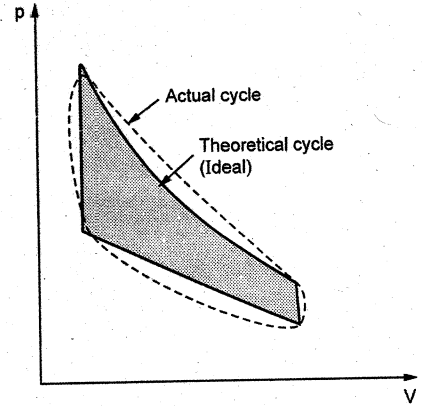
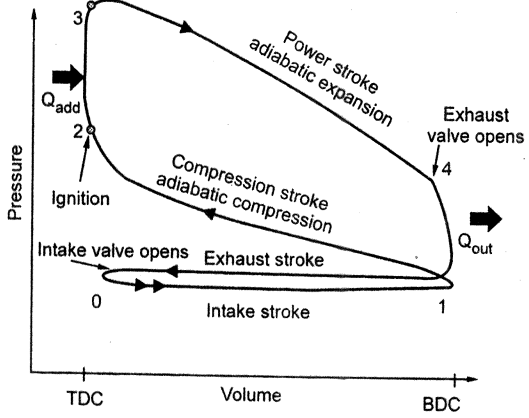


Fig. 4.20 : Actual and ideal p-V diagram for 4 stroke SI engine

The specific heat of fuel varies with temperature. The specific heat increases with increase in temperature of cycle.

- Fig. 4.20 (a) shows the difference in p-V diagram of ideal and actual cycle of S.I. engine.



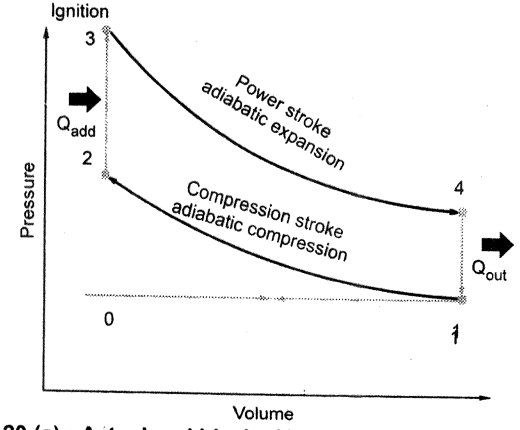


Fig. 4.20 (a) : Actual and ideal p-V diagram for 4 stroke SI engine

4.22 Losses in Actual Cycle

☞ [GTU : Summer-18]

Apart from the losses discussed earlier in case of fuel-air cycle, the other important losses in actual cycles are as follows :

1. Pumping losses
2. Exhaust blow-down losses
3. Time losses
4. Heat losses
5. Rubbing friction losses

4.22.1 Pumping Losses

- Suction and exhaust processes were assumed to be at atmospheric pressure in case of ideal cycles.
- To achieve these processes, there should be some pressure difference between atmosphere and inside the cylinder.
- During suction, the pressure inside the cylinder is lower than the fluid pressure in order to induct the fluid inside the cylinder and the exhaust gases are thrown out at a higher pressure than the atmospheric pressure.
- Therefore, some work is done on the gases during the suction and exhaust stroke. This work is called as pumping work. Pumping work is shown in Fig. 4.9 by shaded area.

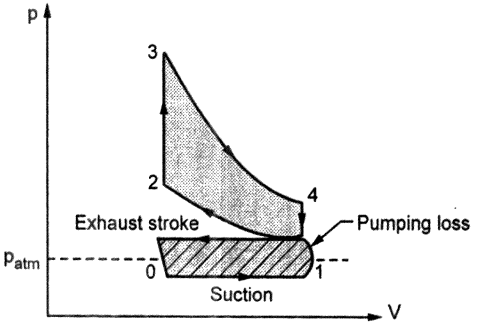


Fig. 4.21 : Pumping losses

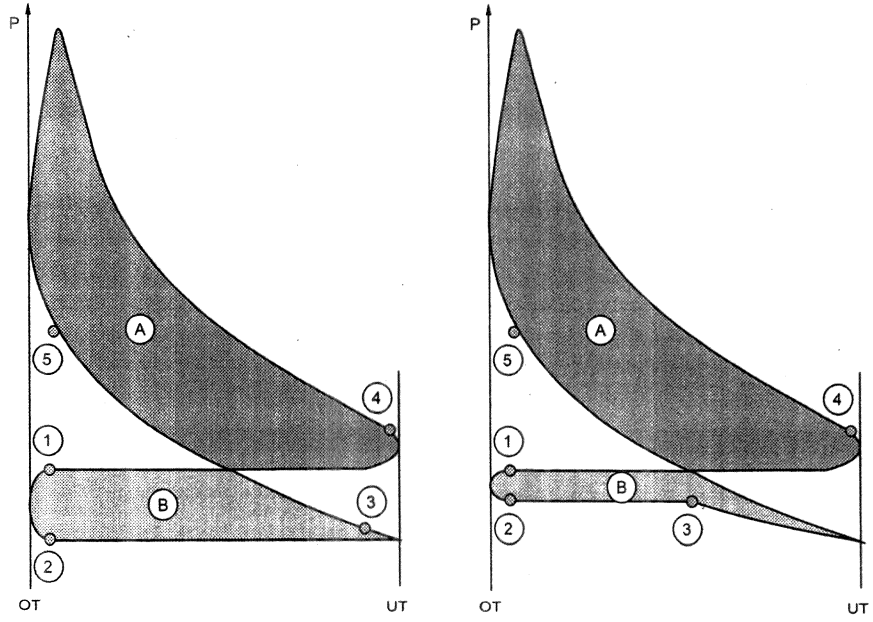


Fig. 4.21 (a) : Pumping losses

4.22.2 Exhaust Blow-Down Losses

[GTU : Summer-16, Winter-18]

- As discussed in valve timing diagram of 4 stroke SI engine, during exhaust stroke, the exhaust valve is opened before BDC and closed after TDC.
- It helps in reducing the pressure in the cylinder during expansion stroke so that the work required in pushing out the exhaust gases by the piston is reduced.
- Therefore, a lot of heat energy is carried away by exhaust gases resulting in loss of work.
- Fig. 4.22 shows the blowdown losses in ideal and actual cycle.

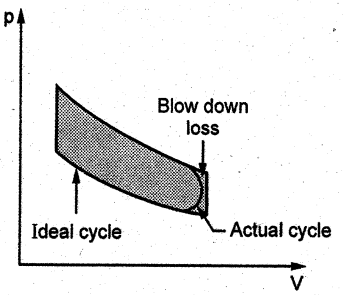


Fig. 4.22 : Blow down losses

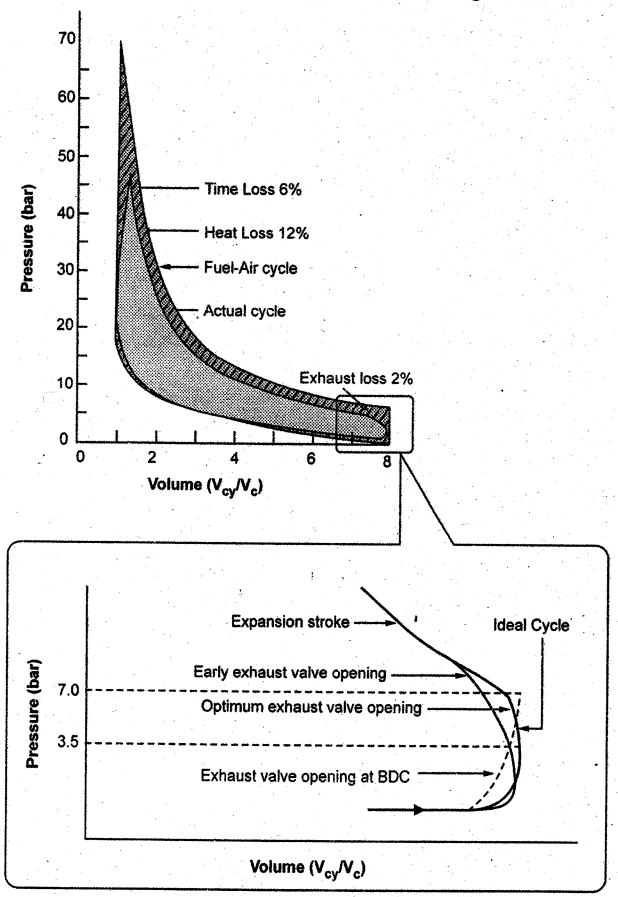


Fig. 4.22 (a) : Blow down losses

- The portion shown by hatching lines represents the exhaust blow-down losses.

4.22.3 Time Losses

[GTU : Summer-18]

- The heat addition process was assumed to be at constant volume in case of theoretical cycle. But in actual cycle, heat addition is done in definite time interval.
- During this period of combustion, the gases experience change in volume.
- Due to motion of piston, the volume is increased which results in lower maximum pressure and less work on the piston.
- Fig. 4.23 represents time losses in otto cycle.
- The hatching lines represents loss of work due to time losses.

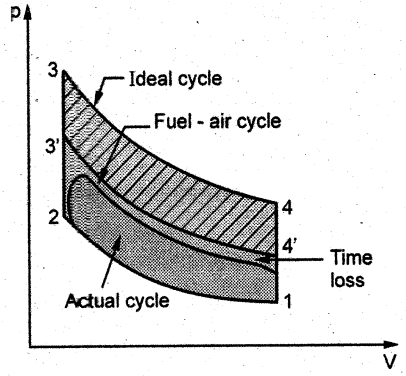


Fig. 4.23 : Time losses

4.22.4 Heat Losses

- The compression and expansion processes are assumed to be adiabatic in case of ideal cycles. But in actual process, there is heat transfer from the working substance to the cylinder walls.
- There is considerable amount of heat loss during combustion and expansion processes due to which there is loss in work and efficiency.
- These losses are called as heat losses.

4.22.5 Rubbing Friction Losses

- These losses occur due to friction between moving engine parts. (i.e. piston and cylinder walls, etc.)
- Friction losses increases with increase in speed of the engine.

4.23 Comparison between Air-Standard Cycle, Fuel-Air Cycle and Actual Cycle

☞ [GTU : Summer-18, Winter-18]

Sr. No.	Air-standard cycle	Fuel-air cycle	Actual cycle
1)	The working substance used is air	The working substance used is mixture of fuel, air and residual gases.	Same as fuel-air cycle.
2)	Specific heat of gases and does not change with change in temperature.	Specific heat and changes with change in temperature.	Same as fuel-air cycle.
3)	Chemical composition of working substance does not change.	Combustion and residual gases changes the number of moles before and after combustion.	Same as fuel-air cycle.
4)	Heat is added by heat reservoirs.	Heat is added due to combustion of fuel and air.	Same as fuel-air cycle.
5)	Gases are in chemical equilibrium.	Chemical equilibrium is maintained and combustion of fuel is complete.	There are losses due to incomplete combustion of fuel.
6)	Compression and expansion processes are isentropic.	Processes are not adiabatic.	Heat losses are considered.
7)	The heat is supplied at constant volume.	The heat is supplied due to burning of air-fuel mixture at constant volume.	Burning of fuel and air is not at constant volume, hence there are time losses.
8)	Valves open and close instantaneously.	Same as air-standard cycle.	Since, there are exhaust blowdown losses, there is a time gap in opening and closing of valve.
9)	Friction in the engine is neglected.	Same as air-standard cycle.	Friction and rubbing losses are considered.
10)	Suction and exhaust strokes are eliminated.	suction and exhaust stroke occurs at atmospheric pressure.	Suction stroke occurs below atmospheric pressure and exhaust stroke occurs above atmospheric pressure in the cylinder.

REVIEW QUESTIONS

Q. 1 What is heat engine ? Give its classification and state advantages.

Q. 2 Compare external combustion engine and internal combustion engine.

Q. 3 State applications of heat engines with suitable examples.

Q. 4 Define and explain the following terms related to IC engines :

a) Compression ratio b) Clearance volume

c) Stroke volume d) Piston speed

Q. 5 Explain construction and working of :

a) Two stroke SI engine b) Two stroke CI engine

c) Four stroke SI engine d) Four stroke CI engine

Q. 6 Give classification of IC engines in detail.

Q. 7 Explain theoretical and actual valve timing diagrams for two stroke SI engine.

Q. 8 Explain theoretical and actual valve timing diagrams for two stroke CI engine.

Q. 9 Explain theoretical and actual valve timing diagrams for four stroke SI engine.

Q. 10 Explain theoretical and actual valve timing diagrams for four stroke CI engine.

Q. 11 State the assumptions made in theoretical otto cycle. Explain how actual diagram differs from it.

Q. 12 State the assumptions made in theoretical diesel cycle. Explain how actual diagram differs from it.

Q. 13 State the assumptions made in theoretical dual cycle. Explain how actual diagram differs from it.

Q. 14 Explain fuel-air cycle in detail. Also state its assumptions.

Q. 15 Explain the following losses in actual cycle :

i) Variation of specific heats ii) Dissociation losses

iii) Pumping losses iv) Time losses

Q. 16 What are the different losses in actual cycle ? Explain any three in detail.

Q. 17 Give the comparison between air-standard cycle, fuel-air cycle and actual cycle.

❑❑❑

Notes

CHAPTER - 5

IC ENGINE PERFORMANCE AND EMISSIONS

Syllabus : Measurement of indicated power, brake power, friction power, fuel consumption and emission, calculation of brake thermal efficiency, brake power and brake specific fuel consumption, variable compression ratio engines, heat balance sheet, principal engine emissions, source of engine emissions, emission measurement instruments like five gas analyzer and smoke meter, Euro and Bharat standards of emissions of I.C. Engines, Emission control methods like Air injection, Exhaust gas recirculation, Catalytic converter, Evaporative emissions control.

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5.2	Various Performance Parameters of IC Engine	5 - 3
5.3	Measurement of Indicated Power (IP)	5 - 6
5.4	Measurement of Brake Power (BP)	5 - 9
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5.1 Objective of IC Engine Testing

☞ [GTU : Summer-13, Winter-16]

- The engine is a device which converts heat energy into mechanical work.
- To get heat energy we need to burn the fuel.
- The fuel is expensive in nature.
- The fuel causes environmental pollution also.
- Hence the termal efficiency of engine should be higher.
- To desing the engine for better performance the testing should be required.
- The testing data is usefull to achive norms and standards of engine.
- The testing is usefull to distinguish the engine.

5.2 Various Performance Parameters of IC Engine

- I.C. engine have got lot of importance due to transportation demand therefore, it is necessary to have efficient engine performance.
- The engine performance can be measured using various performance parameters as follows:

1. Power and Mechanical Efficiency
2. Mean effective pressure and torque
3. Specific Output
4. Volumetric Efficiency
5. Air-fuel ratio
6. Specific fuel consumption
7. Thermal Efficiency and heat balance
8. Exhaust smoke and other emissions
9. Specific weight

1. Power and Mechanical Efficiency

☞ [GTU : Summer-14, Winter-17]

i) Indicated Power :

- It is defined as the total power developed during the combustion of fuel in combustion chamber.
- It is denoted by letters I.P. and is given by,

$$\text{I.P.} = \frac{n \times imep \times l \times A \times N}{60 \times 1000}, \text{kW}$$

where, n = No. of cylinders

imep = Indicated mean effective pressure in N/m^2

l = Length of stroke in m

A = Area of the piston in m^2

N = Speed in rpm

N = $\frac{N}{2}$ for four stroke engine = N for two stroke engine

ii) Brake Power :

- It is defined as the power developed at the output shaft by an engine.
- It is denoted by B.P. and is given by,

$$\text{B.P.} = \frac{2 \times \pi \times N \times T}{60 \times 1000}, \text{kW}$$

where, N = Speed in rpm

T = Torque in Nm

iii) Frictional Power :

- It is the difference between the indicated power and brake power.
- It is denoted by F.P. and is given by,

$$\text{F.P.} = \text{I.P.} - \text{B.P.}$$

iv) Mechanical Efficiency :

- It is the ratio of brake power to indicated power.
- It is denoted by η_m and is given by,

$$\eta_m = \frac{\text{B.P.}}{\text{I.P.}}$$

❑ 2. Mean effective pressure and torque

i) Mean effective pressure (m.e.p.) :

- It is the hypothetical pressure which acts during the power stroke on the piston.
- If the mean effective pressure is based on the indicated power then it is known as indicated mean effective pressure (i.m.e.p.).
- If the mean effective pressure is based on the brake power then it is known as brake mean effective pressure (b.m.e.p.).

❑ 3. Specific Output

- Specific output is defined as the brake power per unit piston displacement. It is given by,

$$\text{Specific output} = \frac{\text{B.P.}}{A \times l}$$

❑ 4. Volumetric Efficiency

- Volumetric efficiency is defined as the actual volume of charge taken during suction stroke to the swept volume of piston.

❑ 5. Air-fuel ratio

- It is the ratio of mass of air to the mass of fuel in air fuel mixture.

❑ 6. Specific fuel consumption (s.f.c.)

- It is the mass of fuel consumed per kW of power developed per hour.
- It represents the criteria for economical power production and it is given by,

$$\text{s.f.c.} = \frac{\dot{m}_f}{\text{B.P.}}, \text{kg / kWh}$$

7. Thermal Efficiency

- Thermal efficiency is the ratio of power developed to the energy supplied by fuel.
- The indicated thermal efficiency is given by,

$$\eta_{in.th.} = \frac{I.P.}{\dot{m}_f \times C.V.}$$

- The brake thermal efficiency is given by,

$$\eta_{br.th.} = \frac{B.P.}{\dot{m}_f \times C.V.}$$

where, $\dot{m}_f$ = Mass of fuel in kg/sec C.V. = Calorific value of fuel

5.3 Measurement of Indicated Power (IP)

☞ **[GTU : Summer-18, Winter-18]**

- The power which is developed in engine cylinder is known as indicated power.
- Various indicaters are used to measure IP.
- The following are the indicators are used for indicated power

1) Mechanical engine indicator 2) Electronic engine indicator

5.3.1 Measurement of IP using Mechanical Engine Indicator

- The following diagram shows mechanical engine indicator.
- The mechanical engine indicator contains the following components cylinder, piston, coupling nut, gas hole tap, piston rod, spring and mechanical linkages.
- The piston sliding inside the cylinder is connected to a straight line mechanism using piston rod.
- The movement of piston is control by spring of known stiffness value.
- The pen is attached at the end of straight line linkage mechanism and graph is plotted on paper wrapped around a rotating dream.
- The paper on which the plot is obtained is called **indicator card**.
- The card which is attached rotates the drum whose motion is same as the motion of engine piston.
- Due to rotary motion of drum, vertical motion of pen and horizontal motion of cord, a closed figure called **indicator diagram** is obtained on the indicator card.

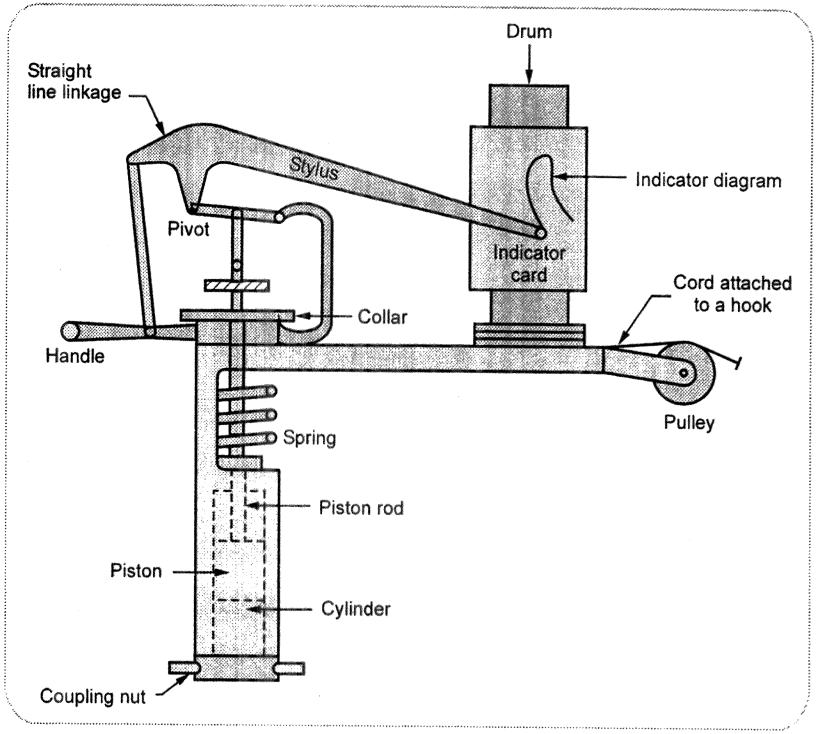


Fig. 5.1 : Mechanical engine indicator

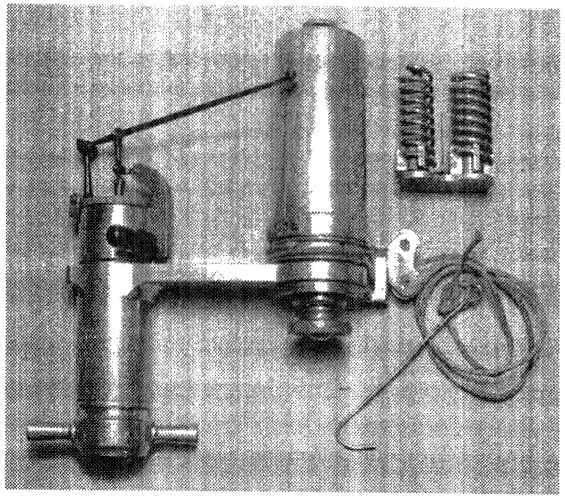


Fig. 5.1(a)

❑ Advantages of mechanical type engine indicator

i) It usefull for low speed engine

ii) Performance is accurate

iii) Zero running cost

iv) Strength and rigidity is high

v) Low initial cost

5.3.2 Measurement of IP using Electronic Engine Indicator

- The electronic indicator is accurate measuring method.
- The following diagram shows electronic engine indicator.
- The indicator mainly contains control grid, heater, electronic beam, focusing coil, deflecting coils.
- Pressure transducer is used in electronic engine indicator.

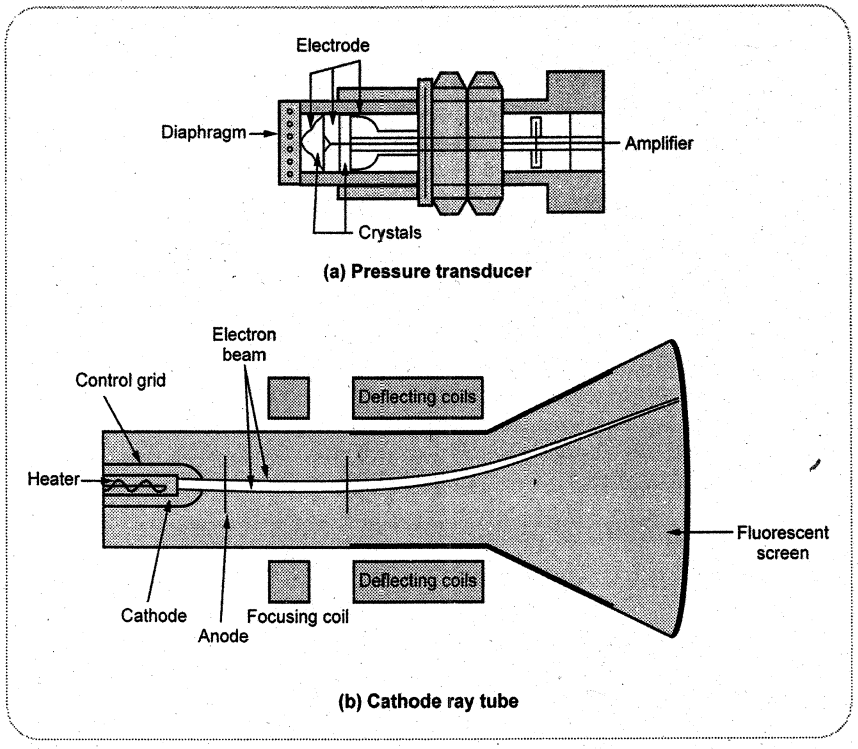


Fig. 5.2 : Electronic indicators

- Pressure transducer is directly inserted into engine cylinder head.
- The diaphoagm of transducer, which works on piezo-electric effect is subjected to engine gas pressures.

❑ Advantages of electronic indicator

- Usefull for slow and high speed engine.
- The accuracy of result is very high.
- Results are accurate in Less trails.

5.4 Measurement of Brake Power (BP)

- The brake power is measured by using a device called dynamometer.

1. Rope brake dynamometer.	2 Prony brake dynamometer
3. Hydraulic dynamometer	4. Eddy current dynamometer

- It is power at crank shaft of the engine.
- The following are the dynamometers are used for finding BP.

5.4.1 Rope Brake Dynamometer [Absorption Type Mechanical Dynamometer]

☞ [GTU : Summer-18]

- It consists of one, two or more ropes wound around the rim of a pulley of flywheel which is keyed to the shaft of an engine.
- The upper end of rope is connected to a spring balance whereas lower end carries a dead weight W as shown in Fig. 5.3.
- To avoid the slipping of the rope over the pulley, 3 or 4 U-shaped wooden blocks are placed at the equal intervals around the circumference of the pulley.
- Due to friction between the rope and drum, high amount of heat is generated, hence the pulley is provided with internal flanges on the rim. It forms a channel for the flow of water to cool the pulley and drum.
- These dynamometers are commonly used for the absorption of wider range of powers.

- Let,

W = Suspended weight or dead weight in N

S = Spring balance reading in N

R_D = Radius of brake drum in m

R_R = Radius of rope in m

R_E = Effective radius of brake drum in m = $R_D + R_R$

N = Speed of engine in rpm

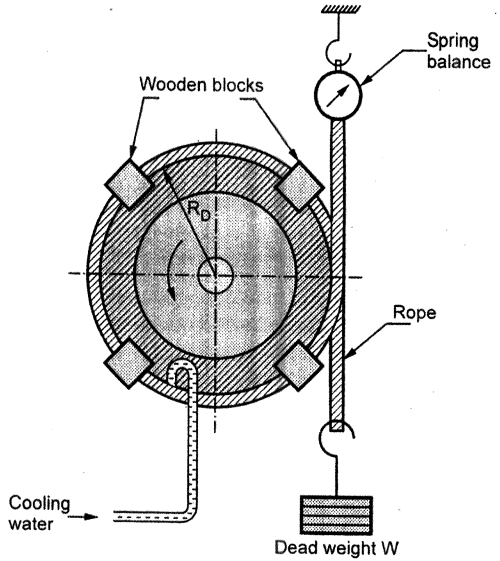


Fig. 5.3 : Rope brake dynamometer

- Net load on the brake is,

Net load = (W – S), N

- Moment of net load or torque on the shaft is,

$$T = (W - S) \times R_e \text{, N-m}$$

- We know that, brake power of the engine is,

$$P = \frac{2\pi NT}{60} = \frac{2\pi N (W - S) \times R_e}{60} \text{ Watts} \quad \ldots (5.1)$$

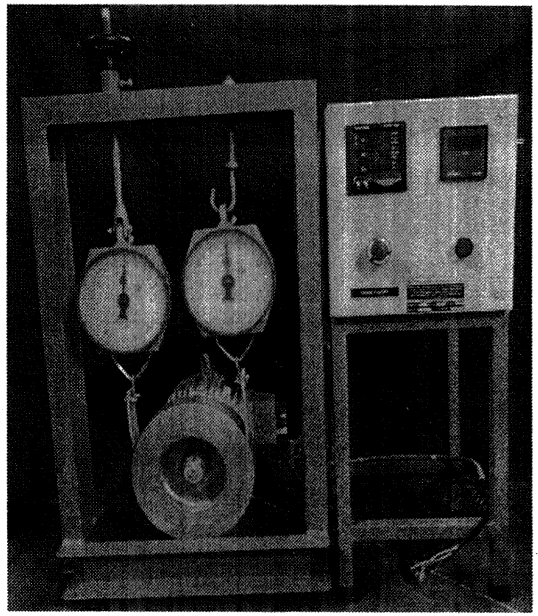


Fig. 5.3(a)

5.4.2 Prony Brake Dynamometer [Absorption Type Dynamometer]

- It is the simplest form of an absorption type dynamometer as shown in Fig. 5.4.

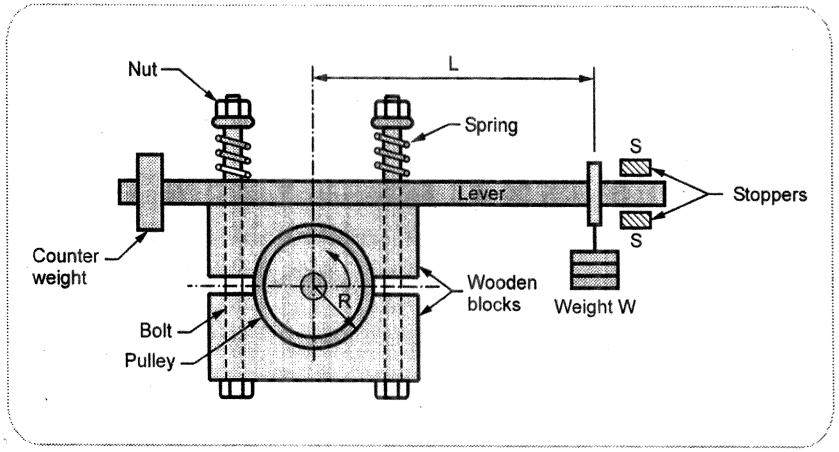


Fig. 5.4 : Prony brake dynamometer

- It consists of two wooden blocks which are clamped together on a revolving pulley (brake drum). This pulley is fixed to the shaft of an engine whose power is to be measured.
- For adjusting the pressure on pulley and to control its speed, helical spring is provided between the nut and upper block.
- The upper block has a long lever attached to it which carries a weight W at its outer end. At the other end of lever a counter weight is placed which balances the brake at unloading position.
- For limiting the motion of the lever, two stoppers are provided on the lever.
- The friction between the blocks and pulley tends to rotate the blocks (lever) in the direction of rotation of shaft.
- Due to friction power is absorbed and the motion is prevented by weight W placed at the end of the lever.

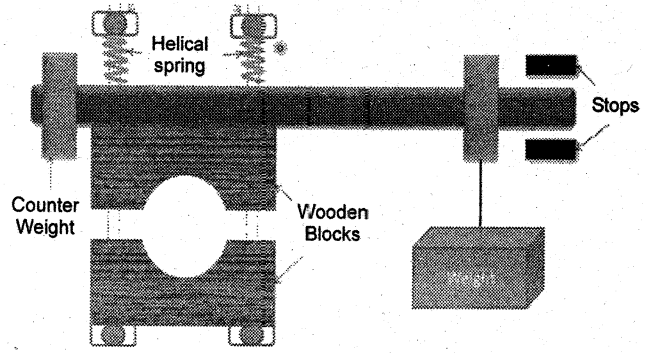


Fig. 5.4(a)

- The weight W is adjusted such that, for desired speed of the engine the lever remains in horizontal position.
- Let, W = Weight at the end of lever in N
  L = Horizontal distance of the weight W from the centre of pulley in m
  N = Speed of the shaft in rpm
  F = Frictional resistance between the blocks and pulley in N
  R = Radius of the pulley in m
- Moment of frictional resistance or torque on the shaft is,

$$T = W \times L = F \times R_f, \text{ N-m}$$

- We know that, brake power of the engine is,

$$P = \frac{2\pi NT}{60} = \frac{2\pi NWL}{60} \quad \text{... (5.2)}$$

- From the above equation (5.2) it is clear that, for calculating the power of the engine it is not necessary to know the radius of pulley (R), coefficient of friction between the blocks and pulley (μ) and pressure exerted by tightening of the nuts (P).

5.4.3 Hydraulic Dynamometer (Absorption TypeWater Brake Dynamometer)

[GTU : Winter-16, Summer-17]

- Froude's hydraulic dynamometer, is the type of absorption dynamometer.
- It consist of shaft, rotor, stator connected to casing, water inlet and outlet connections, etc. Refer Fig. 5.5.

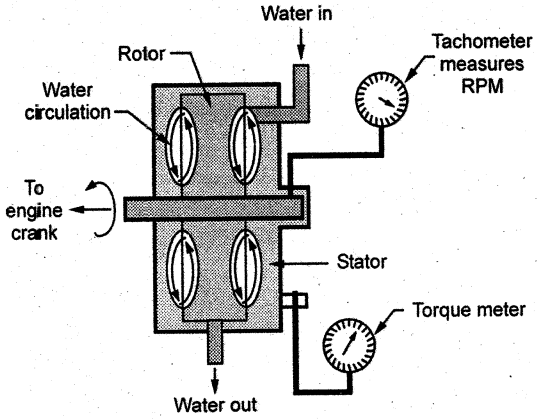


Fig. 5.5 : Froude hydraulic dynamometer

- The shaft is rotated by the prime mover whose power is to be measured.
- This shaft is mounted on the grease lubricated ball bearings to reduce the mechanical friction which affect the input power to be measured.
- The rotor is mounted on the shaft with the help of key. The oil is continuously injected on the shaft for the stress relief purpose.
- The stator and rotor are provided with the semicircular channels or vanes.
- Water is fed to these vanes. As the shaft rotates, it creates the outward flow in the rotor and inward flow in the stator.
- It leads to the vortex flow in the dynomometer compartment. Hence the torque reaction is produced.

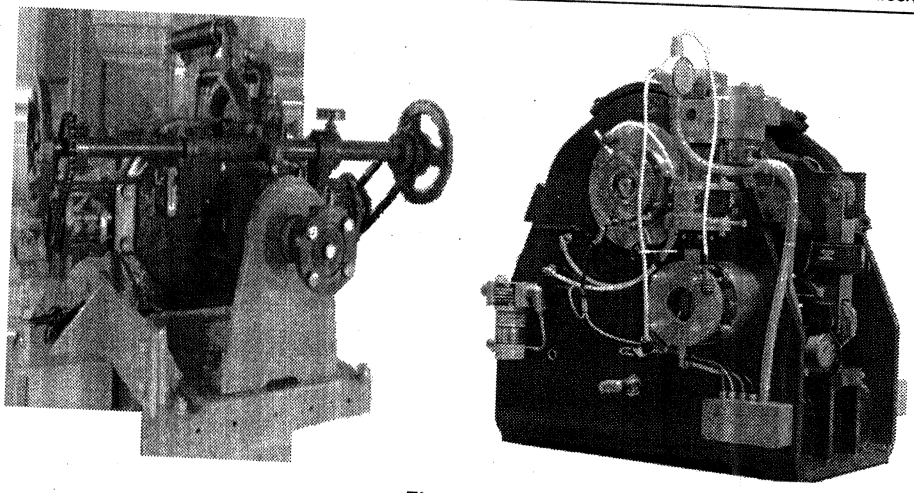


Fig. 5.5(a)

- This torque is measured by the load cell and the reading is shown on the torque meter.
- The water flow is controlled by the butterfly valve and the power absorbed by the hydraulic dynamometer is taken by the water in the form of heat.

❑ Advantages

- Hydraulic dynamometers are robust and compact in design.
- Its maintenance is easy.
- It gives accurate reading as the torque transmitted is fairly accurate.
- Reduced friction losses.
- Wear in the bearing is less.
- It has quick load varying ability.

❑ Disadvantages

- It requires comparatively large amount of oil and oil reservoir.
- Due to requirement of large amount of oil, it is expensive.

❑ Applications

- It is used for the testing of the following :
  - CI and SI engines in automobiles.
  - Turbines in power generation.
  - Electric motors.
  - Marine diesel engines, etc.

5.4.4 Eddy Current Dynamometer

- The schematic diagram of Eddy current dymamometer as shown in following figure.
- It mainly consist of stator and rotor.
- The electromagnets are attached to the periphery of stator, numbers of electromagets are fixed.
- The rotor shaft is directly coupled to engine shaft.
- In antifriction bearing the stator is mounted.
- The stator is free to rotate about the same axis of the shaft.
- The stator casing is connected the load arm which resists the rotation of the stator.
- If the rotor starts rotates eddy current formed due to the rotating magnetic flux.

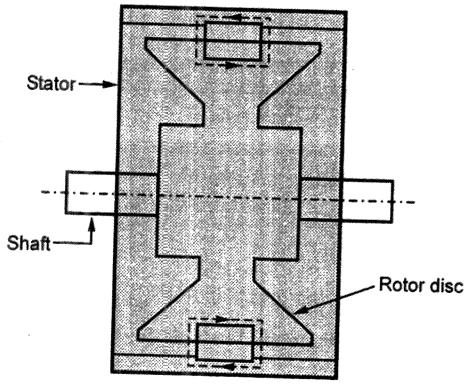


Fig. 5.6 : Eddy current dynamometer

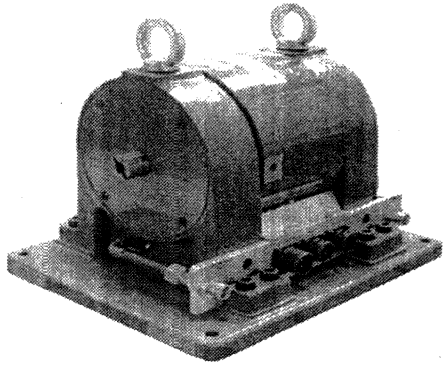


Fig. 5.6(a)

❑ Advantages of eddy current dynamometer

- It is accurate as compared mechanical indicator.
- It can be used for medium speed engine.

5.5 Measurement of Friction Power (FP)

- The frictional power is a difference between indicated power (IP) and brake power (BP), Mathematically,

$$FP = IP - BP$$

- It consist of power loss due to friction.

- The frictional power (FP) affects engine performance.
- The increase in the frictional power the engine life decreases.
- The frictional power can be measured by a following methods :

1. Willan's Line Method 2. Morse test 3. Motoring test

5.5.1 Willan's Line Method

☞ [GTU : Summer-14]

- The Willan's line method includes graphical representation of fuel consumption (FC) and brake power.
- The following graph shows willan's line method.
- In the above graph, we will get a linear curve for FC and BP.
- If this curve in back plotted as shown in figure.
- The back plotted obtained curve indicates frictional power.

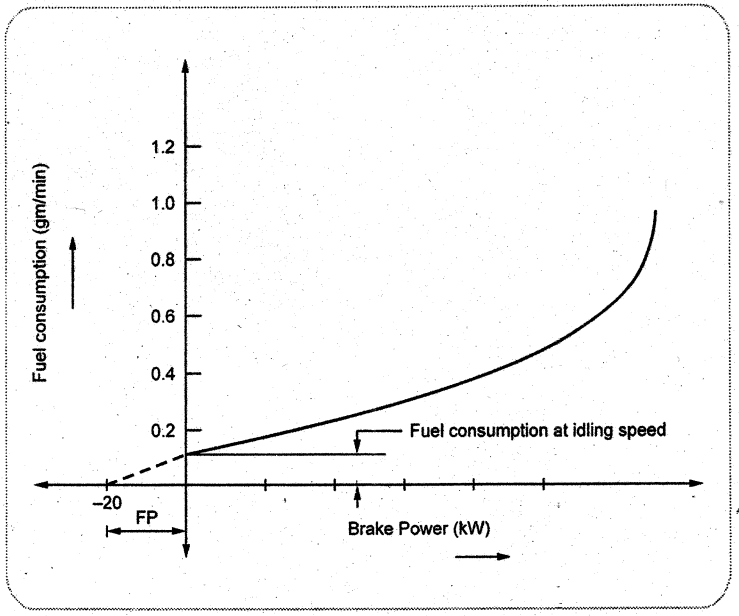


Fig. 5.7 : Willan's line method

5.5.2 Morse Test on Multi-Cylinder Engine

- The Morse test in carried out for multicylinder engine to find frictional power.
- Fig. 5.8 shows set–up for multicylinder engine.

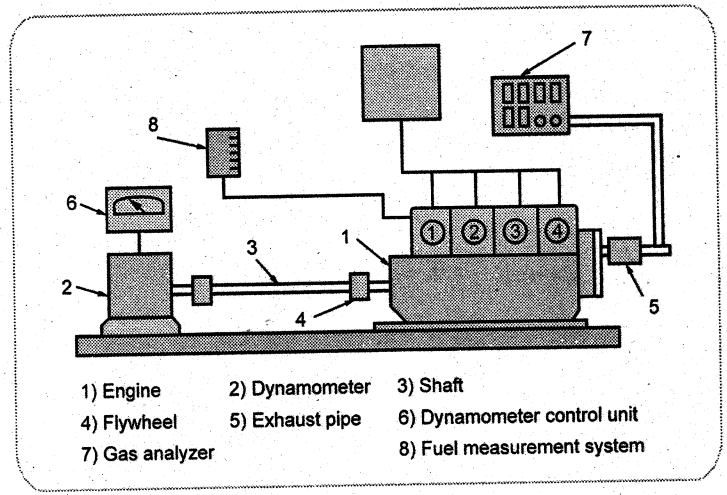


Fig. 5.8 : Morse test setup for multicylinder engine

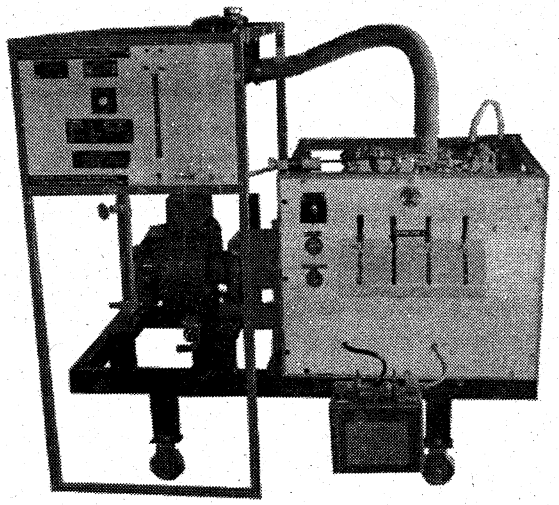


Fig. 5.8(a)

1. Morse Test :

☞ [GTU : Summer-14, 16, Winter-18]

- Morse Test can be used for determining FP/IP of multi-cylinder IC engines, generally 3 cylinders and more by cutting off each cylinder in turn.
- In SI engines, each cylinder is rendered in-operative by short-circuiting the SP or cutting off fuel supply in MPFI systems. In CI engines, fuel supply is cut off.

- Consider 4 strokes, 4 cylinder SI engine coupled with dynamometer.
- Engine is run at constant speed N throughout one set of test parameters, as $FP \propto N^2$
- It is assumed that pumping & mechanical losses are same whether a cylinder is working or not
- Throttle position is kept fixed, however, to attain same speed N, load is decreased by dynamometer

Let B = Brake Power of an engine when all cylinders are working.

BP_1 = BP of engine when Cylinder No 1 is cut off

Similarly,

BP_2 = BP of engine when Cylinder No 2 is cut off

BP_3 = BP of engine when Cylinder No 3 is cut off

BP_4 = BP of engine when Cylinder No 4 is cut off

- Let I_1, I_2, I_3 and I_4 be the IPs developed by Cylinder Nos. 1, 2, 3 and 4 respectively and their corresponding Friction Power be F_1, F_2, F_3 and F_4.

Total BP (B) = $(I_1 + I_2 + I_3 + I_4) - (F_1 + F_2 + F_3 + F_4)$

BP (B) = $(I_1 + I_2 + I_3 + I_4) - (F_1 + F_2 + F_3 + F_4)$

Hence, $B_1 = (I_2 + I_3 + I_4) - (F_1 + F_2 + F_3 + F_4)$

On subtracting; $B - B_1 = I_1$

Similarly, $B - B_2 = I_2$, $B - B_3 = I_3$, $B - B_4 = I_4$

On adding; $IP = I_1 + I_2 + I_3 + I_4$

$IP = 4B - (B_1 + B_2 + B_3 + B_4)$

- B, B_1, B_2, B_3 and B_4 can be measured by Dynamometer, Hence IP can be calculated Therefore, **FP = IP – BP**.

5.5.3 Motoring Test

☞ [GTU : Summer-18]

- The motoring test is one of the better method to find out the friction power [FP]. In this swinging field Dynamometer is used to find out the friction power suitable electric connection of swinging field dynamometer is required to getting, the reversible action i.e. it can be operated as a motor as well as a power absorbing device.
- The engine is tested by using its operated suitable speed and power upto steady state condition are achieved.

- At the time of testing, the swinging field dynamometer is coupled to engine to absorb the power of engine and the speed of dynamometer and engine are noted.
- After that, the engine fuel supply or ignition is cutoff and the dynamometer is switched to run as an electric motor.
- Therefore, the electric motor i.e. swinging field dynamometer maintains the original engine speed which is noted at time of testing.
- By using current and voltage readings, power consumption of the motor is calculated.
- This power represent the frictional power (FP) of the engine at that speed.
- So, this type of test is called as "Motoring" Test; because engine is driven by an electric motor.
- At the time of motoring test engine cooling system must be shut down. This is given idea about the varying of temperature i.e. engine temperature attained during steady state will remain more or less same during motoring test.

5.6 Engine Speed Measurement

- Engine speed is important parameter for testing and performance of the engine.
- The device used for measuring the speed is **Tachometer**.
- The tachometers are two types.

  1) Contact type tachometer 2) Contact less type tachometer
- Contact type tachometer involves direct contact of tachometer with crankshaft of engine.
- Contact type tachometer are less accurate.
- The modern using tachometer are contactless tachometer.
- The contactless tachometer have digital display.
- These tachometer should handle carefully.

5.7 Measurement of Fuel Consumption

☞ [GTU : Summer-18]

- The fuel used in engine is measured by two ways that is interms of volume or interms of mass.
- In case of automobile expressed in kilometers per liter (KMPL).
- The following are the basic method to determine fuel consumption.

1. Buerette Method 2.Rotometer

5.7.1 Volumetric Type Fuel Flowmeter [Buerette Method]

- It is simple method to determine fuel consumption.
- The following Fig. 5.9 shows set–up of volumetric tupe fuel flowmeter.
- The experiment consist of two glass bulbs having a capacity of 100 cc and 200 cc capacity.
- These bulbs are connected to the three way rocks.
- That is one bulb is feed the fuel to engine while other is being filled.
- The time taken by engine and volume of fuel consumed is noted.
- This method is less accurate.

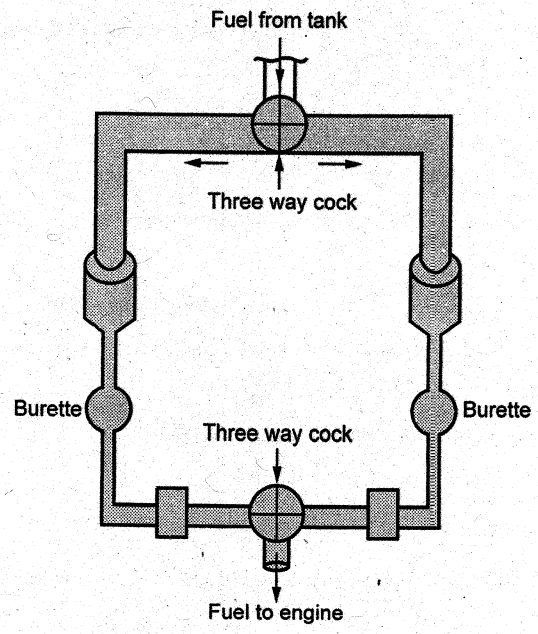


Fig. 5.9 : Flowmeter

5.7.2 Flowmeter or Rotometer

☞ [GTU : Summer-16]

- The flowmeter or Rotometer is a device used to measure fuel consumption in Kilograms per unit time than liters per unit time.
- The engine efficiency directly depends on fuel consumption.

- Fig. 5.10 shows Rotometer or flowmeter
- The flowmeter directly gives the fuel consumption in mass of fuel.
- The rotometer mainly consist of fuel inlet, float and fuel outlet as shown in Fig. 5.10.
- The float is actually a shaped weight, which is pushed up by the force of the fuel flow and pulled down by gravity.
- The fluid density less–than fluid flow rate.
- The rotometer directly gives reading in kg/hr.

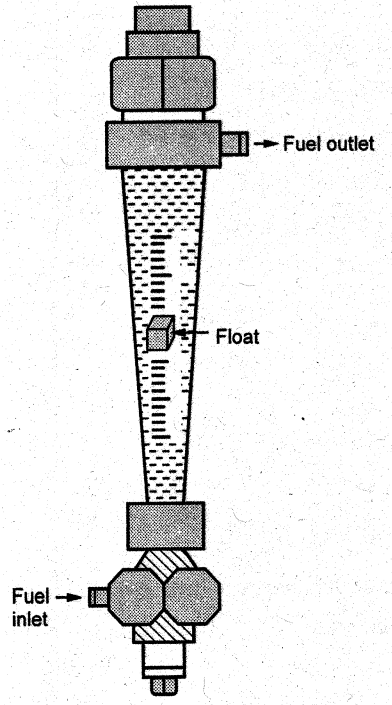


Fig. 5.10 : Rotameter

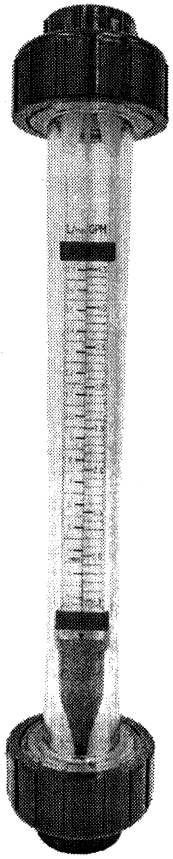


Fig. 5.10(a)

5.8 Measurement of Air Consumption by Air Box Method

[GTU : Summer-14, Winter-16]

- The measurement of air consumption is a important parameter for testing of engine.
- Fig. 5.11 shows a set-up for Air box method.
- The air box method mainly consist of Air tank water manometer, orifice plate and thermometer as shown in Fig. 5.11.
- The manometer is used to measure pressure of air.
- The orifice is used to measure flow rate of air.

- The air tank taken should have suitable volume.
- The orifice plate having a area A and coeficient of discharge is connected to air tank.

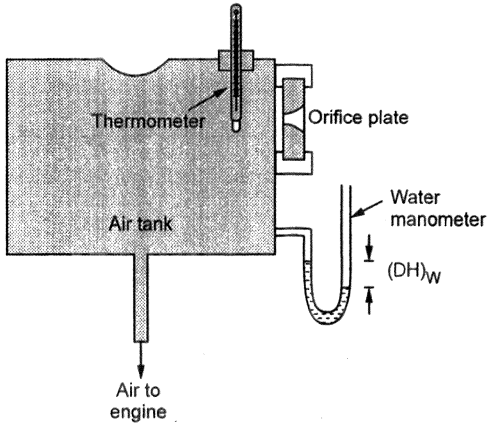


Fig. 5.11 : Measurement of air consumption by air box method

We know that,

$$\Delta H_{air} = H_w \frac{\rho_w}{\rho_a} \quad \ldots(p = \rho g H) \quad \ldots\ldots (5.3)$$

$(\Delta H)_{air}$ = Pressure difference of air

$(\Delta H)_w$ = Pressure difference measured by water manometer.

ρ_w = Density of water in kg/m^3

($\rho_w = 1000\ kg/m^3$)

Rate of discharge of air through orifice is given

$$\dot{Q}_a = A \times V$$

V = Velocity of air in m/s

$$\dot{Q}_a = A \times C_d \times \sqrt{2g\,\Delta H_{air}}$$

From equation (5.3)

$$\dot{Q}_a = A \times C_d \times \sqrt{2g\,\Delta H_w \left(\frac{\rho_w}{\rho_a}\right)}$$

$$\therefore \quad \dot{m}_a = \dot{Q}_a \times \rho_a$$

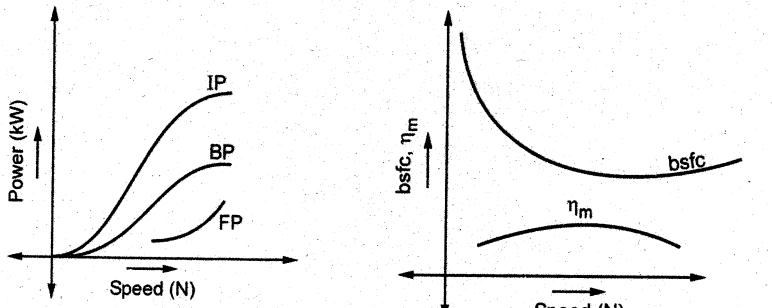
$$\therefore \quad \dot{m}_a = A \times C_d \times \left(\sqrt{2g\,\Delta H_w \times \frac{\rho_w}{\rho_a}} \right) \times \rho_a$$

$$\therefore \quad \boxed{\dot{m}_a = A \times C_d \times \sqrt{2g\,\Delta H_w \times \rho_w \times \rho_a}}$$

5.9 Performance Characteristics of SI Engine

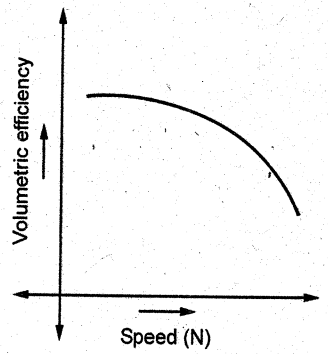
[GTU : Summer-16, Winter-14]

- To find out BP, IP, FP, BSFC and efficiency of SI engines at various speeds and load condition, number of laboratory tests are performed on SI engines.
- The resultant performance curves indicates the optimum speed and load conditions at which engine should be operated so as to get best possible fuel economy.
- The performance curves for SI engines are shown in follow Fig. 5.12.



(a)

(b)



(c)

Fig. 5.12 : Effect of speed on different parameters for SI engines

- To obtaining maximum power test, the engine throttle should be fully opened.

- At the time of test, the speed of engine is varied by applying different values of external loads from collecting data and plotted graphs of performance parameter vs. speed as shown in Fig. 5.12(a, b and c).
- Fig. 5.12(a) shows the graph of IP, BP and FP vs speed. As seen from the graph it is clear that as speed increases, BP also increases i.e. they are directly proportional to each other. It is found through experiments that BP can be doubled by doubling the running speed of engine.
- However, at higher speeds, FP also increases rapidly and results in loss of power. This reduces mechanical efficiency. Hence, engine should be run at high speed which is chosen as optimum speed.
- In Fig 5.12(b) graph of BSFC and mechanical efficiency vs. speed. It is seen that, at low speeds BSFC is high because the engine uses rich mixture. As speed is increases, at point A, it is seen that BSFC is minimum and mechanical efficiency is maximum. At that speed engine fuel consumption. For give rated power output will be minimum. Hence engine should be operated at or near this speed.
- For this optimum speed range for least and highest mechanical efficiency (i.e. best economy) is shown by green colour range on speed indicators for two wheelers.
- If the engine is run beyond this optimum speed, there may be increases BP. However BSFC increases (i.e. more fuel consumption) and mechanical efficiency decrases (i.e. more friction loss).
- Fig. 5.13 (a) shows graph of BSFC vs load.
- Fig. 5.13 (c) shows graph of volumetric efficiency vs. speed. At higher speeds, opening period of inlet valve will be lesser as compared to slow speed running. This implies that lesser charge containing lesser amount of air is inducted into cylinder. Therefore this reduces volumetric efficiency.
- The SI engines are designed such away that BSFC reduces as load varies in the range of 60 % to 100 % of rated load.
- If the engine is overloaded i.e. load beyond 100 % or if engine is run at under loaded (i.e. load below 60 %) BDFC is more.
- If engine 5.13(b) shows graph of mechanical efficiency vs. speed. As load increases mechanical efficiency increases. Engines are designed to give highest mechanical efficiency at full load condition.

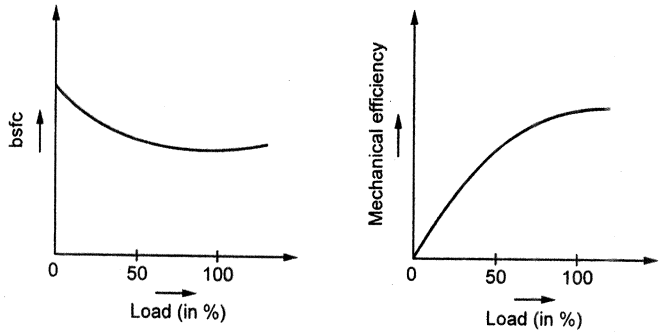


Fig. 5.13 : Effect of load on different parameters for SI engines

5.10 Performance Characteristics of CI Engine

CI Engine Performance Curves

- As FP $\propto N^2$, mechanical efficiency reduces due to steep increase in FP.
- At lower speeds, due to lower charge velocity because of low piston speed, BSFC reduces since volumetric efficiency increases and mechanical efficiency is high.

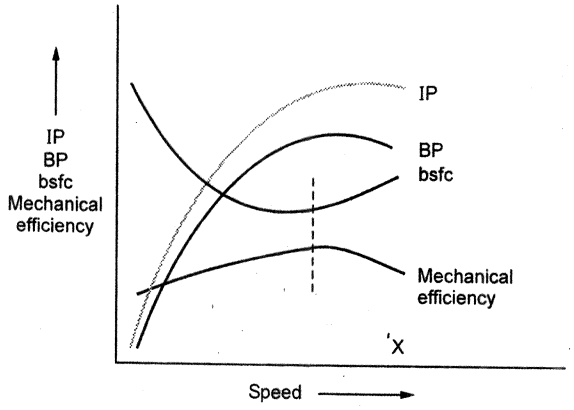


Fig. 5.14

- After certain speed, BSFC increases due to reduction in volumetric efficiency and increase in mechanical losses.
- Point X represents economical speed of engine for minimum fuel consumption.
- Volumetric Efficiency reduces with increase in speed due to increase in intake velocity resulting in drop of suction pressure.

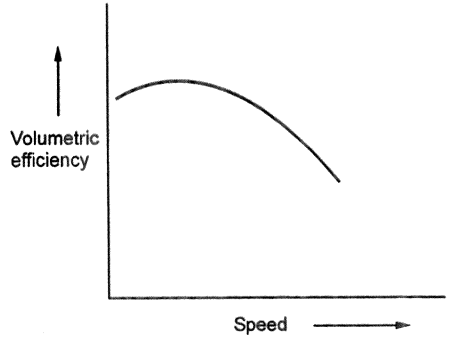


Fig. 5.15

- Higher the speed, lesser the time available for induction of charge.
- Suction valve fully opens only when pressure inside cylinder slightly below the surrounding pressure, thus reducing effective suction stroke.
- IP and BP increase with speed but due to steep increase in FP, IP and BP start coming down.
- For BSFC curve, same reasons as in SI engine.

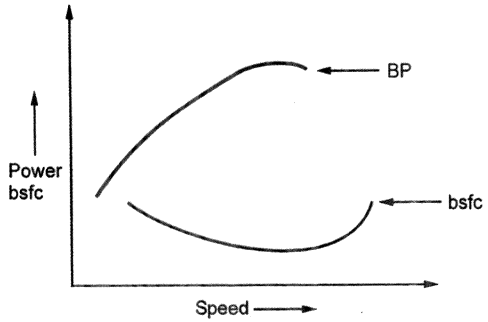


Fig. 5.16

5.11 Solved Examples

Ex. 5.1 : Following data are collected from a 4-stroke single cylinder oil engine at full load. Bore = 200 mm, stroke = 280 mm, speed = 300 rpm, indicated mean effective pressure = 5.6 bar, torque on the brake drum = 250 Nm, oil consumed = 4.2 kg / hour, calorific value of oil = 41,000 kJ/kg. Determine mechanical efficiency, indicated thermal efficiency and brake thermal efficiency. **[GTU : Summer-18, Winter-17, 15]**

Sol. : Given data : d = 200 mm = 0.2 m, l = 280 mm = 0.28 m, N = 300 rpm,

imep = 5.6 bar = 5.6×10^5 N/m^2,

$T = 250$ N-m, $\dot{m}_f = 4.2$ kg/hr $= \frac{4.2}{3600}$ kg/sec. $= 1.6666 \times 10^{-3}$ kg/sec,

C.V. = 41000 kJ/kg, n = 1

To find : i) η_m ii) $\eta_{in.th.}$ iii) $\eta_{br.th.}$

✦ **Step 1 : Calculate the mechanical efficiency of oil engine**

1) Brake Power (B.P.)

$$B.P. = \frac{2\pi \times N \times T}{60 \times 1000} = \frac{2\pi \times 300 \times 250}{60 \times 1000}$$

$$= 7.8539 \text{ kW}$$

2) Indicated Power (I.P.)

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$= \frac{1 \times 5.6 \times 10^5 \times 0.28 \times \left(\frac{\pi}{4}\right) \times (0.2)^2 \times \left(\frac{300}{2}\right)}{60 \times 1000}$$

$$= 12.3150 \text{ kW}$$

- Mechanical efficiency $\eta_m = \frac{B.P.}{I.P.} \times 100 = \frac{7.8359}{12.3150} \times 100$

$$= \mathbf{63.7725\ \%} \quad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the indicated thermal efficiency of oil engine**

$$\eta_{in.th.} = \frac{I.P.}{\dot{m}_f \times C.V.} \times 100 = \frac{12.3150}{1.6666 \times 10^{-3} \times 41000} \times 100$$

$$= \mathbf{18.0226\ \%} \quad \textbf{...Ans.}$$

✦ **Step 3 : Calculate the brake thermal efficiency of oil engine**

$$\eta_{in.th.} = \frac{B.P. \times 100}{\dot{m}_f \times C.V.} = \frac{7.8539}{1.6666 \times 10^{-3} \times 41000} \times 100$$

$$= \mathbf{11.4939\ \%} \quad \textbf{...Ans.}$$

Ex. 5.2 : A four stroke diesel engine has a piston diameter 250 mm and stroke 400 mm. The mean effective pressure is 4 bar and the speed is 500 rpm. The diameter of the brake drum is 1000 mm and the effective brake load is 400 N. Find indicated power, brake power and frictional power.

▣ **Sol. : Given data :**

d = 250 mm = 0.25 m, l = 400 mm = 0.4 m, imep = 4 bar = 4×10^5 N/m^2, N = 500 rpm,

d_b = 1000 mm = 1 m, (W – S) = 400 N, n = 1

To find : i) I.P. ii) B.P. iii) F.P.

✦ **Step 1 : Calculate the indicated power of diesel engine**

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$= \frac{1 \times 4 \times 10^5 \times 0.4 \times \left(\frac{\pi}{4}\right) \times (0.25)^2 \times \left(\frac{500}{2}\right)}{60 \times 1000} = \mathbf{32.7249\ kW} \quad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the brake power of diesel engine**

$$B.P. = \frac{(W - S)\pi d_b N}{60 \times 1000} = \frac{400 \times \pi \times 1 \times 500}{60 \times 1000} = \mathbf{10.4719\ kW} \quad \textbf{...Ans.}$$

✦ **Step 3 : Calculate the frictional power of diesel engine**

$$\mathbf{F.P.} = I.P. - B.P. = 32.7249 - 10.4719$$

$$= \mathbf{22.253\ kW} \quad \textbf{...Ans.}$$

Ex. 5.3 : The following details were noted in a test on a four-cylinder, four-stroke engine, diameter = 100 mm, stroke = 120 mm, speed of the engine = 1600 rpm, fuel consumption 0.2 kg/min, calorific value of fuel is 44000 kJ/kg, difference in tension on either side of brake pulley = 40 kg, brake circumference is 300 cm. If the mechanical efficiency is 80 %, calculate : i) Brake thermal efficiency ii) Indicated thermal efficiency iii) Indicated mean effective pressure iv) Brake specific fuel consumption. **[GTU : Winter-16, Summer-14]**

▣ **Sol. : Given :** d = 100 mm = 0.1 m, l = 120 mm = 0.12 mm, N = 1600 rpm,

$\dot{m}_f = 0.2$ kg/min $= \frac{0.2}{60}$ kg/sec $= 3.333 \times 10^{-3}$ kg/sec, C.V. = 44000 kJ/kg,

n = 4, $(T_1 - T_2) = 40$ kg $= 40 \times 9.81 = 392.4$ N,

Circumference = 300 cm = 3 m, $\eta_1 = 80\ \%$.

To find : i) η_{bth} ii) η_{inth} iii) i.m.e.p. iv) b.s.f.c.

✦ **Step 1 : Calculate brake power and indicated power**

Brake circumference is given by,

$$C_b = 2\pi r_b \qquad \therefore \quad 3 = 2\pi r_b$$

$$\therefore \quad r_b = 0.4774 \text{ m}$$

We know that, torque is

$$T = (T_1 - T_2) \times r_b = 392.4 \times 0.4774 = 187.331 \text{ N-m}$$

Brake power is given by,

$$BP = \frac{2\pi NT}{60 \times 100} = \frac{2\pi \times 1600 \times 187.331}{60 \times 1000} = 31.3876 \text{ kW}$$

Mechanical efficiency is given by,

$$\eta_m = \frac{BP}{IP} \times 100 \qquad \therefore \quad 80 = \frac{31.3876}{IP} \times 100$$

$$\therefore \quad I.P. = 39.2345 \text{ kW}$$

But indicated power can also be given as,

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$39.2345 = \frac{4 \times imep \ \ 0.12 \times \left(\frac{\pi}{4}\right) \times (0.1)^2 \times \frac{1600}{2}}{60 \times 1000}$$

$$\therefore \quad \textbf{i.m.e.p.} = 7.8054 \times 10^5 \text{ N/m}^2 = \textbf{7.8054 bar} \qquad \textbf{...Ans.}$$

✦ **Step 2 : Calculate indicated thermal efficiency and brake thermal efficiency of the engine**

Indicated thermal efficiency is given by,

$$\eta_{in.th.} = \frac{I.P.}{\dot{m}_f \times C.V.} \times 100 = \frac{39.2345}{3.333 \times 10^{-3} \times 44000} \times 100 = \textbf{26.751 \%} \qquad \textbf{...Ans.}$$

$$\eta_{orth.} = \frac{B.P. \times 100}{\dot{m}_f \times C.V.} = \frac{31.3876}{3.333 \times 10^{-3} \times 44000} \times 100 = \textbf{21.2644 \%} \qquad \textbf{...Ans.}$$

✦ **Step 3 : Calculate brake specific fuel consumption**

Brake specific fuel consumption is given by,

$$b.s.f.c. = \frac{\dot{m}_f}{B.P.} = \frac{3.333 \times 10^{-3}}{31.3876}$$

$$= \textbf{1.0618} \times \textbf{10}^{-4} \textbf{ kg/kW-s} \qquad \textbf{...Ans.}$$

Ex. 5.4 : The following observations were recorded during a test on 4 stroke single cylinder diesel engine.

Bore = 200 mm, stroke = 250 mm, mean effective pressure = 0.6 MPa.

Brake drum diameter = 1.2 m, net brake load = 50 kg. Mean piston speed = 300 m/min. Find : i) Friction power ii) Mechanical efficiency.

▣ **Sol. :**

Given data : d = 200 mm = 0.2 m, L = 250 mm = 0.25 m, imep = 0.6 MPa = 0.6×10^6 N/m^2, d_b = 1.2 m, (W – S) = 500 N, V = 300 m/min = $\frac{300}{60}$ m/s

To find : i) F.P. ii) η_m

Speed of crankshaft

$$V = \frac{2LN}{60} \qquad \therefore \quad \frac{300}{60} = \frac{2 \times 0.25 \times N}{60}$$

$$N = 600 \text{ rpm.}$$

✦ **Step 1 : Calculate the indicating power of diesel engine**

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$\textbf{I.P.} = \frac{1 \times 0.6 \times 10^6 \times 0.25 \times \left(\frac{\pi}{4}\right) \times (0.2)^2 \times \frac{600}{2}}{60 \times 1000} = \textbf{23.5619 kW} \qquad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the brake power of diesel engine**

$$\textbf{B.P.} = \frac{(W - S)\, \pi d_b\, N}{60 \times 1000} = \frac{500 \times \pi \times 1.2 \times 600}{60 \times 1000} = \textbf{18.8495 kW} \qquad \textbf{...Ans.}$$

✦ **Step 3 : Calculate the frictional power of diesel engine**

$$\textbf{F.P.} = I.P. - B.P. = 23.5619 - 18.8495 = \textbf{4.7123 kW} \qquad \textbf{...Ans.}$$

✦ **Step 4 : Calculate the mechanical efficiency of diesel engine**

$$\eta_m = \frac{B.P.}{I.P.} \times 100 = \frac{18.8495}{23.5616} \times 100 = \textbf{79.99 \%} \qquad \textbf{...Ans.}$$

Ex. 5.5 : The following results refer to a test on I.C. engine : Indicative power = 42 kW, frictional power = 7 kW, engine speed = 1800 rpm, specific fuel consumption per B.P. = 0.30 kg/kWh, calorific value of fuel used = 43000 kJ/kg. Calculate I) Mechanical efficiency ii) Brake thermal efficiency iii) Indicated thermal efficiency. **[GTU : Summer-16]**

▣ **Sol. : Given data :**

I.P. = 42 kW, F.P. = 7 kW, N = 1800 rpm, sfc = 0.30 kg/kWh, C.V. = 43,000 kJ/kg

To find : i) η_m ii) $\eta_{br.th.}$ iii) $\eta_{in.th.}$

✦ **Step 1 : Calculate the mechanical efficiency of I.C. engine**

$$B.P. = I.P. - F.P.$$

$$= 42 - 7 = 35 \text{ kW}$$

$$\eta_m = \frac{B.P.}{I.P.} \times 100 = \frac{35}{42} \times 100 = \mathbf{83.3334\ \%} \quad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the brake thermal efficiency of I.C. engine**

$$sfc = \frac{\dot{m}_f}{B.P.} \quad \therefore \quad 0.30 = \frac{\dot{m}_f}{35} \quad \therefore \quad \dot{m}_f = 10.5 \text{ kg/h}$$

$$\eta_{br.th} = \frac{B.P.}{\dot{m}_f \times C.V.} \times 100 = \frac{35}{\left(\frac{10.5}{3600}\right) \times 43000} \times 100 = \mathbf{27.90\ \%} \quad \textbf{...Ans.}$$

✦ **Step 3 : Calculate the indicated thermal efficiency of I.C. engine**

$$\eta_{in.th} = \frac{I.P.}{\dot{m}_f \times C.V.} \times 100 = \frac{42}{\left(\frac{10.5}{3600}\right) \times 43000} \times 100 = \mathbf{33.48} \quad \textbf{...Ans.}$$

Ex. 5.6 : A single cylinder four stroke engine runs at 1000 rpm has a bore of 115 mm and has a stroke of 140 mm. The brake load is 6 kg at 600 mm radius and the mechanical efficiency is 80 %. Calculate brake power and the mean effective pressure. ☞ **[GTU : Winter-15]**

▣ **Sol. : Given data :**

n = 1, N = 1000 rpm, d = 115 mm = 0.115 m, l = 140 mm = 0.14 m,

m = 6 kg, r = 600 mm = 0.6 m, η_m = 80 % = 0.8

To find : i) B.P. ii) mep

✦ **Step 1 : Calculate the brake power of engine**

$$B.P. = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times 1000 \times 6 \times 9.81 \times 0.6}{60 \times 1000}$$

$$\mathbf{B.P. = 3.6982\ kW} \quad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the mean effective pressure of engine**

$$\eta_m = \frac{B.P.}{I.P.} \quad \therefore \quad 0.8 = \frac{3.6982}{I.P.}$$

$$I.P. = 4.6228 \text{ kW}$$

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$4.6228 = \frac{1 \times mep \times 0.14 \times \left(\frac{\pi}{4}\right) \times (0.115)^2 \times \left(\frac{1000}{2}\right)}{60 \times 1000}$$

$$\therefore \quad \mathbf{mep = 3.8148 \times 10^5\ N/m^2 \text{ or } 3.8148\ bar} \quad \textbf{...Ans.}$$

Ex. 5.7 : The following observations were recorded during a test on 4-stroke diesel engine : Bore = 200 mm, Stroke = 250 mm, Mean effective pressure = 0.6 MPa, Brake drum diameter = 1.2 m, Net brake load = 500 N, Speed of crank shaft = 600 rpm Find i) Indicating power ii) Brake power iii) Friction power iv) Mechanical efficiency.

▣ **Sol. : Given data :**

d = 200 mm = 0.2 m, l = 250 mm = 0.25 m, N = 600 rpm

imep = 0.6 MPa = 0.6×10^6 N/m^2, d_b = 1.2 m, (W – S) = 500 N

To find : i) I.P. ii) B.P. iii) F.P. iv) η_m

✦ **Step 1 : Calculate the indicating power of diesel engine**

$$I.P. = \frac{n \times imep \times l \times A \times (N/2)}{60 \times 1000}$$

$$I.P. = \frac{1 \times 0.6 \times 10^6 \times 0.25 \times \left(\frac{\pi}{4}\right) \times (0.2)^2 \times \left(\frac{600}{2}\right)}{60 \times 1000} = \mathbf{23.5619\ kW} \quad \textbf{...Ans.}$$

✦ **Step 2 : Calculate the brake power of diesel engine**

$$B.P. = \frac{(W - S)\,\pi d_b N}{60 \times 1000} = \frac{500 \times \pi \times 1.2 \times 600}{60 \times 1000}$$

$$= \mathbf{18.8495\ kW} \quad \textbf{...Ans.}$$

✦ **Step 3 : Calculate the frictional power of diesel engine**

$$F.P. = I.P. - B.P. = 23.5619 - 18.8495$$

$$= \mathbf{4.7123\ kW} \quad \textbf{...Ans.}$$

✦ **Step 4 : Calculate the mechanical efficiency of diesel engine**

$$\eta_m = \frac{B.P.}{I.P.} \times 100 = \frac{18.8495}{23.5619} \times 100 = \mathbf{79.99\ \%} \quad \textbf{...Ans.}$$

Ex. 5.8 : A single cylinder engine running at 2000 rpm develops torque 10 Nm. The indicated power of the engine is 2.3 kW. Find loss due to F.P as the percentage of brake power.

▣ **Sol. :**

Given data : K = 1, N = 2000 rpm, T = 10 N - m, IP = 2.3 kW

To find : i) FP ii) Loss due to F.P. as percentage of B.P.

1) To find FP

$$FP = IP - BP$$

$$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2 \times \pi \times 2000 \times 10}{60 \times 1000} = 2.094 \text{ kW}$$

$$FP = IP - BP = 2.3 - 2.0944 = \mathbf{0.2056\ kW} \quad \text{... Ans.}$$

2) Loss due to F.P. as percentage of B.P.

$$= \frac{FP}{BP} \times 100 = \frac{0.2056}{2.0944} \times 100 = \mathbf{9.817\ \%} \quad \text{... Ans.}$$

Ex.5.9 : A single cylinder 4 stroke engine gave the following results while running on full load ;
Area of indicator card = 300 mm^2 ; Length of diagram = 40 mm
Spring constant = 1 bar/mm ; Speed of engine = 400 rpm
Load on brake = 370 N ; Spring balance reading = 50 N
Diameter of brake drum = 1.2 m ; Fuel consumption = 2.8 kg/h
C.V of fuel = 41800 kJ/kg ; Diameter of cylinder 160 mm
Stroke of piston = 200 mm
Calculate :
a) Indicated mean effective pressure
b) Brake power and brake mean effective pressure
c) b.s.f.c
d) Brake thermal efficiency
e) Indicated thermal efficiency. ☞ **[GTU : Summer-15, Winter-16]**

Sol. :

Given data : K = 1, x = 2, a = 300 mm^2, b = 40 mm, k = 1 bar/mm

N = 400 rpm, W = 370 N, S = 50 N, D_b = 1.2 m, $r = \frac{1.2}{2} = 0.6$ m, $\dot{m}_f$ = 2.8 kg/hr

C.V. = 41800 kJ/kg, D_c = 0.16 m, L_s = 0.2 m

To find : i) p_m ii) BP and bmep iii) b.s.f.c iv) η_{bth} v) η_{ith}

i) To calculate P_m

$$P_m = \frac{a}{b} \times k = \frac{3000}{40} \times 1 = \mathbf{7.5\ bar} \quad \text{...Ans.}$$

ii) BP and bmep

$$BP = \frac{(W - S)\, 2\pi\, rN}{60 \times 1000}$$

$$= \frac{(370 - 50) \times 0.6 \times 2 \times \pi \times 400}{60 \times 1000} = 8.04 \text{ kW}$$

$$BP = \frac{P_m \times L \times A \times N}{60 \times X \times 1000}$$

$$8.04 = \frac{P_m \times L \times A \times N}{60 \times x \times 1000}$$

$$= \frac{P_m \times 0.2 \times \pi \times (0.16)^2 \times 400}{60 \times 2 \times 1000 \times 4}$$

$$\therefore \quad p_m = \mathbf{6 \times 10^5\ N/m^2 = 6\ bar}$$

iii) bsfc

$$bsfc = \frac{\dot{m}_f}{BP} = \frac{2.8}{8.0425}$$

$$= \mathbf{0.3482\ kg/kW.hr} \quad \text{...Ans.}$$

iv) Brake thermal efficiency (η_{bth})

$$\eta_{bth} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.} = \frac{8.04 \times 3600}{2.8 \times 41800}$$

$$= \mathbf{24.74\ \%} \quad \text{...Ans.}$$

v) Indicated thermal efficiency

$$\eta_{ith} = \frac{IP \times 3600}{\dot{m}_f \times C.V.}$$

$$IP = \frac{K\, P_m\, LAN}{60 \times x \times 1000}$$

$$= \frac{1 \times 7.5 \times 10^5 \times 0.2 \times \pi \times (0.16)^2 \times 400}{60 \times 2 \times 1000 \times 4}$$

$$IP = 10.053 \text{ kW}$$

$$\eta_{ith} = \frac{IP \times 3600}{\dot{m}_f \times C.V.} = \frac{10.053 \times 3600}{2.8 \times 41800}$$

$$= \mathbf{30.92\ \%} \quad \text{...Ans.}$$

Ex. 5.10 : A six cylinder gasoline engine operates on four stroke cycle. The bore of each cylinder is 80 mm and stroke 100 mm. The clearance volume per cylinder is 70 cc. At a speed of 4000 rpm the fuel consumption is 20 kg/hr and the torque developed is 150 Nm calculate
a) B.P. b) bmep
c) brake thermal efficiency C.V. given 43000 kJ/kg
d) Relative efficiency on brake power Take γ = 1.4 for air ☞ **[GTU : Winter-17]**

Sol. :

Given data : $D_c = 0.08$ m, $K = 6$, $L = 0.1$ m, $V_c = 70$ cc $= 70$ cm^3

$N = 4000$ rpm, $m_f = 20$ kg/hr, $T = 150$ N - m,

C.V $= 43000$ kJ/kg, $\gamma = 1.4$

To find : i) B.P ii) bmep iii) η_{bth} iv) $\eta_{Relative}$

i) B.P

$$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2 \times \pi \times 4000 \times 150}{60 \times 1000}$$

$$= \mathbf{62.831\ kW} \quad \text{... Ans.}$$

ii) Brake mean effective pressure (bmep)

$$BP = \frac{P_m \times A \times L \times N}{60 \times x \times 1000} = 62.83$$

$$= \frac{P_m \times \pi \times (0.08)^2 \times 4000 \times 0.1 \times 6}{60 \times 2 \times 1000 \times 4}$$

$$\therefore \mathbf{P_m = 6.25 \times 10^5\ N/m^2 \text{ or } 6.25\ bar} \quad \text{... Ans.}$$

iii) Brake thermal efficiency (η_{bth})

$$\eta_{bth} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.} = \frac{62.832 \times 3600}{20 \times 43000}$$

$$\therefore \mathbf{\eta_{bth} = 26.30\ \%} \quad \text{... Ans.}$$

iv) Relative efficiency (η_r)

$$\eta_{re} = \frac{\eta_{bth}}{\eta_{air}} \quad \text{But} \quad \eta_{air} = 1 - \frac{1}{(r_c)^{\gamma - 1}}$$

$$r_c = \frac{V_s + V_c}{V_c}$$

$$V_s = \frac{\pi \times D^2}{4} \times L_s = \frac{\pi}{4} \times \left(\frac{80}{10}\right)^2 \times \frac{100}{10}$$

$$\therefore \mathbf{V_s = 502.65\ cc}$$

$$\therefore \mathbf{r_c} = \frac{502.65 + 70}{70} = \mathbf{8.18}$$

$$\eta_{air} = 1 - \frac{1}{(8.18)^{1.4-1}} = \mathbf{56.86\ \%}$$

$$\eta_{re} = \frac{\eta_{bth}}{\eta_{air}} = \frac{0.2634}{0.5686} = \mathbf{46.32\ \%} \quad \text{... Ans.}$$

Ex. 5.11 : A single cylinder 4-stroke engine gave the following results on full load. Area of indicator card = 300 mm^2, length of diagram = 40 mm, spring constant = 1 bar/mm, speed of engine = 400 rpm, load on brake = 370 N, spring balance reading = 50N, diameter of brake drum = 1.2 m, fuel consumption = 2.8 kg/h, C.V. of fuel = 41800 kJ/kg, diameter of cylinder = 160 mm, stroke of piston = 200 mm.
Calculate : i) Indicated MEP ii) Indicated thermal efficiency iii) Brake power iv) Brake thermal Efficiency. **[GTU : Summer-14, Winter-16]**

Sol. :

Given data : $K = 1$, $x = 2$, $A = 300$ mm^2, $b = 40$ mm, Spring constant $= 1$ bar/mm

$N = 400$ rpm, $W = 370$ N, $S = 50$ N, $D_b = 1.2$ m, $m_f = 2.8$ kg/hr

C.V. $= 41800$ kJ/kg, $D_c = 0.16$ m, $L_S = 0.2$ m

To find : i) P_m ii) BP and bmep iii) b.s.f.c iv) η_{bth} v) η_{ith}

i) P_m

$$\mathbf{P_m} = \frac{a}{b} \times k = \frac{300}{40} \times 1 = \mathbf{7.5\ bar} \quad \text{...Ans.}$$

ii) BP and bmep

$$BP = \frac{(W - S)\, 2\pi\, rN}{60 \times 1000}$$

$$= \frac{(370 - 50) \times 2 \times \pi \times 0.6 \times 400}{60 \times 1000} = 8.04\ kW$$

$$BP = \frac{P_m \times L \times A \times N}{60 \times x \times 1000}$$

$$8.04 = \frac{P_m \times L \times A \times N}{60 \times x \times 1000}$$

$$= \frac{P_m \times 0.2 \times \pi \times (0.16)^2 \times 400}{60 \times 2 \times 1000 \times 4}$$

$$\therefore \mathbf{P_m = 6 \times 10^5\ N/m^2 = 6\ bar}$$

iii) bsfc

$$bsfc = \frac{\dot{m}_f}{BP} = \frac{2.8}{8.0425}$$

$$= \mathbf{0.3482\ kg/kW.hr} \quad \text{...Ans.}$$

iv) Brake thermal efficiency (η_{bth})

$$\eta_{bth} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.} = \frac{8.04 \times 3600}{2.8 \times 41800} = \mathbf{24.74\ \%} \quad \text{...Ans.}$$

v. Indicated thermal efficiency

$$\eta_{ith} = \frac{IP \times 3600}{\dot{m}_f \times C.V.}$$

$$IP = \frac{K\,P_m\,LAN}{60 \times x \times 1000}$$

$$= \frac{1 \times 7.5 \times 10^5 \times 0.2 \times \pi \times (0.16)^2 \times 400}{60 \times 2 \times 1000 \times 4}$$

$$IP = 10.053 \text{ kW}$$

$$\eta_{ith} = \frac{IP \times 3600}{\dot{m}_f \times C.V.} = \frac{10.053 \times 3600}{2.8 \times 41800}$$

$$= \mathbf{30.92\ \%}$$ **...Ans.**

Ex. 5.12 : A 8 cylinder four stroke engine of 9 cm bore and 8 cm stroke with compression ratio of 7 is tested at 4500 rpm on a dynamometer which has 50 cm arm; During 10 minutes test the dynamometer reading was 42 kg and consumed 4.4 kg of pertrol having C.V. of 44 MJ/kg; Air at 27 ° C and 1 bar was supplied to carburettor at the rate of 6 kg/min

Find : a) B.P. b) b.m.e.p c) b.s.f.c d) η_v e) η_b f) Brake specific air consumption g) Air fuel ratio

Sol. :

Given data : K = 8, D_c = 0.9 m, L = 0.8 m , r_c = 7, N = 4500 rpm,

W = 42 kg = 42 × 9.81 = 412 N, l_b = 0.5 m, $\dot{m}_f$ = 4.4 kg for 10 min

T = W × l_b = 412 × 0.5 = 206 N - m,

C.V = 44000 kJ/kg, T_1 = 27 + 273 = 300 K, $P = 1 \times 10^5$ N/m^2, m_a = 6 kg/ min.

To find : i) B.P ii) b.m.e.p iii) b.s.f.c iv) η_{vol} v) η_{bth} vi) b.s.a.c vii) A/F

i) B.P

$$B.P = \frac{2\pi NT}{60 \times 1000} = \frac{2 \times \pi \times 206 \times 4500}{60 \times 1000} = \mathbf{97.1\ kW}$$ **... Ans.**

ii) b.m.e.p

$$B.P = \frac{P_m \times L \times A \times N \times K}{60 \times x \times 1000}$$

$$\therefore \quad 97.1 = \frac{P_m \times 0.08 \times \pi (0.09)^2 \times 4500 \times 8}{60 \times 2 \times 1000 \times 4}$$

$$\mathbf{P_m = 6.36 \times 10^5\ N/m^2 = 6.36\ bar}$$ **... Ans.**

iii) b.s.f.c

$$bsfc = \frac{\dot{m}_f}{B.P.} = \frac{4.4 \times 60}{10 \times 97.1} = \mathbf{0.2718\ kg/kW.hr}$$ **... Ans.**

iv) Volumetric efficiency

$$\eta_{vol} = \frac{V_a}{V_s} = \frac{\text{Actual volume}}{\text{Stroke volume}} = \frac{m_a}{\dot{m}_a}$$

$$V_s = \frac{\pi D^2}{4} \times L_s \times K = \frac{\pi \times (0.09)^2}{4} \times (0.08) \times 8$$

$$V_s = 4.071 \times 10^{-3}\ m^3$$

Now, $$\dot{V}_s = \frac{V_s \times N}{2} = \frac{4.071 \times 10^{-3} \times 4500}{2}$$

$$\dot{V}_s = 9.159\ m^3/\text{min}$$

$$\therefore \quad \dot{m}_a = \frac{P\dot{V}_s}{RT}$$

$$= \frac{1 \times 10^5 \times 9.160}{287 \times 300} = 10.6 \text{ kg/min}$$

But, m_a = 6 kg/min

$$\eta_v = \frac{m_a}{\dot{m}_a} = \frac{6}{10.6} = \mathbf{56.60\ \%}$$ **... Ans.**

vi) b.s.a.c

$$b.s.a.c = \frac{m_a}{BP} = \frac{6 \times 60}{97.1} = \mathbf{3.707\ kg/kW.hr}$$ **... Ans.**

vii) η_{bth}

$$\eta_{bth} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.}$$

$$= \frac{97.1 \times 3600}{26.4 \times 44000} = \mathbf{30.09\ \%}$$ **... Ans.**

viii) A/F

$$A/F = \frac{\dot{m}_a}{\dot{m}_f} = \frac{6}{\frac{4.4}{10}} = \mathbf{13.636}$$ **... Ans.**

Ex. 5.13 : The following details were noted in a test on a 4-cylinder, 4-stroke engine, diameter = 100 mm; stroke = 120 mm; speed of the engine = 1600 rpm; fuel consumption = 0.2 kg/min; calorific value of fuel is 44000 kJ/kg; difference in tension on either side of the brake pulley = 40 kg; brake circumference is 300 cm. If the mechanical efficiency is 80%, Calculate :

(i) Brake thermal efficiency

(ii) Indicated thermal efficiency

(iii) Indicated mean effective pressure

(iv) Brake specific fuel consumption.

Sol. :

Given data : $K = 4$, $x = 2$, $d = 100$ mm, $L_S = 120$ mm, $N = 1600$ rpm,

$m_f = 0.2$ kg/min $= 3.33 \times 10^{-3}$ kg/s, C.V. $= 44000$ kJ/kg

$(W - S) = 40$ kg $= 40 \times 9.81 = 392.4$ N

$\eta_{mech} = 80\%$, $2\pi R_e = 300$ cm $= 3$ m

To find : 1) η_{bth} 2) η_{ith} 3) imep 4) bsfc

i) η_{bth} :

$$BP = \frac{2\pi N (W - S) \times R_e}{60} = \frac{3 \times 392.4 \times 1600}{60}$$

BP = 31.392 kW

Brake Thermal efficiency

$$\eta_{bth} = \frac{B.P.}{\dot{m}_f \times C.V.} \times 100 = \frac{31.392}{3.33 \times 10^{-3} \times 44000} \times 100$$

η_{bth} = 21.425 % **...Ans.**

ii) η_{ith} :

$$\eta_{mech} = \frac{B.P.}{I.P.}$$

$$0.8 = \frac{31.392}{IP}$$

IP = 39.24 kW

$$\eta_{ith} = \frac{IP}{\dot{m}_f \times C.V.} \times 100 = \frac{39.24}{3.33 \times 10^{-3} \times 44000} \times 100$$

η_{ith} = 26.781 % **...Ans.**

3) imep

$$IP = \frac{K P_m \times L_S \times A_S \times N}{60 \times 1000 \times x}$$

$$39.24 = \frac{4 \times P_m \times 0.12 \times \frac{\pi}{4} \times (0.1)^2 \times 1600}{60 \times 1000 \times 2}$$

$\therefore$ **p_m = imep = 7.81 bar** **... Ans.**

4) bsfc

$$bsfc = \frac{\dot{m}_f}{BP} = \frac{0.2 \times 60}{31.392}$$

bsfc = 0.3482 kg/kW.hr **...Ans.**

Ex. 5.14 : The air flow to 4 cylinder 4-s gasoline engine was measured by means of 8 cm diameter sharp edge orifice with $C_d = 0.65$ during a test the following data was recorded:

Bore = 10 cm, Stroke = 15 cm, N = 2500 rpm, B.P. = 36 kW,

Fuel consumption = 10 kg/hr, Calorific values of fuel = 42000 kJ/kg

Pressure drop across orifice = 4 cm of water Atomspheric temperature and pressure are and 1 bar respectively.

Calculate : i) η_b ii) bmep iii) η_v

Sol. :

Given data : $K = 4$, $x = 2$, $l = 8$ cm $= 0.08$ m, $C_d = 0.65$, $D_c = 10$ cm $= 0.10$ m

$L_s = 0.15$ m, $N = 2500$ rpm, B.P $= 36$ kW, $m_f = 10$ kg/hr, C.V $= 42000$ kJ/kg

$\Delta h = 0.04$ m, $T = 17 + 273 = 290$ K, $P = 1 \times 10^5$ N/m^2

To find : i) η_{bth} ii) b.m.e.p iii) η_{vol}

i) Brake thermal efficiency (η_b)

$$\eta_b = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.} = \frac{36 \times 3600}{10 \times 42000} = 0.308 = 30.8\ \%$$ **... Ans.**

ii) Brake mean effective pressure (bemp)

$$BP = \frac{p_m \times L \times A \times N \times K}{60 \times x \times 1000}$$

$\therefore$ $$36 = \frac{p_m \times 0.15 \times \pi \times (0.10)^2 \times 2500 \times 4}{60 \times 2 \times 1000 \times 4}$$

p_m $= 3.67 \times 10^5$ N / m^2 **= 3.67 bar** **... Ans.**

iii) Volumetric efficiency (η_v)

$$\eta_{vol} = \frac{V_a}{V_s} = \frac{m_a}{\dot{m}_a}$$

$$V_s = \frac{\pi D^2}{4} \times L_s = \frac{\pi \times (0.10)^2}{4} \times 0.15 = 1.178 \times 10^{-3} \text{ m}^3$$

$$\dot{V}_s = \frac{V_s \times N}{2} = \frac{1.178 \times 10^{-3} \times 2500}{2} = 1.472 \text{ m}^3/\text{min}$$

$$\therefore \quad \dot{m}_a = \frac{P\dot{V}_s}{RT} = \frac{1 \times 10^5 \times 1.472}{287\,(290)} = 1.768 \text{ kg/min}$$

$$\dot{V}_a = C_d \times A_0 \sqrt{2g\,\Delta h}$$

$$= 0.65 \times \frac{\pi\,(0.08)^2}{4} \sqrt{2 \times 9.81 \times 0.04}$$

$$= 2.89 \times 10^{-3} \text{ m}^3/\text{sec} = 2.89 \times 10^{-3} \times 60 = 0.17 \text{ m}^3/\text{min}$$

$$\therefore \quad \eta_{vol} = \frac{V_a}{V_s} = \frac{0.17}{1.472} = \mathbf{11.54\%}$$ **... Ans.**

Ex. 5.15 : A six cylinder 4S, SI engine running at 4000 rpm. The bore of each cylinder is 100 mm and stroke is 120 mm. The clearance volume of each cylinder is 100 cc. The fuel consumption is 20 kg/hr and torque develop is 150 Nm. Calculate :

i) BP ii) BMEP

iii) Break thermal efficiency,

iv) Relative efficiency based on brake power.

Assume calorific value of fuel as 43 MJ/kg.

Sol. :

Given data : K = 6, x = 2, N = 4000 rpm, D = 0.1 m, L = 0.12 m

$V_c = 100$ cc, $\dot{m}_f = 20$ kg/hr, T = 150 Nm $= 100 \times 10^{-6}$ m^3

i) Brake Power (B.P.)

$$\text{B.P.} = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times 4000 \times 150}{60 \times 1000}$$

$$\mathbf{B.P. = 62.83 \text{ kW}}$$ **... Ans.**

ii) Brake Mean Effective Pressure (bmep)

$$\text{B.P.} = \frac{K \times P_m\,LAN}{60 \times x \times 1000} = \frac{6 \times (BMEP) \times 0.12 \times \frac{\pi}{4} \times (0.1)^2 \times 4000}{60 \times 2 \times 1000} = 62.83$$

$$\mathbf{P_m = 3.3 \times 10^5 \text{ N/m}^2 = 3.3 \text{ bar}}$$ **... Ans.**

iii) Brake thermal efficiency (η_{bth})

$$\eta_{bth} = \frac{3600 \times \text{B.P.}}{\dot{m}_f \times \text{C.V.}} = \frac{3600 \times 62.83}{20 \times 43000}$$

$$\therefore \quad \mathbf{\eta_{bth} = 26.3\ \%}$$ ... Ans.

iv) Relative efficiency based on Break Power (R_c)

$$R_c = \frac{V_c + V_s}{V_c}$$

$$V_s = \frac{\pi}{4} D^2 \times L = \frac{\pi}{4} \times (0.1)^2 \times (0.12)$$

$$\therefore \quad \mathbf{V_s = 9.42 \times 10^{-4} \text{ m}^3}$$

$$R_c = \frac{100 \times 10^{-6} + 9.42 \times 10^{-4}}{100 \times 10^{-6}}$$

$$\mathbf{R_c = 10.42}$$

$$\eta_{air} = 1 - \frac{1}{(R_c)^{\gamma - 1}} = 1 - \frac{1}{(10.42)^{1.4-1}}$$

$$\mathbf{\eta_{air} = 60.84\ \%}$$ **... Ans.**

$$\eta_R = \frac{\eta_{bth}}{\eta_{air}} = \frac{26.3}{60.84}$$

$$\mathbf{\eta_R = 43\ \%}$$ **... Ans.**

5.12 Heat Balance or Energy Balance in IC Engine

- The heat supplied IC engine obtained by burning the fuel
- The fuel is expensive in nature and creats environmental polution
- The affective utilization of heat or fuel is required
- The heat balance sheet or energy balance sheet gives proper distribution heat in IC engine.
- The heat balance sheet is usefull to study the performance of IC engine.

5.11.1 Steps Involved in Heat Balance Sheet

The following are the steps involved in heat balance

✦ **Step I : Heat supplied (Q_A)**

$$Q_A = \dot{m}_f \times C.V$$

$\dot{m}_f$ = Mass of fuel in kg/hr

C.V = Calorific value of fuel kJ/kg

✦ **Step II : Heat used for mechanical work (Q_B)**

$$Q_B = B.P \text{ (kJ/s)}$$

✦ **Step III : Heat carried away by cooling water (Q_c)**

$$Q_c = \dot{m}_{cw} \times C_{pw} \times [T_{co} - T_{ci}]$$

Where, $\dot{m}_{cw}$ = Mass of cooling water kg/hr

C_{Pw} = Specific heat of cooling water kJ/kg

T_{co} = Outlet temperature of cooling water 'K'

T_{ci} = Inlet temperature of cooling water in 'K'

✦ **Step IV : Heat carried away by exhoust gasses (Q_D)**

$$Q_D = \dot{m}_g \times C_{Pg} \times [T_{go} - T_{gi}]$$

Where, $\dot{m}_g$ = Mass of gas in kg/hr

C_{Pg} = Specific heat of gas kJ/kg

T_{go} = Outlet temperature of gas in 'K'

T_{gi} = Inlet temperature of gas in 'K'

Note : If '$\dot{m}_g$' is not given $\dot{m}_g = \dot{m}_a + \dot{m}_f$

✦ **Step V : Heat unaccounted losses (Q_E)**

$$Q_E = Q_A - [Q_B + Q_c + Q_D]$$

- **Heat balance sheet**

Heat supplied	kJ/hr	Heat used	kJ/hr	%
Heat supplied $Q_A = \dot{m}_f \times C.V.$	...	i. Heat used to convert mechanical work (Q_B)		
		ii. Heat carried by cooling water (Q_c)		
		iii. Heat carried by exhaust gasses (Q_D)		
		iv. Heat unaccounted (Q_E)		
	100%			100%

❑ **Solved examples on heat balance sheet**

Ex. 5.16 : The following data were recorded in a test one hour duration on single cylinder oil engine working on 4 stroke cycle bore = 300 mm; Stroke 450 mm ; Fuel used = 8.8 kg; C.V. = 41800 kJ/kg; Average speed = 200 rpm; m.e.p. = 5.8 bar; Brake friction load = 1860 N; Quantity of cooling water = 650 kg, Temperature rise = 22°C, Diameter of brake wheel = 1.22 m

Calculate : a) Mechanical efficiency b) Brake thermal efficiency

Draw heat balance sheet. ☞ **[GTU : Summer-16, 18, Winter-17]**

▣ **Sol. :**

Given data : Time of trail = 1 hr , D_c = 300 mm = 0.3 m, L = 450 mm = 0.45,

$\dot{m}_f$ = 8.8 kg/hr, C.V = 41800 kJ/kg, N = 200 rpm, P_m = 5.8 bar, (W – S) = 1860 N

$\dot{m}_w$ = 650 kg/hr, $\Delta t = (T_{co} - T_{ci}) = 22°C$, D = 1.22 m, $\therefore r = \frac{D}{2} = 0.61$ m

To find : 1) η_{mech} 2) η_{bth} 3) Heat balance sheet

1) To calculate mechanical efficiency (η_{mech})

$$IP = \frac{K P_m LAN}{60 \times x \times 1000}$$

$$= \frac{1 \times 5.8 \times 10^5 \times 0.45 \times \pi \times (0.3)^2 \times 200}{60 \times 2 \times 1000 \times 4}$$

$$\mathbf{IP = 30.74 \text{ kW}}$$

$$BP = \frac{(W - S)\, 2\pi rN}{60 \times 1000} = \frac{1860 \times 2\pi \times 0.61 \times 200}{60 \times 1000}$$

$$\mathbf{BP = 23.76 \text{ kW}}$$

$$\therefore \eta_{mech} = \frac{BP}{IP} = \frac{23.76}{30.74} = \mathbf{0.773 \text{ or } 77.3\ \%}$$

2) Brake thermal efficiency (η_{bth})

$$\eta_b = \frac{3600 \times B.P.}{\dot{m}_f \times C.V.} = \frac{3600 \times 23.76}{8.8 \times 41800} = \mathbf{23.25\ \%}$$

3) Heat balance sheet

✦ **Step I : Heat supplied (Q_A)**

$$Q_A = \dot{m}_f \times C.V = 8.8 \times 41800$$

$$= \mathbf{367840\ kJ/hr}$$

✦ **Step II : Heat used to get mechanical work (Q_B)**

$$Q_B = BP = 23.76\ kJ/sec = 23.76 \times 60 \times 60$$

$$= \mathbf{85536\ kJ/hr}$$

✦ **Step III : Heat carried away by cooling water (Q_C)**

$$Q_C = \dot{m}_{cw} \times C_{Pw} \times [T_{co} - T_{ci}]$$

$$Q_C = 650 \times 4.187 \times [22] = \mathbf{59874.1\ kJ/hr}$$

✦ **Step IV : Heat unaccounted losses (Q_E)**

$$Q_E = Q_A - [Q_B + Q_C]$$

$$= 367840 - [84132 + 59874.1]$$

$$\mathbf{Q_E = 223833.9\ kJ/hr}$$

Heat supplied	kJ/hr	Heat used	kJ/hr	%
Heat supplied $Q_A = \dot{m}_g \times C.V.$	367840	i) Heat used to convert mechanical work (Q_B)	84132	22.87
		ii) Heat carried by cooling water (Q_C)	59874.1	16.27
		iii) Heat carried by exhaust gases (Q_D)		–
		iv) Heat unaccounted (Q_E)	223833.9	60.85
	100 %			100 %

Ex. 5.17 : In a test of an oil engine under full load condition, the following results were obtained.

Frictional power = 10 kW, rpm = 1750, fuel used = 15 kg/hr, brake torque = 327.5 Nm, Calorific value of fuel used = 42000 kJ/kg, air supplied = 4.75 kg/min, volume flow rate of cooling water = 16 lit/min, room temperature = 20.8 °C, rise in temperature of cooling water = 45 °C, exhaust gas temperature = 400 °C C_{Pw} = 4.2 kJ/kg K, C_{Pg} = 1.25 kJ/kg K, Find ISFC.

Also draw heat balance sheet on kW basis. **[GTU : Summer-16, Winter-16]**

Sol. : Given data :

F.P = 10 kW, N = 1750 rpm, m_f = 15 kg/hr,

T = 327.5 N-m, C.V = 42000 kJ/kg, m_a = 4.75 kg/min,

ΔT = 45 °C, T_R = 20.8 °C, T_{go} = 400 °C,

C_{Pw} = 4.2 kJ/kg K, C_{Pg} = 1.25 kJ/kg K, $\dot{m}_{cw}$ = 16 kg/min

To find : i) BP ii) η_{mech} iii) b.s.f.c iv) Heat balance sheet on 'kW' basis

i) Calculate brake power (B.P)

$$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times (1750) \times (327.5)}{60 \times 1000} = \mathbf{60.017\ kW}$$

ii) Mechanical efficiency (η_{mech})

$$\eta_{mech} = \frac{BP}{IP} \times 100$$

But, $IP = BP + FP = 60.017 + 10$

$$IP = 70.017\ kW$$

$$\eta_{mech} = \frac{60.017}{70.017} = \mathbf{85.717\ \%}$$

iii) b.s.f.c

$$b.s.f.c = \frac{\dot{m}_f}{BP} = \frac{15}{60.017} = \mathbf{0.2499\ kg/kWh}$$

iv) Heat balance sheet on kW and % basis

✦ **Step I : Heat supplied (Q_A)**

$$Q_A = \dot{m}_f \times C.V = 15 \times 42000 = 630000\ kJ/hr$$

$$\mathbf{Q_A = 175\ kJ/sec\ or\ 175\ kW}$$

✦ **Step II : Heat used to get mechanical work (Q_B)**

$$Q_B = B.P = 60.017\ kW$$

✦ **Step III : Heat carried away by cooling water (Q_C)**

$$Q_C = \dot{m}_{cw} \times C_{Pw} \times [T_{Co} - T_{Ci}]$$

$$Q_C = \frac{16}{60} \times 4.2 \times 45$$

$$Q_C = 50.4 \text{ kJ/sec}$$

✦ **Step IV : Heat carried by flue gasses (Q_D)**

$$Q_D = \dot{m}_g \times C_{Pg} \times [T_{go} - T_{gi}]$$

$$\dot{m}_g = \dot{m}_a + \dot{m}_f$$

$$\dot{m}_g = 4.75 + \frac{15}{60} = 5 \text{ kg/min}$$

$$Q_D = 5 \times 1.25 \times (400 - 20.8) = 2370 \text{ kJ/min}$$

$$\mathbf{Q_D = 39.5 \text{ kJ/sec or } 39.5 \text{ kW}}$$

✦ **Step V : Heat losses unaccounted**

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$Q_E = 175 - [60.017 + 50.4 + 39.5] = \mathbf{25.083 \text{ kJ/sec}}$$

• **Heat balance sheet on 'kW' basis**

Heat supplied	kW	Heat used	kW	%
$Q_A = \dot{m}_f \times C.V$	175	i) Heat by B.P	60.017	34.29
		ii) Heat by cooling water	50.4	28.8
		iii) Heat by flue gases	39.5	22.57
		iv) Heat losses unaccounted	25.08	14.33
	100%			100 %

Ex. 5.18 : The following data was recorded from a test on a single cylinder four-stroke oil engine :

Cylinder bore = 0.15 m, Engine stroke = 0.25 m, Indicated mean effective pressure = 7.355 bar, Engine speed = 400 rpm, Brake torque = 225 N-m, Fuel consumption = 3 kg/hr, CV = 44200 kJ/kg, Cooling water flow rate = 4 kg/min, Cooling water temperature rise = 42 °C, Cp water = 4.187 kJ/kgK. Calculate : i) Mechanical efficiency ii) Brake thermal efficiency iii) Specific fuel consumption. **☞ [GTU : Winter-14]**

▣ **Sol. : Given data**

$x = 2$, $K = 1$, $D = 0.15$ m, $L_S = 0.25$ m, $P_m = 7355 \times 10^5$ N/m

$N = 400$, $T = 225$ N-m, $\dot{m}_f = 3$ kg/hr, C.V. = 44200 kJ/kg

$\dot{m}_{cw} = 4$ kg/min

$T_{wo} - T_{wi} = 42$ °C $\quad C_{pw} = 4.187$ kJ/kgK

$$\eta_{mech} = \frac{B.P.}{I.P.}$$

$$I.P. = \frac{K P_m L_S A N}{60 \times 1000 \times x} \quad \text{But, } A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

$$= \frac{1 \times 7355 \times 10^5 \times 0.25 \times 0.01767 \times 400}{60 \times 1000 \times 2} = 10.830 \text{ kW}$$

$$B.P. = \frac{2\pi NT}{60 \times 1000}$$

$$= \frac{2\pi \times 400 \times 225}{60 \times 100} = 9.424 \text{ kW}$$

$$\eta_{mech} = \frac{B.P.}{I.P.} = \frac{9.424}{10.830} \times 100$$

$$= 87.2 \%$$

$$\eta_{mech} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.} = \frac{9.424 \times 3600}{3 \times 44200}$$

$$= 25.58 \%$$

$$bsfc = \frac{\dot{m}_f}{B.P.}$$

$$= \frac{3}{9.424} = 0.318 \text{ kg/kW/hr}$$

$$ISFC = \frac{\dot{m}_f}{I.P.} = \frac{3}{10.830} = 0.277 \text{ kg/kWhr}$$

Heat balanced sheet

1) Heat supplied (Q_S)

$$Q_S = \dot{m}_f \times C.V. = \frac{3}{60} \times 44200 = 2210 \text{ kJ/min}$$

Heat used in mechanical work Q_A

$$Q_A = B.P.$$

$= 9.424 \times 60$

$= 565.44$ kJ/min

Heat carried by cooling water (Q_B)

$$Q_B = \dot{m}_w \times C_{Pw} \times [T_{wo} T_{wi}]$$

$= 4 \times 4.187 \times 42 \times 60$

$= 11.7236 \times 60$ kJ/min $= 703.416$ kJ/min

Heat used in frictional power (Q_C)

$$Q_C = FP = IP - BP = (10.89 - 9.424) \times 60$$

$= 84.36$ kJ/min

Heat unaccounted (Q_D)

$$Q_D = Q_S - (Q_A + Q_B + Q_C)$$

$= 856.784$ kJ/min

Heat balanced sheet

Heat supplied	kJ/min	Heat used	kJ/min	%
Q_S	2210	Q_A	565.44	25.57
		Q_B	703.416	31.82
		Q_C	8436	3.81
		Q_D	856.784	38.77
	100 %			99.97

Ex. 5.19 : During the trial of a single-cylinder, four stroke oil engine, the following results were obtained :

Cylinder diameter = 20 cm ; Stroke = 40 cm

Mean effective pressure = 6 bar ; Torque = 407 Nm

Speed = 250 rpm ; Oil consumption = 4 kg/hr

Calorific value of fuel = 43 MJ/kg ; Cooling water flow rate = 4.5 kg/min

Air used per kg of fuel = 30 kg ; Rise in cooling water temperature = 45°C

Temperature of exhaust gases = 420°C ; Room temperature = 20°C

Mean specific heat of exhaust gas = 1 kJ/kg K ; Specific heat of water = 4.18 kJ/kg K

Find the IP, BP and draw up a heat balance sheet for the test in kJ/hr. **[GTU : Winter-14]**

Sol. : Given :

$k = 1$, $x = 2$, $D = 20$ cm, $L_S = 40$ cm, mep = 6 bar, $T = 407$ Nm,

$N = 250$ rpm. $m_f = 4$ kg/hr, $(CV)_f = 43 \times 10^3$ kJ/kg, $\dot{m}_w = 4.5$ kg/min $= 4.5 \times 60$ kg/hr

$\frac{\dot{m}_a}{\dot{m}_f} = 30$ kg, $(\Delta T)_w = 45°C$, $T_{go} = 420°$, $T_R = 20$ °C

$C_{pg} = 1$ kJ/kg K, $C_{pw} = 4.18$ kJ/kg K

To find : 1) IP 2) BP 3) Heat balance sheet in kJ/hr

1) $$IP = \frac{k P_m \times L_S A \times N}{60 \times 1000 \times x}$$

$$= \frac{1 \times 60 \times 10^5 \times 0.4 \times \frac{\pi}{4} \times (0.2)^2 \times 250}{60 \times 1000 \times 2}$$

IP = 15.7 kW

2) $$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times 250 \times 407}{60 \times 1000}$$

BP = 10.655 kW

3) Heat balance sheet :

a) Heat supplied (Q_A) :

$$Q_A = \dot{m}_f \times C_v = 4 \times 43000$$

Q_A = 172000 kJ/hr

b) Heat used to get mechanical work – (Q_B)

$$Q_B = BP = 10.655 \text{ kW}$$

$$BP = 10.655 \times 60 \times 60$$

BP = 38358 kJ/hr

c) Heat carried away by cooling water (Q_C)

$$Q_C = \dot{m}_w + (P_w + \Delta T)_w = 4.5 \times 60 \times 4.18 \times 45$$

Q_C = 50787 kJ/hr

d) Heat carried by exhaust gases (Q_D)

$$Q_D = \dot{m}_f \times C_{pg} \times (T_{go} - T_R) = (\dot{m}_a + \dot{m}_f) \times C_{pg} \times (T_{go} - T_R)$$

$$= (30 + 1)\, 4 \times 1 \times (420 - 20)$$

$$\mathbf{Q_D = 49600\ kJ/hr}$$

e) Heat losses unaccounted (Q_E)

$$Q_E = Q_A - (Q_B + Q_C + Q_D) = 172000 - (38358 + 50787 + 49600)$$

$$\mathbf{Q_E = 33255\ kJ/hr}$$

Heat balance sheet on kJ/hr basis

Heat supplied	kJ/hr	Heat used	kJ/hr	%
$Q_A = \dot{m}_f \times C_v$	172000	1) Heat used to get mechanical work	38358	22.30
		2) Heat by cooling water	50787	29.52
		3) Heat by exhaust gases	49600	28.83
		4) Heat losses unaccounted	33255	19.33
	100 %			99.98

Ex. 5.20 : The following observations are made during trial on an oil engine. Motor power to start the engine = 10 kW ; rpm = 1750; Fuel used = 15 kg/hr; Brake torque = 327.5 Nm; C.V. of fuel = 42000 kJ/kg ; Air supplied = 4.75 kg/min; Temperature of cooling water = 65.8°C, mass of cooling water is 16 kg/min; Room temperature = 20.8°C; Exhaust gas temperature = 400°C;
Take C_{Pw} = 4.2 kJ/kgK and C_{Pg} = 1.25 kJ/kgK
Find : 1) BP 2) η_m (Mechanical efficiency) 3) b.s.f.c. 4) Draw heat balance sheet on kW basis and percentage basis.

Sol. :

Given data : F.P = 10 kW, N = 1750 rpm, m_f = 15 kg/hr , T = 327.5 N-m

C.V = 42000 kJ/kg , $\dot{m}_a$ = 4.75 kg/min , T_{co} = 65.8°C, T_{go} = 400°C,

T_R = 20.8°C, C_{Pw} = 4.2 kJ/kg K, C_{Pg} = 1.25 kJ/kg K,

To find : i) BP ii) η_{mech} iii) b.s.f.c iv) Heat balance sheet on 'kW' basis

i) Calculate brake power (B.P)

$$BP = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times (1750) \times (327.5)}{60 \times 1000}$$

$$= \mathbf{60.017\ kW} \quad \text{... Ans.}$$

ii) Mechanical efficiency

$$\eta_{mech} = \frac{BP}{IP} \times 100$$

But, $IP = BP + FP = 60.017 + 10$

$$IP = 70.017\ kW$$

$$\eta_{mech} = \frac{60.017}{70.017} = \mathbf{85.717\ \%} \quad \text{... Ans.}$$

iii) b.s.f.c

$$\text{b.s.f.c} = \frac{\dot{m}_f}{BP} = \frac{15}{60.017} = \mathbf{0.2499\ kg/kWh} \quad \text{... Ans.}$$

iv) Heat balance sheet on kW and % basis

✦ **Step I : Heat supplied (Q_A)**

$$Q_A = \dot{m}_f \times C.V = 15 \times 42000$$

$$= 630000\ kJ/hr$$

$$\mathbf{Q_A = 175\ kJ/sec\ or\ 175\ kW}$$

✦ **Step II : Heat used to get mechanical work (Q_B)**

$$Q_B = B.P = 60.017\ kW$$

✦ **Step III : Heat carried away by cooling water (Q_C)**

$$Q_C = \dot{m}_{cw} \times C_{Pw} \times [T_{Co} - T_{Ci}]$$

$$Q_C = 60 \times 4.2 \times [65.8 - 20.8]$$

$$\mathbf{Q_C = 50.4\ kJ/sec}$$

✦ **Step IV : Heat carried by flue gasses (Q_D)**

$$Q_D = \dot{m}_g \times C_{Pg} \times [T_{go} - T_{gi}]$$

$$\dot{m}_g = \dot{m}_g + \dot{m}_f$$

$$\dot{m}_g = 4.75 + \frac{15}{60} = 5\ kg/min$$

$$Q_D = 5 \times 1.25 \times (400 - 20.8) = 2370\ kJ/min$$

$$\mathbf{Q_D = 39.5\ kJ/sec\ or\ 39.5\ kW}$$

✦ **Step V : Heat losses unaccounted (Q_E)**

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$Q_E = 175 - [60.017 + 50.4 + 39.5] = \mathbf{25.083\ kJ/sec}$$

- **Heat balance sheet on 'kW' basis**

Heat supplied	kW	Heat used	kW	%
$Q_A = \dot{m}_f \times C.V$	175	i) Heat by B.P	60.017	34.29
		ii) Heat by cooling water	50.4	28.8
		iii) Heat by flue gases	39.5	22.57
		iv) Heat losses unaccounted	25.08	14.33
	100 %			100 %

Ex. 5.21 : In a test of an oil engine under full load condition, the following results were obtained I.P. = 33 kW, B.P. = 27 kW, Fuel used = 8 kg/h ; Rate of flow of water through gas calorimeter = 12 kg/min ; Cooling water flow rate = 7 kg/min ; C.V. of fuel = 43 MJ/kJ ; Inlet temperature of cooling water = 15°C ; Outlet temperature of cooling water = 75°C ; Inlet temperature of water to exhaust gas calorimeter = 15°C ; Outlet temp of water to exhaust gas calorimeter = 55°C ; Final temperature of exhaust gases = 80°C after calorimeter ; Room temperature = 17°C ; A.F. ratio on mass basis = 20 ; Mean specific heat of exhaust gas = 1 kJ/kgK ; Specific heat of water = 4.18 kJ/kgK

Draw heat balance sheet on kJ/min basis and estimate the brake thermal and mechanical efficiencies.

☞ **[GTU : Summer-14, Winter-16]**

Sol. :

Given data : I.P = 33 kW, B.P = 27 kW, $\dot{m}_f$ = 8 kg/hr, $\dot{m}_{cw}$ = 7 kg/min, C.V = 43000 kJ/kg, T_{ci} = 15°C, T_{co} = 75°C, T_{gcp} = 15°C, T_{gco} = 55°C, T_{go} = 80°C, T_R = 17°C, A/F = 20, C_{Pg} = 1 kJ/kg, C_{Pw} = 4.18 kJ/ kg

To find :

i) Heat balance sheet on kJ/min basis

ii) η_{bth} iii) η_{mech}

ii) Brake thermal efficiency (η_{bth})

$$\eta_{bth} = \frac{B.P. \times 3600}{\dot{m}_f \times C.V.}$$

$$\eta_{bth} = \frac{27 \times 3600}{8 \times 43000} = \mathbf{28.26\ \%} \quad \text{... Ans.}$$

iii) Mechanical efficiency (η_{mech})

$$\eta_{mech} = \frac{BP}{IP} \times 100 = \frac{27}{33} \times 100 = \mathbf{81.818\ \%} \quad \text{... Ans.}$$

i) Heat balance sheet on kJ/min basis

✦ **Step I : Heat supplied (Q_A)**

$$Q_A = \dot{m}_f \times C.V = 8 \times 43000 = 344000\ kJ/hr$$

$$\mathbf{Q_A = 5733.34\ kJ/min}$$

✦ **Step II : Heat used to get B.P (Q_B)**

$$Q_B = BP = 27 \times 60 = \mathbf{1620\ kJ/\ min}$$

✦ **Step III : Heat carried away by cooling water (Q_C)**

$$Q_C = \dot{m}_{cw} \times C_{Pw} \times [T_{co} - T_{ci}]$$

$$Q_C = 7 \times 4.18\ [75 - 15] = \mathbf{1755.6\ kJ/min}$$

✦ **Step IV : Heat carried by exhaust gasses (Q_D)**

$$\dot{m}_g = \dot{m}_a + \dot{m}_f = \dot{m}_f \left(1 + \frac{\dot{m}_a}{\dot{m}_f}\right) = \frac{8}{60}(1 + 20)$$

$\dot{m}_g$ = 2.8 kg/min

Q_D = Heat gained by water in exhaust gas calorimeter + Heat carried by flue gases.

$$Q_D = \dot{m}_w \times C_{Pw} \times [T_{gco} - T_{gci}] + \dot{m}_g \times C_{Pg} \times [T_{go} - T_{gi}]$$

$$Q_D = 12 \times 4.18\ (50 - 15) + 2.8 \times 1 \times [80 - 17]$$

$$\mathbf{Q_D = 2182.8\ kJ/min}$$

✦ **Step V : Heat unaccounted losses (Q_E)**

$$Q_E = Q_A - [Q_B + Q_C + Q_D]$$

$$Q_E = 5733.33 - [1620 + 1755.6 + 2182.8]$$

$$\mathbf{Q_E = 174.93\ kJ/min}$$

- **Heat balance sheet on kJ/min basis**

Heat supplied	kJ/min	Heat used	kJ/min	%
$Q_A = \dot{m}_f \times C.V$	5733.33	i) Heat to BP	1620	28.2
		ii) Heat by cooling water	1755.6	30.62
		iii) Heat by exhaust gasses	2182.8	38.07
		iv) Heat unaccounted	174.93	3.05
	100 %			99.94 %

Ex. 5.22 : During trial on single cylinder 4-S oil engine, the following results were obtained :
Cylinder diameter = 20 cm, Stroke = 40 cm
Mean effective pressure = 6 bar
Torque = 407 N-m, Speed = 250 rpm
Oil consumption = 4 kg/hr
Calorific value of fuel = 43 MJ/kg
Cooling water flow rate = 270 kg/h
Rise in cooling water temperature = 45 °C
Temperature of exhaust gases = 420 °C
Room temperature = 20 °C
Air used per kg of fuel = 30 kg
Mass flow rate of water through exhaust gas calorimeter = 8 kg/min
Rise in temperature of calorimeter water = 8 °C
Mean specific heat of exhaust gas = 1 kJ/kgK
Find IP, BP and draw heat balance sheet for the test in kW. ☞ **[GTU : Summer-14, 16]**

Sol. : Given data : $D = 0.2$ m, $L = 0.4$ m, $p = 6 \times 10^5$ N/m^2, $T = 407$ Nm

$N = 250$ rpm, $\dot{m}_f = 4$ kg/min $= \frac{4}{60}$ kg/sec $= 0.0667$ kg/sec

$\Delta T_c = 45°C$, $T_{go} = 420°C$, C.V. = 43000 kJ/kg,

$\dot{m}_w = 270$ kg/hr $= \frac{270}{60}$ kg/min $= 4.5$ kg/min

$T_{ci} = 0°C$, $T_{gi} = 20°C$, $\dot{m}_a = 30$ kg/kg of fuel

$\dot{m}_{wc} = 8$ kg/min, $T_{co} - T_{ci} = 8°C$, $C_{pg} = 1$ kJ/kg K

i) Indicated Power (I.P.)

$$\text{I.P.} = \frac{K \times p \times L \times A \times N}{60 \times x \times 1000} = \frac{1 \times 6 \times 10^5 \times 0.4 \times \frac{\pi}{4} \times (0.2)^2 \times 250}{60 \times 2 \times 1000}$$

I.P. = 15.70 kW

ii) Brake Power (B.P.)

$$\text{B.P.} = \frac{2\pi NT}{60 \times 1000} = \frac{2\pi \times 250 \times 407}{60 \times 1000}$$

B.P. = 10.66 kW

iii) Heat balance sheet :

Heat supplied	kJ/min	Heat used	kJ/min	%
$Q_a = \dot{m}_f \times C.V$ $= 0.0667 \times 43000$ $= 28681$	28681	i) Heat to BP Q_b = B.P.	10.66	0.037
		ii) Heat by cooling water $Q_c = \dot{m}_w \times C_{Pw} \times (T_{co} - T_{ci})$ $= 4.5 \times 4.187 \times (45 - 0)$	847.87	2.96
		iii) Heat by exhaust gasses $Q_d = \dot{m}_g \times C_{Pg} \times (T_{go} - T_{gi})$ $= 2.067 \times 1 \ (420 - 20)$	826.8	2.88
		iv) Heat by calorimeter water $Q_e = \dot{m}_{cw} \times C_{pw} \times (T_{co} - T_{ci})$ $= \frac{8}{60} \times 4.187 \times 8$	4.47	0.016
		iv) Heat unaccounted $Q_f = Q_a - (Q_b + Q_c + Q_d + Q_e)$ $= 28681 - (10.66 + 847.87 + 826.8 + 4.47)$	26991.2	94%
	100%			99.89 %

Ex. 5.23 : A four stroke cycle four cylinder petrol engine was tested at full load condition by maintaining constant speed. The cylinder have diameter of 80 mm and stroke of 100 mm. The fuel was supplied at rate of 5.44 kg/hr and plug of 4 cylinder were successfully cut out w.r.t change of speed. The power measured was as follows.
Assume compression ratio is 6.02
With all cylinder working = 14.7 kW Cylinder 1 is cut off = 10.1 kW
Cylinder 2 is off = 10.3 kW Cylinder 3 is cut off = 10.4 kW
Cylinder 4 is cut off = 10.2 kW C. V of fuel = 41900 kJ/kg
Clearance volume = 100 cc
Determine 1) η_{mech} 2) η_{ith} 3) Air std. efficiency 4) Relative efficiency ☞ **[GTU : Winter-17]**

Sol. :

Given : $K = 4$, $x = 2$, $D = 80$ mm $= 0.08$ m, $L = 100$ mm $= 0.1$ m

$\dot{m}_f = 5.44$ kg/hr, $BP_n = 14.7$ kW, $BP_1 = 10.1$ kW,

$BP_2 = 10.3$ kW, $BP_3 = 10.4$ kW, $BP_4 = 10.2$ kW,

$r_c = 6.02$, $CV = 41900$ kJ/kg,

To find : i) η_{mech} ii) η_{ith} iii) η_{air} iv) η_{re}

$$V_s = \frac{\pi}{4} \times D^2 \times C = \frac{\pi}{4} \times 0.08^2 \times 0.1 = 5.02 \times 10^{-4}$$

1) $IP_1 = BP_n - BP_1 = 14.7 - 10.1 = 4.6$ kW

2) $IP_2 = BP_n - BP_2 = 14.7 - 10.3 = 4.4$ kW

3) $IP_3 = 14.7 - 10.4 = 4.3$ kW

4) $IP_4 = 14.7 - 10.2 = 4.5$ kW

$$IP_n = IP_1 + IP_2 + IP_3 + IP_4 = 4.6 + 4.4 + 4.3 + 4.5 = 17.8$$

1) $$\eta_{mech} = \frac{BP}{IP} \times 100 = \frac{14.7}{17.8} \times 100 = \mathbf{82.58\ \%}$$ **... Ans.**

2) $$\eta_{ith} = \frac{3600 \times IP}{\dot{m}_f \times C.V.} = \frac{3600 \times 17.8}{5.44 \times 41900} \times 100 = \mathbf{28.11\ \%}$$ **... Ans.**

3) Air std. efficiency

$$r_c = \frac{V_s + V_c}{V} = 6.02$$

$$\eta = 1 - \frac{1}{(r_c)^{\gamma - 1}} = 1 - \frac{1}{(6.02)^{1.4-1}} = \mathbf{51.22\ \%}$$ **... Ans.**

4) Relative efficiency $= \frac{\eta_{ith}}{\eta_{air}} = \frac{28.11}{51.22} \times 100 = \mathbf{54.80\ \%}$ **... Ans.**

Ex. 5.24 : A gasoline engine working on four stroke develops a brake power of 20.9 kW. A Morse test was conducted on this engine and the brake power obtained when each cylinder was made inoperative by short circuiting the spark plug are 14.9, 14.3, 14.8 and 14.5 respectively. The test was conducted at constant speed of 3000 rpm. Find the indicated power, mechanical efficiency and bmep when all cylinders are firing. The bore of the engine is 75 mm and the stroke is 90 mm. ☞ **[GTU : Summer-14]**

Sol. :

Given data : $BP_n = 20.9$ kW, $k = 4$, $BP_1 = 14.9$ kW, $BP_2 = 14.3$ kW, $BP_3 = 14.8$ kW $BP_4 = 14.5$ kW, $N = 3000$ rpm, $D = 75 \times 10^{-3}$ m, $L_S = 90 \times 10^{-3}$ m

To find : i) IP_n ii) η_{mech} iii) bmep

i) To calculate indicated power (IP)

$$IP_n = IP_1 + IP_2 + IP_3 + IP_4$$

$IP_1 = BP_n - BP_1 = 20.9 - 14.9 = 6$ kW

$IP_2 = BP_n - BP_2 = 20.9 - 14.3 = 6.6$ kW

$IP_3 = BP_n - BP_3 = 20.9 - 14.8 = 6.1$ kW

$IP_4 = BP_n - BP_4 = 20.9 - 14.5 = 6.4$ kW

$IP_n = 6 + 6.6 + 6.1 + 6.4$

$\mathbf{IP_n = 25.1\ kW}$ **... Ans.**

ii) To calculate (η_{mech})

$$\eta_{mech} = \frac{BP_n}{IP_n} = \frac{20.9}{25.1}$$

$\mathbf{\eta_{mech} = 83.26\ \%}$ **... Ans.**

iii) To calculate b.m.e.p

$$BP_n = \frac{KP_m LAN}{60 \times x \times 1000}$$

$$\therefore \quad 20.9 = \frac{4 \times P_m \times 90 \times 10^{-3} \times \frac{\pi}{4} \times (75 \times 10^{-3}) \times 3000}{60 \times 1000 \times 2}$$

$$p_m = 5.25 \times 10^5\ N/m^2$$

$\mathbf{p_m = 5.25\ bar}$ **... Ans.**

Ex. 5.25 : A petrol engine having four cylinder and runs at 3000 rpm develops a brake power of 21 kW. A morse test was conducted on a engine and B.P obtained when each cylinder was made inoperative by short circuiting the spark plug are 15, 14.2, 14.7 and 14.4 kW respectively. The test conducted at constant speed. Find IP, η_{mech} and bmep when all cylinders are firing. The bore and stroke of engine are 8 cm and 9 cm respectively.

Sol. :

$BP_n = 21$ kW, $BP_1 = 15$ kW, $BP_2 = 14.2$ kW, $BP_3 = 14.7$ kW, $BP_4 = 14.4$ kW

To find : i) IP_n ii) η_{mech} iii) bmep

i) To calculate indicated power (IP)

$$IP_n = IP_1 + IP_2 + IP_3 + IP_4$$

$$IP_1 = BP_n - BP_1 = 21 - 15 = 6 \text{ kW}$$

$$IP_2 = BP_n - BP_2 = 21 - 14.2 = 6.8 \text{ kW}$$

$$IP_3 = BP_n - BP_3 = 21 - 14.7 = 6.3 \text{ kW}$$

$$IP_4 = BP_n - BP_4 = 21 - 14.4 = 6.6 \text{ kW}$$

$$IP_n = 6 + 6.8 + 6.3 + 6.6 = \mathbf{25.7 \text{ kW}} \quad \text{... Ans.}$$

ii) To calculate (η_{mech})

$$\eta_{mech} = \frac{BP_n}{IP_4} = \frac{21}{25.7} = \mathbf{81.71\ \%} \quad \text{... Ans.}$$

iii) To calculate b.m.e.p

$$BP_n = \frac{KP_m LAN}{60 \times x \times 1000}$$

$$\therefore \quad 21 = \frac{4 \times P_m \times 0.09 \times \pi(0.08)^2 \times 3000}{60 \times 2 \times 1000 \times 4}$$

$$\therefore \quad \mathbf{p_m = 4.638 \times 10^5 \text{ N/m}^2 = 4.64 \text{ bar}} \quad \text{... Ans.}$$

Ex. 5.26 : A six-cylinder, gasoline engine operates on the four-stroke cycle. The bore of each cylinder is 80 mm and the stroke is 100 mm. The clearance volume per cylinder is 70 cc. At the speed of 4100 rpm, the fuel consumption is 19.8 kg/hr. and the torque developed is 160 Nm. Calculate :

i) Brake power

ii) The brake mean effective pressure

iii) Brake thermal efficiency if the calorific value of the fuel is 44,000 kJ/kg. Take $\gamma = 1.4$ for air.

Sol. :

Given data : $K = 6$, $x = 2$, $D_b = 0.08$ m, $L = 0.1$ m, $V_c = 70$ cc

$N = 4100$ rpm, $\dot{m}_f = 19.8$ kg/hr, $T = 160$ N - m, $C.V = 44000$ kJ/kg, $r = 1.4$

To find : i) B.P ii) bmep iii) η_{bth}

i) To calculate B.P

$$B.P = \frac{2\pi NT}{60 \times 1000} = \frac{2 \times \pi \times 4100 \times 160}{60 \times 1000}$$

$$= 68.69 \text{ kW}$$

ii) To calculate bmep

$$BP = \frac{K_1 P_m LAN}{60 \times x \times 1000}$$

$$68.69 = \frac{6 \times p_m \times 0.1 \times \pi(0.08)^2 \times 4100}{60 \times 2 \times 1000 \times 4}$$

$$\mathbf{p_m = 6.67 \times 10^5 \text{ N/m}^2 = 6.67 \text{ bar}} \quad \text{... Ans.}$$

iii) To calculate brake thermal efficiency

$$\eta_{bth} = \frac{3600 \times BP}{\dot{m}_f \times C.V.} = \frac{3600 \times 68.69}{19.8 \times 44000} = \mathbf{28.38\ \%} \quad \text{... Ans.}$$

5.13 Introduction to Emission of Engine

- In the vast atmosphere and relatively huge land share of the universe, about half to one kg of pollutant is emitted over by an automobile in a day.
- The same fact becomes alarming when crores of vehicles in the world do the same addition everyday which causes air pollution in the form of million tonnes of pollutants like Hydrocarbons (HC), Carbon Monoxide (CO), and oxides of nitrogen (NOx).
- Because of these harmful polutants emitted by an automobile, many countries laid emission norms on automobile to control their emission within the specified range.
- In February 1999, persons performing traffic control in Delhi indicated that about 90 % of them had unacceptable level of lead content in their blood. A very high percentage of persons were found to be suffering from respiratory diseases and eye problems.
  Such dangerous effects of I.C. engine emissions forced the Supreme Court to ban the use of vehicles having more than 15 years of service.

- In earlier days, addition of something to air was considered as air pollution. The air pollution changed air natural quality and consequently polluting ecosystem and environment.
- The driving cycle is a means of representing the road driving conditions of a specific country or region. They were developed to take into account various road driving conditions (Idling, cruising, acceleration and deceleration), and as an important input for testing Comparison of tailpipe emissions and fuel efficiency of different types of vehicles.
- They have also helps to evaluate transportation management strategies. The driving cycle is determined by statistical analysis of the vehicle travel database, which is derived through extensive road driving experiments under different conditions (such as different roads and traffic densities) and finally represents the working day driving pattern of an area.
- The driving cycle can be used to indicate the average driving pattern in a certain area or the specific driving conditions to be studied by the chassis dynamometer (for example, cold start).
- The Indian Driving Cycle (IDC) is the first driving cycle in India. Since then, some changes have been made to the loop.
- However, the increases in the number of vehicles and the limited growth of road infrastructure have led to an increase in traffic congestion in the country's urban centers. (WSA) released a congestion index for different cities in 2008, highlighting the highest levels of congestion faced by cities such as Delhi, Mumbai, Varanasi and Bangalore.
- A research report by the Central Road Research Institute (CRRI) stated that the annual fuel loss caused by idling in Delhi is as high as 9.94 billion rupees.
- Congestion will not only reduce the speed of vehicles, but also cause Distortion of the driving cycle specified in the emission test procedure. On the other hand, the development of road and highway infrastructure has led to an increase in speed and acceleration distribution in non-crowded driving mode. The shift to high-power cars
- It also indicates the shift speed and acceleration curve. Overall, the driving style in India has changed significantly since 1985 due to the increase in vehicles and road infrastructure.

5.14 Indian Driving Cycles

- India's mandatory vehicle emission control, this is India's first driving cycle (IDC) was first developed in 1985 by the Automotive Research Association of India (ARAI).
- According to India's Central Automobile Rules, the cycle is based on actual road time Measurement performed by ARAI in Pune.
- In India, IDC is still tracking and testing the emissions of two-wheel/three-wheelers, which accounts for 73% of the total number of registered vehicles in India.
- IDC is actually a very short cycle (although it includes six driving cycle modes), only 108 seconds, and does not cover all the different driving conditions observed on the road.
- IDC's average speed is 21.9 km/h (covering 3.94 km), which seems to be high considering the increase in congestion in Indian cities.
- In addition, with the exception of diesel vehicles, all two-wheeled and three-wheeled vehicles were idling for 40 seconds as a pretreatment, and then sampled by a chassis dynamometer (MoRTH, 2000), which did not properly consider cold start emissions.
- Recently in 2012, the Indian government launched WMTC (World Motorcycle Test Cycle) as alternative regulations for BS III two-wheelers in India. It is expected that WMTC will the BS IV two-wheeler regulation came into effect after 2015, the regulation became a mandatory requirement.
- Although IDC is still used to test two-wheeled and three-wheeled vehicles in India, MIDC (modified India Driving Cycle) is used to assess emissions from automobiles and light commercial vehicles.
- MIDC was adopted in 2000 and later modified through better cold start test procedures. It is basically the same as NEDC (New European Driving Cycle) in Europe. NEDC consists of four ECE-15 urban driving cycles (UDC1) and an out-of-city driving cycle (EUDC2).
- However, considering that the top speed in the MIDC cycle has been reduced to 90 km/h
- The situation in India (Fig. 5.17 (b)). NEDC's test procedures have been standardized by the UNECE World Vehicle Regulations Coordination Forum. Please note that IDC and MIDC includes cold start procedure testing.
- A comparison of Indian driving cycles (IDC and MIDC) with others in the world is presented in Fig. 5.17 (a), which shows that Indian cycles assume lower speeds in comparisonto cycles followed in other countries. Fig. 5.17 (b) shows the extensive speed profiles coveredby MIDC in comparison to the IDC.

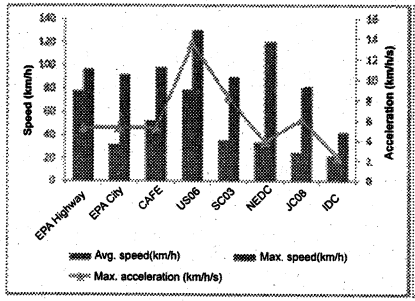


Fig. 5.17 (a) : Comparison of average and maximum speeds and maximum accelerations under different driving cycles followed across the world

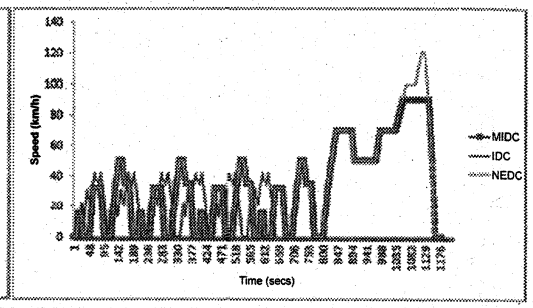
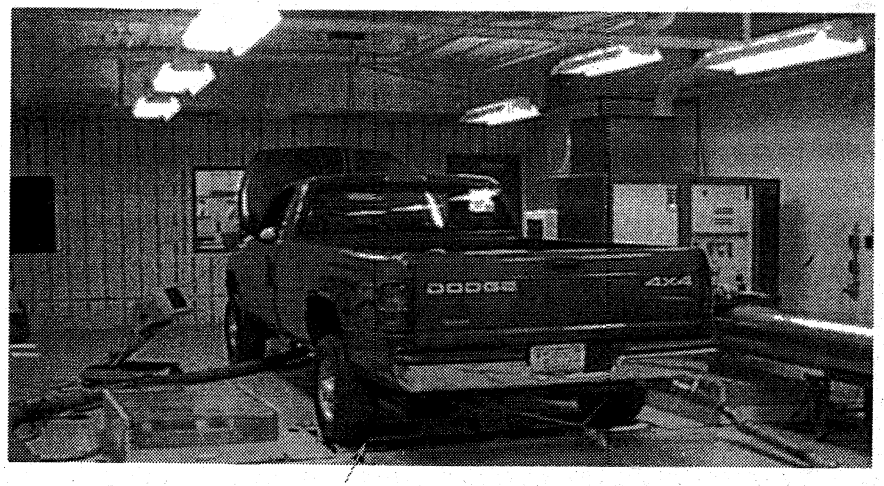


Fig. 5.17 (b) : Indian Driving Cycle and Modified Indian Driving Cycle (IDC and MIDC)

5.14.1 Emission Test Cycles :

- The following describes the driving cycles in the United States and Europe for emission measurement. Except for Japan, which has its own driving cycle, most other countries/regions follow European or American test methods. In India, except for motorcycles and three wheelers, European test methods are used.

5.14.2 Driving Cycles for Light Duty Vehicles :



Chassis Dynamometer Rolls

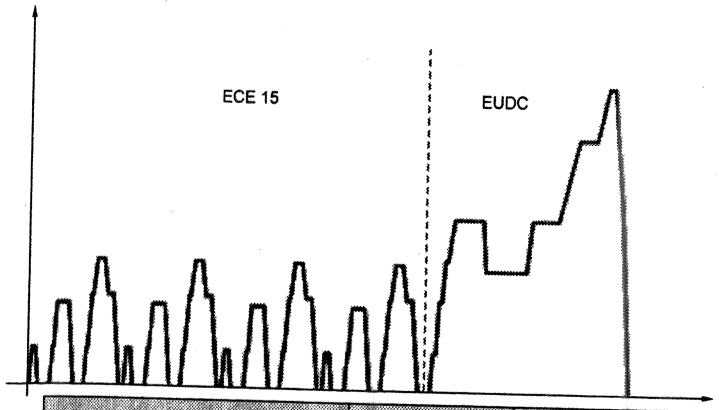
Fig. 5.18 : View of a vehicle emission test and chassis dynamometer Facility

- The emission test driving cycle includes a cold start period, idling, moderate acceleration and deceleration and cruise mode. The test period is given as the relationship between vehicle speed and time.
- Light-duty and medium-duty vehicles are driven on a chassis dynamometer with a prescribed driving cycle. During operation, the engine is required to produce horsepower that depends on the speed of a given vehicle.
- The road power requirements and speed data provided by the vehicle manufacturer or determined by the vehicle coasting test are stored in the chassis dynamometer controller to simulate the actual road operation of the vehicle.
- The vehicle weight is simulated by mechanically changing the rotating mass or electronically changing the inertia on the roller dynamometer, that is, the inertia required during the transition mode of the driving cycle. Fig. 5.18 shows a typical emission test facility for a vehicle with a chassis dynamometer.

5.14.3 European Test Driving Cycle :

- Fig. 5.19 shows the driving cycle followed by light and medium vehicles in Europe. The European test cycle consists of stable operating patterns derived from actual vehicle operating data on the road. Although it consists of acceleration and deceleration modes, since all modes represent average operating conditions, this cycle is often called a steady mode cycle. The European test cycle given in Fig. 5.19 is implemented according to the 1992 Euro 1 emission standard.
- It has two parts;

  (i) an urban driving cycle (ECE 15) and (ii) an extra-urban driving cycle (EUDC)
- The break-up of different operating modes in the two parts of the cycle is also given in Fig. 5.19.



	ECE-15 cycle	EUDC Cycle
Distance, km	4.052	6.955
Time, s	780	400
Average speed, km/h	19	62.5
Maximum speed, km/h	50	120
Acceleration, % time	21.16	–
Maximum Acceleration, m/s^2	–	0.833
Deceleration, % time	13.8	–
Maximum Deceleration, m/s^2	–	– 1.389
Idle, % time	35.4	–
Steady speed, % time	29.3	–

Fig. 5.19 : European driving cycle for light and medium duty vehicles: ECE 15 cycle followed by extra urban driving cycle (EUDC)

- Before the European 1 regulation, only ECE 15 cycles were used. The low-speed city test cycle includes four and 15 mode cycles without interruption, and the total duration is 780 seconds.
- The average speed is 19 km/h and the total distance is 4.052 km. After the ECE-15 cycle, a high-speed test cycle called "Extra Urban Driving Cycle (EUDC)" is performed.
- The maximum speed of this high-speed loop is 120 km/h. The EUDC part contributes more to NOx, while the ECE-15 cycle contributes more to CO and HC emissions. The emission measurement starts when the engine is cold started at the beginning of the first ECE-15 mode cycle itself.

5.15 Air Pollution

- The atmospheric air is made up of 21 % oxygen (O_2), 78 % Nitrogen (N_2) and remaining other gases and particulates like carbon dioxide, dust particles, etc.
- Air present around us should be healthy as it is required for breathing. But some toxic elements like carbon monoxide, nitrogen oxide, hydrocarbons etc are mixed with the atmospheric air which is very harmful for human beings, plants and animals.
- These undesirable gases and substances in the air are called as air pollutants, and the effect produced by these gases on the environment is called as air pollution.

5.15.1 Major Sources of Air Pollutants

- The major sources of air pollutants are as follows :

i) Automobiles
ii) Factories
iii) Thermal power plants
iv) Petrol and diesel engines
v) Refineries
vi) Explosive materials

5.15.2 Major Air Pollutants and their Harmful Effects on Human Health and Environment

☞ **[GTU : Summer-16, Winter-18]**

- The major air pollutants and their harmful effects on human beings and environment are discussed below :

1) Carbon Monoxide (CO)

Causes

- CO is produced by incomplete combustion of fuel.
- Generally caused by insufficient oxygen.

Effects

- CO has strong affinity to combine with hemoglobin in the blood and reduce its oxygen carrying capacity.
- Al low level also, CO affects mental function and visual ability.
- It causes headache, nausea as well as breathing probems.

2) Hydrocarbons (HC)

Causes

- Hydrocarbon (HC) gas is produced if the mixture is too rich or too lean.
- It is also produced by incomplete combustion of fuel.

Effects

- Hydrocarbon causes irritation in respiratory system.
- Low molecular weight HC causes coughing eye irritation, sneezing, etc.
- High molecular weight HC causes cancer and mutagenic (DNA change) effect.

❑ 3) Oxides of Nitrogen

Causes

- NO_x are produced when nitrogen and oxygen combine at higher temperature (above 1100 °C).
- NO_x is formed during complete combustion as highest temperature is reached during this period.

Effects

- NO reacts with atmospheric O_2 and produce NO_2, which is an insidious poisonous gas.
- NO_x causes irritation of eyes, nose and throat.
- It also causes coughing, headache and damage to lungs.
- NO_2 is most poisonous with penetrating odour which can damage lung's tissues.

❑ 4) SO_2, H_2S and Lead

Causes

- Toxic gas SO_2 and bad small of H_2S gas are formed on burning of sulphur present in petroleum products.

Effects

- SO_2 derived particulate reduces visibility and its toxicity damages sensitive plants.
- Increases mortality from bronchitis, lung cancer.

❑ 5) Particulate Matters (PM)

Causes

- Particulate includes ash, solid carbon and liquid like sulphuric acid vapours. These are produced on burning of diesel fuel.
- The particulates are also formed due to incomplete combustion of fuel.

Effects

- The particulate matters enters the body through respiratory systems. Fine particles of size less than reach respiroty system and particles of size less than 1 μm reach lungs and causes lung problems.
- The inhalation of these particles may also cause bronchitis and cardiac diseases.

❑ 6) Soot

Causes

- Maximum amount of soot is produced at full load by diesel engines.
- They are produced at high temperature and high pressure and due to lack of oxygen.

Effects

- Soot particles get suspended in the air and are breathed in by humans and animals.
- They are very dangerous to health as it produces lung cancer.

❑ 7) Smoke (Black, Blue and White)

Causes

- Smoke is usually emitted by diesel engines. It is formed due to insufficient oxygen.
- Smoke is formed when lubricating oil is burned during combustion.

Effects

- It may cause reduction in visibility.
- It may lead to problems like bronchitis.
- It increases the chances of lung cancer.

❑ 8) Aldehydes

Causes

- Aldehydes are formed is diesel engines mostly at idling speed due to cold flames.

Effects

- Aldehydes causes unpleasant smell due to odour from diesel fuel.
- It may cause irritation to eyes, nose as well as throat.

5.15.3 Effect of Pollutants on Environment

The harmful effect of pollutants on environment are discussed below :

5.15.4 Acid Rain and Thermal Pollution

- Various human activities such as burning of fossile fuels for different purposes, oxides of sulphur, nitrogen, carbon release into atmosphere.
- When the water vapour in the atmosphere condenses into water droplets, they fall as a rain.
- During raining, the polluting gases such as sulphur dioxide, nitrogen oxides, carbon dioxide mixes with the rainwater.
- This forms mild solutions of sulphuric nitric and carbonic acid which falls as rain and this is known as acid rain.
- The wind may spread these acidic solutions across the atmosphere and over hundreds of miles including forests, farms, homes, buildings and structures.
- These acidic solutions are very corrosive in nature and everything which comes in contact with the acid rain is subjected to corrosive effects of acids.

Effects of Acid Rain

- Acid rain has many ecological effect. But the most prominent in that is impact of acid rain on aquatic life and food chain. Acid rain makes reservoir water acidic and toxic and affects the aquatic life and ecosystem.
- Acid rain also affects the forests and trees and damages them badly. Acid rain absorbs the essential nutrients in the soil and leaves the content of aluminium in the soil.
- The corrosive action of acid rain also damages the buildings and structures and monuments like Taj Mahal. It also damages the protective coatings of paints and exterior metallic parts.
- The acidified drinking water causes damage to kidneys and urinary tracts.

Control of Acid Rain

- The only way to reduce these harmful effects of acid rain is to reduce or control the release of pollutant that causes acid rain.
- Also promoting the use of alternative energy source can help in reducing harmful emissions.
- The individuals can also help to prevent acid rain by conserving energy.

5.15.5 Greenhouse Effect and Global Warming

- A greenhouse consists of transparent roof, walls made of glass that allows the sunlight to reach the plants kept inside.

- Due to the transparent covering the radiation gets trapped inside the greenhouse and keeps the plants warm. So in the bad weather outside plants remain lush and green inside.
- The greenhouse effect related to earth works in the same way as that for plants.
- Some gases from the atmosphere forms the transparent covering. These gases are known as greenhouse gases.
- The greenhouse gases include carbon dioxide, methane, carbon monoxide, hydro-carbons, chlorofluro carbons, etc.
- The solar radiation from the sunlight passes through a transparent medium to warm the planetary surface.
- The earth's surface absorbs this energy and and get heated.
- The Earth cools down by giving off the infrared radiation. But before all this, the radiation from the earth can escape the outer space, the greenhouse gases absorb some of the radiation and makes the atmosphere warmer.
- The energy absorbed and trapped by these gases warms the atmosphere.
- Without this natural greenhouse effect, the earth would be much colder than it is now.
- Human activities such as burning of fossile fuel and clearing the forests have intensified the natural greenhouse process.
- The increase in amount of greenhouse gases due to human activities causes trapping of extra heat and hence there is rise in earth's temperature causing global warming.
- The global warming is defined as the increase in temperature of earth that causes change in climate.
- With increase in energy needs of growing population, the consumption of fossile fuel has also increased.
- This causes the level of CO_2 to rise which contributes to the greenhouse effect and global warming.
- This severely affects earths atmosphere in the form of melting of polar ice and increase in level of sea, heavy rainfalls, droughts, etc.
- This results in the severe flooding and consequently shortage of habitats and agricultural land.
- According to the reports, during the 21st century the global temperature can rise by 1.7°C to 4.8°C and ocean level will rise by 2 to 2.5 meters.

5.15.6 Ozone Depletion

- We know that, there is a layer of ozone (O_3) gas in stratosphere upto 50 km from earth's surface called as ozone layer.

- This ozone layer protects us against the harmful effects like skin cancer due to UV rays coming from sun since ozone layer absorbs the harmful UV rays and allows only useful rays to reach the earth's surface.
- The harmful pollutants like NO_x, HC produced by engine exhaust causes the depletion of ozone at particular places which is called as ozone depletion.

5.15.7 Pollution Control Systems in Automobiles

There are two approahces / measures for controlling the IC engine emissions. These are as follows :

1) Non-technical Measures for Emission Control (Human Dependant)

- Proper traffic pattern and road networking.
- Phasing out old vehicles.
- Proper maintenance of vehicle.
- Public awareness (e.g. energy efficient purchase habbits).

2) Technical Measures for Emission Control (Design Dependant)

- Use of clean fuels (i.e. CNG, LPG, etc.)
- Modification in engine design / optimal design of engine.
- Post-treatment of exhaust gases (i.e. 3-way catalytic converter).

5.15.8 Emission Control Devices

- As discussed in previous section, some of the technical measures for controlling emission are as follows :

1. Positive Cranckcase Ventilation (PCV) System
2. Catalytic Converter

5.15.9 Positive Cranckcase Ventilation (PCV) System

☞ **[GTU : Summer-14, 16]**

- A gap is created between piston rings and cylinder due to wear and unburnt air-fuel mixture from combustion chamber leaks out to cranckcase from this gap (called blowby action).
- Major portion of these blowby gases in crankcase is thus HC (unburnt fuel). These blowby gases are released to atmosphere through a pipe from the crankcase called breather.
- These blowby gases are recirculated to combustion chamber for burning by connecting crankcase (breather) to intake manifold through a pipe and valve combination. Refer Fig. 5.20.

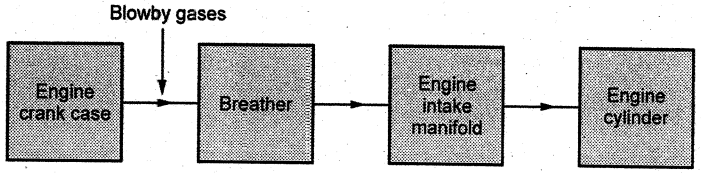


Fig. 5.20 : Positive Crankcase Ventilation (PCV) system

Positive Crankcase Ventilation System

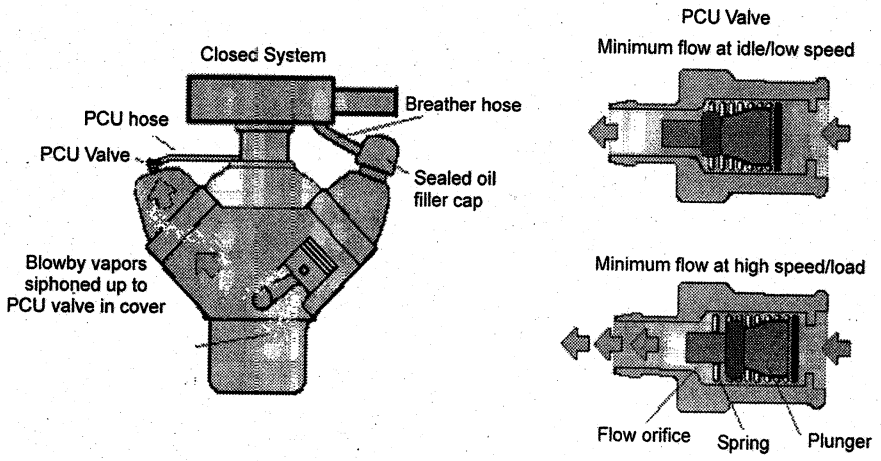


Fig. 5.20 (a)

5.15.10 Catalytic Converter

☞ **[GTU : Summer-14, 16, Winter-16]**

- The main aim of the catalytic converter is to convert the harmful gases (i.e. CO, HC and NO_x) from an automobile to harmless compounds (i.e. CO_2, H_2O, N_2 and O_2).
- A catalyst is a material which promotes chemical change without entering into chemical reaction. The materials which are used as catalysts are Platinum (Pt), Palladium (Pd) and Rhodium (Rh).
- Chemical reaction of conversion of CO to CO_2 and HC to H_2O, which normally takes place at temperatures of 500 °C to 600 °C occurs at 300 °C to 350 °C in the presence of catalyst.
- Platinum and Palladium are good catalysts for conversion of CO and HC and Rhodium is used for reduction of NO_x.
- Catalytic converter is a tubular device integrated into exhaust system. It is filled with thin walled mesh coated with catalysts.

- As the exhaust gases passes through this mesh, HC gets converted to H_2O and CO gets converted to CO_2. When the mesh is coated with Rhodium, NO_x gets converted to N_2 and O_2.
- As all the three pollutants are to be converted into harmless substances, two wire meshes i.e. one coated with Platinum and other with Rhodium, are used in one converter only.
- As three pollutants converted by one catalytic converter, it is called as 3-way catalytic converter as shown in Fig. 5.21.

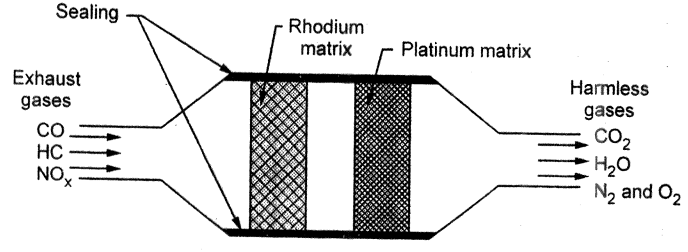


Fig. 5.21 : Three way catalytic converter

- Catalyst does not take part in chemical reaction, they have a long life. But if gasoline contains lead, then the mesh gets coated with this poisonous lead and the converter becomes useless.
- Hence, the automobiles fitted with catalytic converter should always use unleaded fuel.

5.15.11 Exhaust Gas Recirculation (EGR)

☞ **[GTU : Summer-16, Winter-17]**

- Exhaust Gas Recirculation is used to reduce the production of NO_x.
- We know that, NO_x is produced by the combustion of Nitrogen and oxygen of air at high temperature (above 1100 °C) in the combustion chamber of the engine.
- In EGR, the portion of exhaust gases are recirculated and mixed with fresh intake air to lower the combustion temperature. Fig. 5.22 shows the typical exhaust gas recirculation system.
- The recirculated gas is hotter than the intake air. It provides inert gases to combustion process which act as absorbent of combustion heat which reduces cylinder temperature.
- Due to this, NO_x formation in combustion chamber reduces.
- The amount of recirculated exhaust gas is controlled by the EGR valve. The EGR valve is actuated by the signal of ECU (Electronic Control Unit).

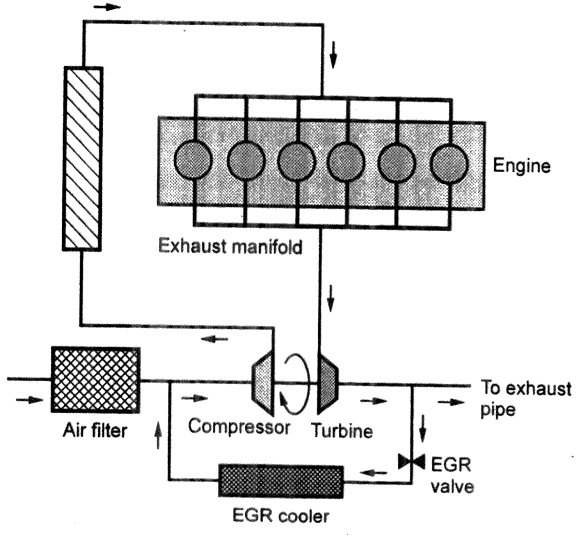
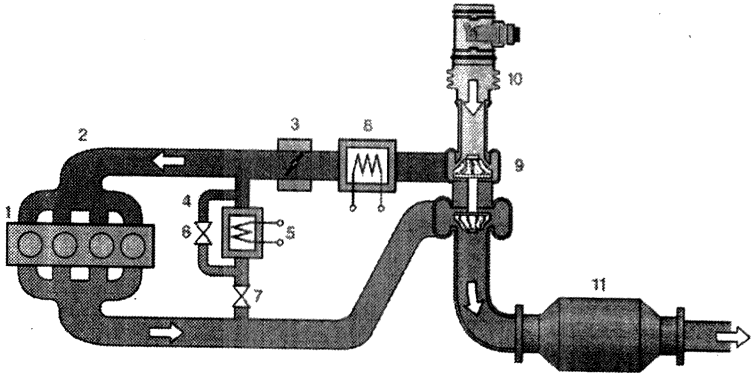


Fig. 5.22 : Exhaust gas recirculation

- From Fig. 5.23 it can be seen that NO_x formation is reduced with increase in EGR percentage.



1) Internal combustion engine
2) Intake manifold
3) Intake throttle
4) EGR cooler bypass
5) EGR cooler
6) EGR cooler bypass valve
7) EGR valve intercooler (turbocharger)
8) Turbocharger
9) Intake air-mass meter
10) Diesel oxidation catalyst (DOC) and diesel particulate filter (DPF)

Fig. 5.23 (a)

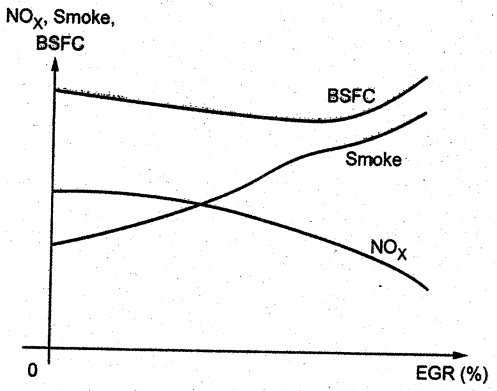


Fig. 5.23 : NO_x Reduction with EGR

5.15.12 Evaporative Emission Control Systems

- In this system, the evaporative losses of fuel from fuel tank and carburetor are controlled.
- The vapours of fuel from fuel tank and carburetor are collected and they are routed to the combustion chamber.
- The schematic diagram of evaporative emission control system is as shown in Fig. 5.24.
- It consists of charcoal canister, vapour separator, purge control valve etc.
- The vapours from fuel tank are collected and separated from liquid fuel in the vapour separator. This is done to avoid reaching of liquid fuel particles into crankcase.
- The vapours from vapour separator are routed to charcoal canister, which acts a fuel vapour absorber.

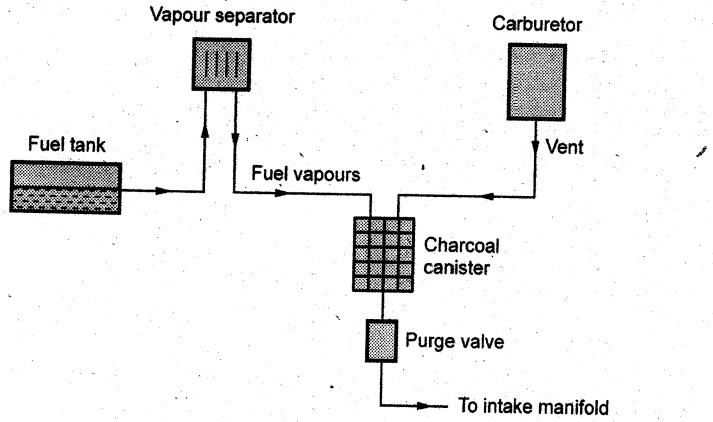


Fig. 5.24 : Evaporative emission control system

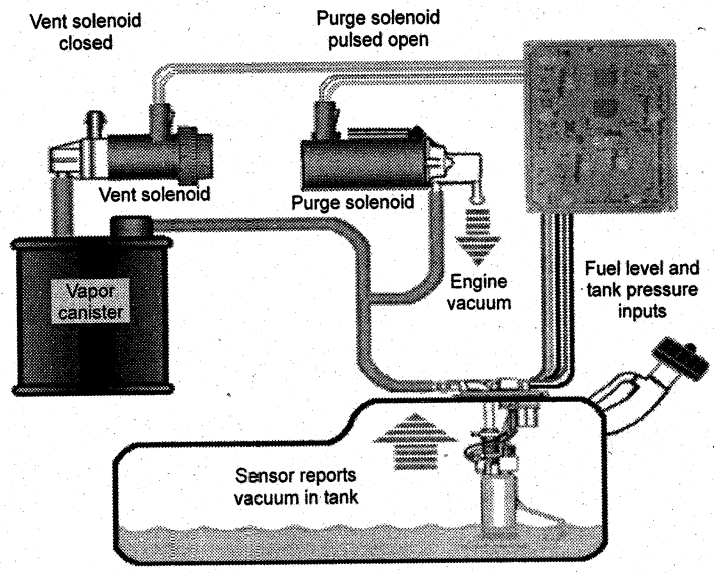


Fig. 5.24 (a)

- The charcoal canister connected to the carburetor vent. The vapours from carburetor are also sent to charcoal canister.
- The charcoal canister is connected to the intake manifold. When the cylinder sucks air from intake manifold, the air passes from charcoal canister which removes the fuel vapours from canister.

5.16 Emission Measurement :

- The emission regulations stipulate the types, working principles and general structure of exhaust gas analyzers that can be used for vehicle and engine emission certification. Table 5.1 lists the types of analyzers used to measure different exhaust gas components.

Table 5.1 : Measurement Principles of Exhaust Gas Analyzers

Gas component	Measurement Principle
CO	NDIR (Non-dispersive infrared)
HC	FID (Flame Ionization detector)
NO_x	CLD (Chemilumiescence detector)

5.17 Non Dispersive Infrared (NDIR) Analyzer :

- The Beer-Lambert law is used in the operation of NDIR analyzers by measuring the degree of absorption of infrared rays (IR) when passing through a column of gases. The fraction of incident radiation absorbed is given by :

$$\frac{I}{I_0} = \left(1 - e^{-k \cdot c \cdot d}\right) \quad \ldots(5.1)$$

Where, I = Radiation energy absorbed

I_0 = Incident radiation energy

k = characteristic absorption constant for the gas, m^2 /gmol

c = concentration of the gas, gmol/m^3

d = length of the gas column, m

- The NDIR analyzer is shown in Fig. 8.3. Since the absorption of IR radiation is measured only in a narrow wavelength range (rather than the entire range of IR radiation wavelengths) that has a particularly high absorption rate for a specific gas, this technique is called "non-dispersive infrared". For example, carbon monoxide has strong absorption in the 4.5-5 μm wavelength band.
- The analyzer measures the difference in energy absorbed from the two columns of gas; (i) the gas to be analyzed in the "sample cell", and (ii) the gas containing a fixed component (such as N2) in the reference cell, without the target gas, and in the infrared The area is relatively non-absorbing.
- Usually the infrared beam from a single light source is divided into two beams with the same intensity, which are used in the sample cell and the reference cell respectively.
- The detector is divided into two compartments, separated by a flexible membrane between the two compartments. one period Receive the transmitted infrared energy from the sample cell and another infrared energy from the reference cell.
- The detector is filled with the gas of interest, so the energy transmitted to the detector is completely absorbed.
- The flexible diaphragm of the detector can sense the pressure difference between the two parts
- The difference in the amount of absorbed infrared energy transmission causes the angle of the detector to change of the deflection in the diaphragm is used to generate an electrical signal that determines the concentration of the target gaseous substance.

- A rotating arc extinguishing chamber placed in the path of the infrared beam to generate an amplifiable AC signal output. NDIR analyzer can accurately measure CO and CO_2 in exhaust gas.

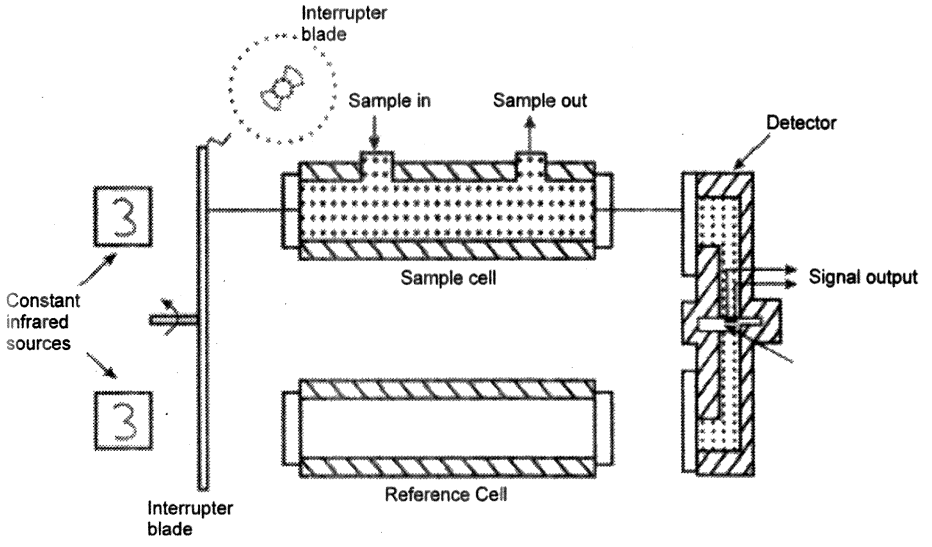


Fig. 5.25 : An NDIR analyzer for measurement of CO and CO_2 concentration

- NDIR instruments are rarely used for hydrocarbon measurement, except for garage types. The infrared absorption rate of the analyzer for different hydrocarbons is very different.
- Unsaturated Hydrocarbons are the main cause of photochemical smog, but they do not have sufficient absorption in the infrared wavelength range specific to saturated hydrocarbons, and vice versa.
- The sensitivity and response of NDIR to exhaust gas HC are usually only half of the true value. NO is only absorbed in the infrared region once a week.
- Moreover, CO, CO_2 and water vapor will seriously interfere. Therefore, the NDIR analyzer is also not used for NO measurement.

5.18 Flame Ionization Detector (FID) :

- The pure hydrogen air flame contains almost no ions, but introduces a small amount of Hydrocarbons cause the flame to ionize and become conductive.
- The ionization current is proportional to the number of carbon atoms present in the hydrocarbon molecule.

- Therefore, FID is actually a carbon atom counter. For example, a molecule of propane produces three times the response of a molecule of methane.
- The measurement of HC by FID is expressed as one part per million of methane, that is, ppmC 1, that is, ppm of hydrocarbon equivalent. The HC concentration is usually written as ppmC.
- The HC concentration measured in ppm propane (C3) should be multiplied by 3 to convert it to ppmC.
- All kinds of hydrocarbons, namely paraffins, alkenes, aromatics, etc., react almost the same to FID. Oxygen-containing compounds, such as aldehydes and alcohols, however, are less responsive.
- FID is mainly composed of hydrogen burner and ion collector components, as shown in Fig. 5.26.
- The sample gas is introduced into the burner assembly together with hydrogen, and the mixture is burned in a diffusion flame.
- An electric potential is applied between the collector plates to cause ionization current to flow and generate a signal proportional to the HC concentration in the sample gas. The current is amplified and the output signal is measured.
- A well-designed burner will produce an ionization current linearly proportional to the hydrocarbon content in a dynamic range of almost 1 to 106.
- The most sensitive range of commercial FID analyzers is set to approximately 0-50 ppmC, and the maximum range reaches 0-100,000 ppmC.
- Hydrogen and helium are mixed in a ratio of 40:60 to lower the flame temperature and improve flame stability. The FID analyzer uses a mixture of propane or methane in nitrogen for calibration.
- In order to measure the hydrocarbons in diesel exhaust, the sampling line and FID were heated to 191±11°C to minimize the condensation of heavy hydrocarbons in the diesel exhaust in the sampling system.
- Currently, emission standards are based on non-methane hydrocarbons. Methane The content of HC emissions is determined by one of the following methods:
- Gas chromatography (GC) method or non-methane cutter (NMC) method
- In gas chromatography, a sample is injected into a gas chromatography column, which divides the sample into two parts:

(i) CH_4 -air-CO, and (ii) NMHC–CO_2 –H_2O.

- The molecular sieve column separates methane from air and carbon monoxide before passing it to the FID. Therefore the measured methane content is deducted from the total hydrocarbon content.
- In the NMC method, all hydrocarbons except CH_4 are oxidized to CO_2 and water on the catalyst. Therefore, when the gas sample passes through the NMC, HFID can only detect CH_4.
- The NMC cutter has been calibrated to calibrate the catalytic effect of a mixture of CH_4 and higher hydrocarbons (ethane) in the presence of water vapor (typically 600 K or higher).
- Samples can pass NMC or bypass NMC. In this way, the only total HC and methane present in the exhaust gas sample were determined.

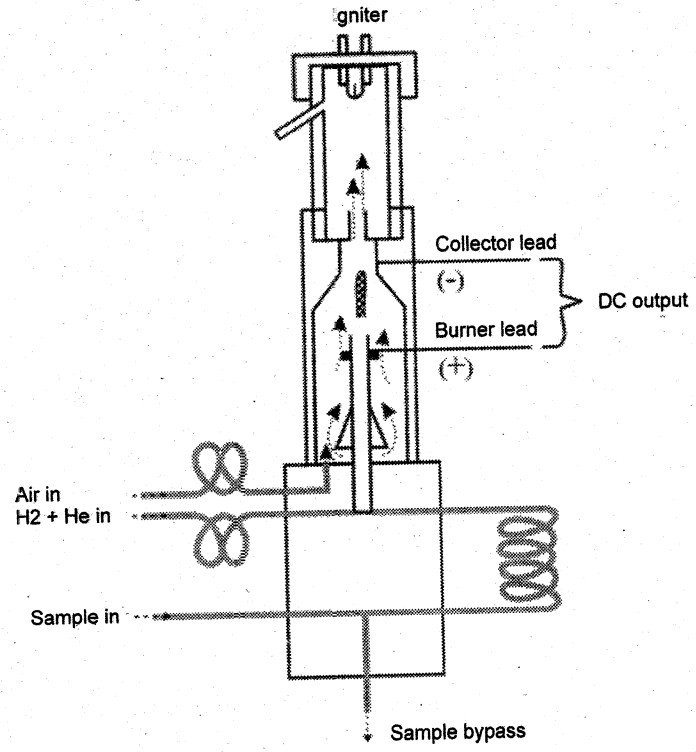


Fig. 5.26 : FID for HC measurement

5.19 Chemiluminescence Analyzer (CLA) :

- When NO and ozone (O_3) react a small fraction (about 10% at 26.7° C) of excited NO_2* molecules isproduced as per the following reactions :

$$NO + O_3 \leftrightarrow NO_2^* O_2 \quad \ldots(5.2)$$

$$NO_2^* \leftrightarrow NO_2 + hv \quad \ldots(5.3)$$

- As the excited NO_2^* molecules decay to the ground state, light in the wavelength range of 0.6-3.0 μm is emitted. At a given reaction temperature, the amount of excited NO_2 generated is fixed, and the light intensity generated during the decay of excited NO_2 is proportional to the concentration of NO in the sample.
- The schematic diagram of the chemiluminescence NO_x analyzer is shown in Fig. 5.27.

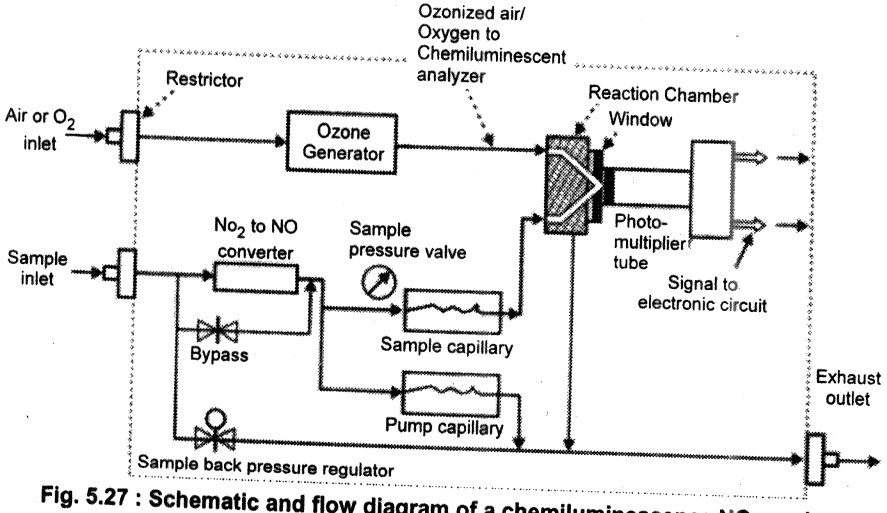


Fig. 5.27 : Schematic and flow diagram of a chemiluminescence NO_x analyzer

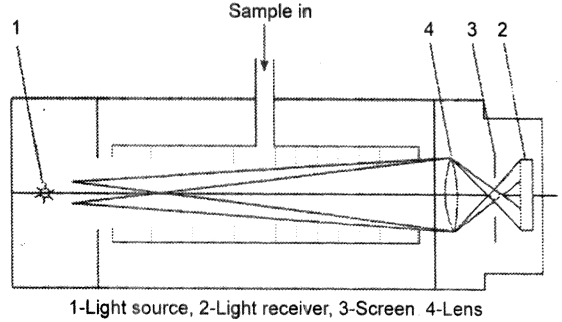
- The sample containing NO flows into the reactor and reacts with the ozone produced by oxygen in the "ozone generator". In the reactor, NO is converted to NO_2.
- The photomultiplier tube detects the light emitted by the excited NO_2. The signal is then amplified and fed to a recorder or pointing device.
- For the measurement of nitrogen oxides (NO_x), first the NO_2 in the sample is converted into NO by heating in the NO_2-NO converter, and then it is introduced into the reactor. At 315°C, approximately 90% of NO_2 is converted to NO_2. Therefore, the total concentration of NO_x in the sample is measured as NO. When the sample bypasses the NO_2-NO converter and is introduced into the reactor, only the concentration of NO

needs to be determined. The difference between the two measurements provides the NO_2 concentration in the sample.

- The response of the instrument has a linear relationship with the concentration of NO. The technology is very sensitive and can detect NO_x up to 10-3 ppm.
- The output signal is proportional to the product of sample flow rate and NO concentration. Since this method is sensitive to flow, precise flow control is required. Calibration and operation are performed at the same flow rate and reactor temperature.

5.20 Smokemeter :

- In filter smoke meter like Bosch smoke meter, a certain quality of white filter paper will inhale a certain amount of exhaust gas.
- Use a reflectometer to evaluate the smoke concentration obtained on the filter paper, which can measure the smoke concentration of diesel exhaust.
- Therefore, extinction/absorption smoke meters based on Beer-Lambert's law are mostly used. Extinction smoke meters are often referred to as "smoke meters" because they can more realistically measure the visible smoke emissions of diesel engines.
- Both sampling type and full flow type opacity meters are in use. The relevant international standards describe the structural requirements, installation and operation details of the opacity meter.
- The sampling smoke meter is shown in Fig. 5.28.
- Incandescent lamps with a color temperature in the range of 2800 K to 3250 K or green light-emitting diodes (LEDs) with a spectral peak between 550 nm and 570 nm are used as light sources.
- The transmitted light is received on a phototube or photodiode (with filter if necessary). When the light source is an incandescent lamp, the receiver should have the same maximum response as the human eye, in the wavelength range of 550 nm to 570 nm.



1-Light source, 2-Light receiver, 3-Screen 4-Lens

Fig. 5.28 : Schematic of a sampling type lightextinction smoke meter

- When the light from the light source is transmitted through a certain length of exhaust gas, the smoke opacity refers to the part of the light absorbed in the exhaust gas column and not reaching the light detector of the smoke meter.
- The absolute smoke density is given by the absorption coefficient ks, the unit of which is m -1, and is given by:

$$k_s = -\frac{1}{L} ln\left(\frac{I}{I_0}\right)$$

- Where L is the length of the smoke column (in meters) through which the light from the light source passes, I_0 is the intensity of the incident light, and I is the transmitted light falling on the smoke meter receiver.
- In a full-flow smoke meter, the light source and detector are usually placed directly across the exhaust flow at the end of the exhaust pipe. In this case, the path length of the smoke measurement varies with the cross-sectional size of the exhaust gas flow or tail pipe.
- Therefore, for the full-flow smoke meter, the conversion diagram from the measured value of different exhaust pipe diameters or path lengths to the absolute smoke density k_s can be obtained.

5.21 Emission Norms

☞ [GTU : Summer-14, Winter-17]

- Emission norms are nothing but the maximum prescribed level of CO, HC and NO_x set by the Government, which a vehicle is permitted to emit from its exhaust while running.
- All the manufacturers are required to ensure that their vehicles do not exceed the permissible limit of pollutants in exhaust as laid down.

5.21.1 EURO Norms

- Euro norms are the permissible emission levels for both petrol and diesel engines, which have been implemented in European countries.
- Euro Norms I were first implement in the year 1992.
- Five such norms (Euro I to Euro V) are planned for implementation. We can see that the permissible level of pollutants is kept reducing from Euro I to Euro V. This ensures less air pollution with better engine design.
- Table 5.2 shown below indicates sample emission levels that are permitted in engine exhaust.

Table 5.2 : Euro norms

Table 5.2 (a) : European norms for petrol vehicles

No.	Regulation/norms	Effective date	CO (gm/km)	HC (gm/km)	NO_x (gm/km)	HC + NO_x (gm/km)
1	Euro I	1-Jul-1992	2.72	-	-	0.97
2	Euro II	1-Oct-1996	2.20	-	-	0.50
3	Euro III	1-Jan-2000	1.30	0.20	0.1	-
4	Euro IV	1-Jan-2005	1.00	0.10	0.08	-
5	Euro V	1-Sept-2009	1.00	0.10	0.06	-

Table 5.2 (b) : European norms for diesel vehicles

No.	Regulation/norms	Effective date	CO (gm/kWh)	HC (gm/kWh)	NO_x (gm/kWh)	PM (gm/kWh)
1	Euro I	1-Jul-1992	4.50	1.10	8.00	0.36
2	Euro II	1-Oct-1996	4.00	1.10	7.00	0.15
3	Euro III	1-Jan-2000	2.10	0.66	5.00	0.10
4	Euro IV	1-Jan-2005	1.50	0.46	3.50	0.02
5	Euro V	1-Oct-2008	1.50	0.46	2.00	0.02

5.21.2 Bharat Stage or Indian Norms

- Emission norms followed in our country are called as Bharat Stage Norms or Indian Norms. These were implemented with effect from the year 1992.
- As per Supreme Court Ruling, Euro I norms were implemented in Delhi from 1st June 1999 which was notified by Government of India through Motor Vehicles Regulation. These norms were known as Bharat I norms.
- Bharat II norms were implemented from the year 2002.
- Table 5.3 shown below indicates sample emission levels that are permitted in engine exhaust.

Table 5.3 : Bharat stage norms

Table 5.3 (a) : BS norms for petrol vehicles

No.	Regulation/norm	Reference	Effective date	CO (gm/km)	HC (gm/km)	NO_x (gm/km)	HC + NO_x (gm/km)
1	India 2000	Euro - I	2000	2.27 - 6.9	-	-	0.97
2	BS-II	Euro - II	2005	2.20 - 5.00	-	-	0.50
3	BS-III	Euro - III	2010	2.30 - 4.17	0.2	0.15	-
4	BS-IV	Euro - IV	2010*	1.00	0.1	0.8	-

Note : * Only applicable in NCR (National Capital Region-Delhi) and 13 metro cities.

Table 5.3 (b) : BS norms for diesel vehicles

No.	Regulation/norm	Reference	Effective date	CO (gm/kWh)	HC (gm/kWh)	NO_x (gm/kWh)	PM (gm/kWh)
1	India 2000	Euro - I	2000	4.50	1.10	8.00	0.6
2	BS-II	Euro - II	2005	4.00	1.10	7.00	0.15
3	BS-III	Euro - III	2010	2.10	0.66	5.00	0.10
4	BS-IV	Euro - IV	2010#	1.50	0.46	3.50	0.02

Note : # Only applicable in NCR and 13 metro cities.

5.21.3 Testing and Certification Authorities of Pollution Norms for Automobiles

- Automobiles are required to be tested for mass emission tests at manufacturing stage and certification has to be done for emission levels within prescribed limits set by Government of India.
- In India, following agencies are authorised to set the prescribed limits for various automobiles.

1) Indian Institute of Petroleum (IIP), Dehradun.
2) Vehicles Research and Development Establishment (VRDE), Ahmednagar.
3) Automotive Research Association of India (ARAI), Pune.

REVIEW QUESTIONS

Q. 1 Discuss the various methods for measurement of Indicated Power (IP).

Q. 2 Give detailed classification of dynamometers.

Q. 3 Explain construction and working of any one type.

Q. 4 Write short note on Morse test.

Q. 5 Explain Willian's line method for engine testing.

Q. 6 Explain the performance characteristic of S.I. engines.

Q. 7 Explain the performance characteristic of C.I. engines.

Q. 8 Define and explain : BSFC, ISFC, Air standard efficiency, BHP and Mechanical efficiency.

Q. 9 Discuss the methods of improving the engine power output.

Q. 10 Discuss various methods of supercharging along with neat sketch.

Q. 11 Explain the following methods of turbo-charging :

a) Constant pressure turbo-charging,

b) Pulse turbo-charging and

c) Pulse converter turbo-charging.

Q. 12 Explain Bharat norms in detail.

Q. 13 Explain Exhaust Gas Recirculation System with neat sketch.

Q. 14 Explain catalytic converter used in S.I. engines.

Q. 15 Write short notes on :

i) Acid rain

ii) Green house effect and global warming

iii) Ozone depletion

Q. 16 Explain the causes and effects of following gases :

i) Carbon Monoxide (Co) ii) Oxides of Nitrogen (NO_x)

iii) Particulate Matters (PM) iv) Hydrocarbons (HC).

Q. 17 List down the major sources of air pollutants.

❑❑❑

Notes

CHAPTER - 6

FUNDAMENTALS OF COMPRESSIBLE FLOW

Syllabus : Ideal gas relationship, Adiabatic energy equation, Mach number and its significance, Mach waves, Mach cone and Mach angle, static and stagnation states, relationship between stagnation temperature, pressure, density and enthalpy in terms of Mach number, stagnation velocity of sound, reference speeds, various regions of flow, Effect of Mach number on compressibility, Area velocity relationship.

Section No.	Topic Name	Page No.
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6.2	Basic Thermodynamic Relations	6 - 2
6.3	Basic Thermodynamic Processes	6 - 4
6.4	Basic Equations for Compressible Fluid Flow	6 - 5
6.5	Velocity of Sound or Pressure Wave in a Fluid	6 - 9
6.6	Mach Number	6 - 14
6.7	Propagation of Pressure Waves (Disturbances) in a Compressible Fluid	6 - 15
6.8	Stagnation Properties	6 - 17
6.9	Area Velocity Relationship for Compressible Flow	6 - 21
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	Review Questions	6 - 35

6.1 Introduction

- A compressible flow is a flow in which density of the fluid changes during the flow significantly. Generally gases shows such behaviour.
- If the relative change in density is small, the fluid can be treated as incompressible.
- To distinguish between the compressible and incompressible fluid flow, the Mach number must be sufficiently greater. Mach number is given by,

$$\text{Mach number} = \frac{\text{Velocity of flow}}{\text{Velocity of sound}} = \frac{V}{C}$$

- The Mach number should be greater than 0.3 to identify the fluid as compressible.
- The study of compressible flow is important in the following applications :
  - Flow of gases through nozzles, pipelines.
  - High speed air crafts, jet engines.
  - Water hammer and acoustics.
  - Commercial applications such as abrasive blasting, etc.
- Most problems in compressible flow have pressure and velocity unknown along with the density and temperature unlike incompressive flow.
- Hence we need two additional equations to solve equation of state for gas and conservation of energy equation.

6.2 Basic Thermodynamic Relations

☞ [GTU : Summer-16]

- The following are the basic thermodynamic relations useful for gases :

1. The characteristic equation of state
2. Specific heat
3. Internal energy
4. Enthalpy

1. The characteristic equation of state

- The characteristic equation of state for a perfect gas correlates the density of gas, absolute pressure and temperature.
- It is given by,

$$pv = mRT \qquad \ldots (6.1)$$

Where,

p = Absolute pressure in N/m^2

v = Volume of a gas in m^3

m = Mass of the gas in kg

R = Gas constant in J/kgK

T = Absolute temperature in K

2. Specific heat

- Specific heat for a solid or liquid is defined as the amount of heat required to raise the temperature of unit mass through one degree.
- For gases there may be infinite number of specific heats but only the following two are defined :
  - Specific heat at constant volume, C_v
  - Specific heat at constant pressure, C_p
- The relation between C_p and C_v is given by,

$$C_p - C_v = R \qquad \ldots (6.2)$$

and

$$\frac{C_p}{C_v} = \gamma \qquad \ldots (6.3)$$

3. Internal energy

- Internal energy is defined as the amount of heat energy stored in the gas.
- For a perfect gas internal energy is a function of temperature only and is not affected by change in pressure and temperature.

4. Enthalpy

- Enthalpy is the amount of heat content used or released in the system at constant pressure.
- It is expressed as the sum of internal energy and product of pressure and volume.

$$H = U + pv \qquad \ldots (6.4)$$

6.3 Basic Thermodynamic Processes

- The basic thermodynamic processes related to the gases are given as follows :

1. Constant volume process
2. Constant pressure process
3. Isothermal process
4. Adiabatic process

1. Constant volume process (C_v = Constant)

- It is also known as ischoric process.
- In this process, system changes its state at constant volume. For example, heating or cooling of a gas in an enclosed space.

$$\therefore \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \qquad \ldots (6.5)$$

- In this system,

$$\text{Heat added } Q = m\, C_v\, (T_2 - T_1) \qquad \ldots (6.6)$$

$$\text{Work done } W = 0 \qquad \ldots (6.7)$$

2. Constant pressure process (p = Constant)

- It is also known as isobaric process.
- In this process, system changes its state at constant pressure. For example, heating of a gas in cylinder and piston moves against deadweight due to work done by the gas and keeps the pressure constant.

$$\therefore \quad \frac{T_2}{T_1} = \frac{v_2}{v_1} \qquad \ldots (6.8)$$

- For this system,

$$\text{Heat added } Q = C_p\, (T_2 - T_1) \qquad \ldots (6.9)$$

$$\text{Work done } W = p\, (v_2 - v_1) \qquad \ldots (6.10)$$

3. Isothermal process (T = Constant or pv = Constant)

- The process is called as isothermal when the expansion or compression is takes place at constant temperature.
- For example, during isothermal expansion, heat must be added continuously to keep the temperature constant and during isothermal compression heat must be removed continuously.

- In this process gas obeys Boyle's law

$$\therefore \quad pv = \text{Constant} \qquad \ldots (6.11)$$

$$\therefore \quad p_1 v_1 = p_2 v_2 \qquad \ldots (6.12)$$

$$\text{Also,} \quad \frac{p}{\rho} = \text{Constant} \qquad \ldots \left(\because \rho = \frac{1}{v}\right)$$

$$\text{Heat added} \quad Q = p_1 v_1 \log_e\left(\frac{v_2}{v_1}\right) = p_1 v_1 \log_e\left(\frac{p_1}{p_2}\right) \qquad \ldots (6.13)$$

4. Adiabatic process (pv^γ = C)

- An adiabatic process is the process in which there is no heat transfer from system to surrounding and vice-versa.

$$\therefore \quad \text{Heat added, } Q = 0 \qquad \ldots (6.14)$$

$$\text{Work done } W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1} \qquad \ldots (6.15)$$

- For this process,

$$pv^\gamma = C$$

$$\therefore \quad p_1 v_1^\gamma = p_2 v_2^\gamma$$

$$\text{Also,} \quad \frac{p}{\rho^\gamma} = C \qquad \ldots \left(\because V = \frac{1}{\rho}\right) \ldots (6.16)$$

- If the system is reversible, then this process is called as isentropic process.
- If the pressure and density are related in such a way that $\gamma \neq \frac{C_p}{C_v}$ but is equal to any positive value then the process is called as polytropic process.

$$\therefore \quad \frac{p}{\rho^n} = \text{Constant} \qquad \ldots (\text{where } n \neq \gamma) \ldots (6.17)$$

6.4 Basic Equations for Compressible Fluid Flow

- The basic equations of the compressible fluid flow are as follows :

1. Continuity equation
2. Bernoulli's equation
3. Momentum equation
4. Equation of state

6.4.1 Continuity Equation

- Continuity equation is based on the law of conservation of mass.
- For one dimensional steady flow,

Mass per second = ρAV

where, ρ = Mass density of fluid

A = Cross-sectional area

V = Velocity of flow

But, $\rho AV = \text{Constant}$ ... ($\because m = C$)

Differentiating above equation,

$$d(\rho AV) = 0$$

$$\therefore \quad \rho\, d(AV) + AV(d\rho) = 0$$

$$\therefore \quad \rho[AdV + VdA] + AVd\rho = 0$$

$$\therefore \quad \rho AdV + \rho VdA + AVd\rho = 0$$

Dividing by ρAV we get,

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \qquad \ldots (6.18)$$

Equation (6.18) is known as continuity equation in differential form.

6.4.2 Bernoulli's Equation

☞ **[GTU : Summer-16]**

- Bernoulli's equation is obtained by integrating an Euler's equation along a stream line.

$$\frac{dp}{\rho} + VdV + g\,dz = 0 \qquad \ldots \text{(Euler's equation)} \quad \ldots (6.19)$$

On integrating above equation,

$$\int \frac{dp}{\rho} + \int V \cdot dV + \int g\,dz = 0$$

$$\therefore \quad \int \frac{dp}{\rho} + \frac{V^2}{2} + gZ = 0 \qquad \ldots (6.20)$$

- In case of compressible flow the density ρ is not constant. Hence ρ can not be taken outside the integration sign.
- As the density of fluid changes, pressure also changes hence from equations (6.19) and (6.20), the value of ρ is obatined in terms of p and integration is done.

1. Bernoulli's equation for isothermal process :

- For isothermal process,

$$\frac{p}{\rho} = \text{Constant} = C_1 \qquad \ldots \text{(from equation)} \ldots (i)$$

$$\therefore \quad \rho = \frac{p}{C_1}$$

Hence $$\int \frac{dp}{\rho} = \int \frac{dp}{p/C_1} = \int \frac{C_1\,dp}{p} = C_1 \int \frac{dp}{p}$$

$$\therefore \quad \int \frac{dp}{\rho} = C_1 \log_e p = \frac{p}{\rho} \log_e p$$

Therefore substituting above value in equation (6.20) we get,

$$\frac{p}{\rho} \log_e p + \frac{V^2}{2} + gZ = 0$$

$$\therefore \quad \frac{p}{\rho g} \log_e p + \frac{V^2}{2g} + Z = 0 \qquad \ldots (6.21)$$

Equation (6.21) represents the Bernoulli's equation for compressible flow undergoing isothermal process. For the fluid passing from section 1 to section 2, the Bernoulli's equation is,

$$\frac{p_1}{\rho_1 g} \log_e p_1 + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho_2 g} \log_e p_2 + \frac{V^2}{2g} + Z_2 \qquad \ldots (6.22)$$

2. Bernoulli's equation for adiabatic process

- For an adiabatic process,

$$\frac{p}{\rho^\gamma} = \text{Constant} = C_2 \qquad \ldots (ii)$$

$$\therefore \quad \rho^\gamma = \frac{p}{C_2} \quad \therefore \quad \rho = \left(\frac{p}{C_2}\right)^{1/\gamma}$$

Hence $$\int \frac{dp}{\rho} = \int \frac{dp}{\left(\frac{p}{C_2}\right)^{1/\gamma}} = \int \frac{C_2^{1/\gamma}}{p^{1/\gamma}} dp = C_2^{1/\gamma} \int \frac{dp}{p^{1/\gamma}}$$

$$\therefore \int \frac{dp}{\rho} = C_2^{1/\gamma} \int p^{-1/\gamma} \cdot dp = C_2^{1/\gamma} \frac{p^{\left(-\frac{1}{\gamma}+1\right)}}{\left(-\frac{1}{\gamma}+1\right)}$$

$$\therefore \int \frac{dp}{\rho} = \frac{\gamma}{(\gamma-1)} C_2^{1/\gamma} \cdot p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \frac{\gamma}{(\gamma-1)} \cdot \left(\frac{p}{\rho^\gamma}\right)^{1/\gamma} \cdot p^{\left(\frac{\gamma-1}{\gamma}\right)} \quad \text{... from equation (ii)}$$

$$= \frac{\gamma}{(\gamma-1)} \times \frac{p^{1/\gamma}}{\rho^{\gamma \times \frac{1}{\gamma}}} \times p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$= \left(\frac{\gamma}{\gamma-1}\right) \frac{p^{\left(\frac{1}{\gamma}+\frac{\gamma-1}{\gamma}\right)}}{\rho}$$

$$\int \frac{dp}{\rho} = \left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho}$$

Substituting above value in equation (6.20)

$$\left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho} + \frac{V^2}{2} + gZ = \text{Constant}$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant} \quad \text{... (6.23)}$$

Equation (6.23) represents the Bernoulli's equation for compressible flow under adiabatic process for the fluid passing from section 1 to section 2.

$$\left(\frac{\gamma}{\gamma-1}\right) \frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 = \left(\frac{\gamma}{\gamma-1}\right) \frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2 \quad \text{... (6.24)}$$

6.5 Velocity of Sound or Pressure Wave in a Fluid

- The disturbance in a solid or fluid (liquid or gas) is transmitted from one point to other.
- The velocity of disturbance transmission depends on the molecules of the medium.
- As in solids the molecules are closely packed, the disturbance is instantaneouly transmitted.
- But, in case of fluids the molecules are at some distance. Hence, each molecule will have to travel a certain distance before it can transmit the disturbance.
- It means the velocity of disturbance in case of fluids is less than the solids.
- The velocity of disturbance mainly depends upon the change of pressure and mass density of the fluid.

6.5.1 Expression for Velocity of Sound Wave in a Fluid

- We know that, any disturbance create the pressure wave in a fluid. These pressure waves travel with a velocity of sound waves in all directions.
- Consider a pressure wave which travels along x-direction. Fig. 6.1 shows the model for propagation of pressure waves along x-direction (one-dimensional).
- Consider a long pipe of uniform cross-sectional area fitted with a piston.

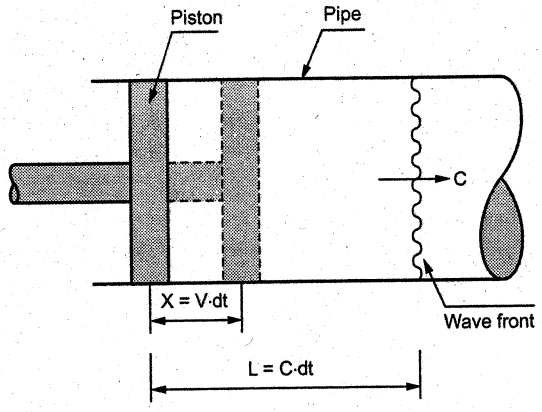


Fig. 6.1 : Propogation of pressure wave or sound wave

- Let, the pipe be filled with the compressible fluid. Initially, the fluid is at rest.
- Now, the piston is moved towards right and a disturbance is created in the fluid.

- This disturbance is in the form of pressure wave which travels with a velocity of sound in the fluid.

Let,

V = Velocity of piston

A = Cross-section area of the pipe

p and ρ = Pressure and density of fluid in pipe before the movement of the piston

C = Velocity of pressure wave or sound wave

dt = Small internal of time in which piston moves

L = Distance travelled by the pressure wave in time dt

X = Distance travelled by the piston in time dt

- Now, the distance travelled by the piston in time dt,

$$X = V \times dt \quad \text{... (V = Dist/Time) ... (i)}$$

Similarly, the distance travelled by the pressure wave in time dt,

$$L = C \times dt$$

- As $C > V$, $L > X$. Thus in the length of the tube $(L - X)$, the fluid will be compressed. Due to this compression, the pressure as well as density of the fluid will change.

Let,

$p + dp$ = Pressure after compression

$\rho + d\rho$ = Density after compression

- The mass of fluid before compression (for length L),

m_1 = $\rho \times$ Volume of fluid upto length L

$$\therefore \quad m_1 = \rho \times AL = \rho \times A \times C \cdot dt \quad \text{... (From equation (ii)) ... (iii)}$$

Similarly, the mass of fluid after compression (for length $(L - X)$),

$$m_2 = (\rho + d\rho) \times A \times (L - X)$$

$$\therefore \quad m_2 = (\rho + d\rho) \times A \times (C - V)\, dt \quad \text{... (iv)}$$

- But, the mass of fluid before compression must be equal to the mass of fluid after compression. Hence, equating equations (iii) and (iv),

$$\rho AC \cdot dt = (\rho + d\rho)\, A\, (C - V)\, dt$$

$$\rho C = (\rho + d\rho)\,(C - V) = \rho C - \rho V + C \cdot d\rho - V \cdot d\rho$$

$$\therefore \quad C \cdot d\rho = \rho V + V \cdot d\rho$$

As $V < C$ and $d\rho$ is very small, $V \cdot d\rho$ can be neglected.

$$\therefore \quad C \cdot d\rho = \rho V \quad \text{... (v)}$$

- When the piston is moved with a velocity V for time dt, the fluid (initially at rest) also moves with velocity V. Hence, the pressure of fluid increases from 'p' to 'p + dp'.
- Now, applying the impulse momentum equation,

Net force on the fluid = Rate of change of momentum

$$\therefore \quad (p + dp)\, A - p \times A = \text{Total mass} \times \frac{\text{Change in velocity}}{\text{Time}}$$

$$\therefore \quad p \times A + A \cdot dp - p \times A = \frac{\rho AL}{dt}(V - 0)$$

$$\therefore \quad A \cdot dp = \frac{\rho AC \cdot dt}{dt}(V) = \rho ACV$$

$$\therefore \quad dp = \rho CV$$

$$\rho V = \frac{dp}{C} \quad \text{... (vi)}$$

Substituting equation (vi) in equation (v),

$$\therefore \quad C \cdot d\rho = \frac{dp}{C}$$

$$\therefore \quad C^2 d\rho = dp$$

$$\text{or} \quad C^2 = \frac{dp}{d\rho}$$

$$\text{or} \quad C = \sqrt{\frac{dp}{d\rho}} \quad \text{... (6.25)}$$

- From the above equation (6.25) it is clear that the velocity of sound wave is the square root of the ratio of the change in pressure to the change in density of a fluid due to disturbance.

6.5.2 Velocity of Sound in Terms of Bulk Modulus

- We know that, bulk modulus of elasticity is given by,

$$K = \frac{\text{Increase in pressure}}{\text{Volumeteric strain}} = \frac{dp}{\left(-\frac{dv}{v}\right)} \quad \ldots (i)$$

where, dv = Decrease in volume

v = Original volume

Negative sign indicates that, with increase in pressure, volume of fluid decreases.

- As mass of fluid is constant,

$$\rho \times v = \text{Constant} \quad \ldots (M = \text{Density} \times \text{Volume})$$

Differentiating the above equation w.r.t. ρ and v,

$$\rho \cdot dv + v \cdot d\rho = 0$$

or $$-\frac{dv}{v} = \frac{d\rho}{\rho} \quad \ldots (ii)$$

Substituting equation (ii) in equation (i), we get

$$K = \frac{dp}{(d\rho/\rho)} = \rho \frac{dp}{d\rho}$$

or $$\frac{dp}{d\rho} = \frac{K}{\rho}$$

But, $$C = \sqrt{\frac{dp}{d\rho}}$$

$\therefore$ $$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \quad \ldots (6.26)$$

- The above equation (6.26) represents the velocity of sound wave in terms of bulk modulus of elasticity and mass density of fluid. This equation can be used for liquids as well as gases.

6.5.3 Velocity of Sound for Isothermal Process

☞ [GTU : Summer-18]

- We know that, for isothermal process (T = C),

$$\frac{p}{\rho} = \text{Constant or } p\rho^{-1} = \text{Constant}$$

Differentiating the above equation w.r.t. p and ρ,

$$p(-1) \times \rho^{-2} \times d\rho + \rho^{-1} \cdot dp = 0$$

$\therefore$ $$p \times \rho^{-1}\, d\rho = dp$$

$\therefore$ $$\frac{dp}{d\rho} = p \times \rho^{-1} = \frac{p}{\rho}$$

But, $$\frac{p}{\rho} = RT$$

$\therefore$ $$\frac{dp}{d\rho} = \frac{p}{\rho} = RT$$

$\therefore$ $$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{p}{\rho}} = \sqrt{RT} \quad \ldots (6.27)$$

6.5.4 Velocity of Sound for Adiabatic Process

- We know that for adiabatic process,

$$\frac{p}{\rho^{\gamma}} = \text{Constant} \quad \text{or} \quad p\rho^{-\gamma} = \text{Constant}$$

Differentiating the above equation w.r.t. p and ρ,

$$p(-\gamma) \times \rho^{-\gamma-1} \times d\rho + \rho^{-\gamma} \times dp = 0$$

$\therefore$ $$p \times \rho^{-1} \times \gamma\, d\rho = dp$$

$\therefore$ $$\frac{dp}{d\rho} = \frac{p}{\rho}\gamma = \gamma RT \quad \ldots \left(\frac{p}{\rho} = RT\right)$$

$\therefore$ $$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\gamma RT} \quad \ldots (6.28)$$

Note : For propagation of small disturbances through air, the process is considered as adiabatic. Isothermal process is considered for the calculation of the sound waves only when it is given in the examples. If nothing is mentioned about the process, then consider it as an adiabatic process.

6.6 Mach Number

☞ [GTU : Summer-14, Winter-17]

It is the square root of the ratio of inertia force to the elastic force. It is denoted by M.

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} \quad \ldots (i)$$

As, Inertia force $= \rho A V^2 = \rho L^2 V^2$ $\quad \ldots (A = L^2)$

Elastic force = Bulk modulus of elasticity × Area

$= K \times A = KL^2$

Substituting these values in equation (i),

$$\therefore \quad M = \sqrt{\frac{\rho A V^2}{KL^2}} = \sqrt{\frac{\rho L^2 V^2}{KL^2}}$$

$$\therefore \quad M = \sqrt{\frac{\rho V^2}{K}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \quad \ldots (6.29)$$

where, $C = \sqrt{\frac{K}{\rho}}$ = Sonic velocity

- Mach number differentiates the flow as incompressible and compressible : (Refer Fig. 6.2).

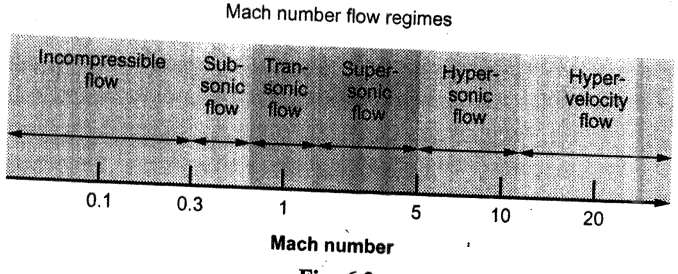


Fig. 6.2

- On the basis of Mach number compressible fluid flow can be divided as follows :

1. Sub-sonic flow
2. Sonic flow
3. Super-sonic flow

1. Sub-sonic flow

- Subsonic flow is defined as the flow at which Mach number is less than 1 ($M < 1$)
- It means that velocity of flow is less than the velocity of sound.

2. Sonic flow

- Sonic flow is defined as the flow at which mach number is equal to 1 ($M = 1$).
- It means that velocity of flow is equal to the velocity of sound.

3. Super-sonic flow

- Super sonic flow is defined as the flow at which Mach number is greater than 1 ($M > 1$).
- It means that velocity of flow is greater than the velocity of sound.

6.7 Propagation of Pressure Waves (Disturbances) in a Compressible Fluid

- In case of fluid whenever there is a minor disturbance, the change in position of molecules before propagation of disturbance depends on the changes in pressure and density.
- The propagation of pressure wave or disturbance is similar to the propagation of sound through media and it depends upon the difference in pressure.
- When some disturbance is produced in a compressible fluid, it is propagated in all directions with a velocity of sound.
- The nature of propagation of disturbance changes according to Mach number (M).
- Now, consider a small projectile moving from left to right (i.e. A to B) with velocity 'V' through a compressible fluid.
- When the projectile moves from point A where time $t = 0$ to point B, it will covers distance AB i.e. equal to $V \times t$.
- This movement of projectile will create a disturbance and it will grow into surface of sphere of radius Ct.
- The following are the various cases according to the Mach number which shows different nature of propagation of disturbance.

Case 1 : Subsonic flow, M < 1 and V < C

- In this case, as the V is less than C, the projectile will lag behind disturbance or pressure wave. Refer Fig. 6.3 (a).
- The projectile at B lies inside the sphere of radius Ct.

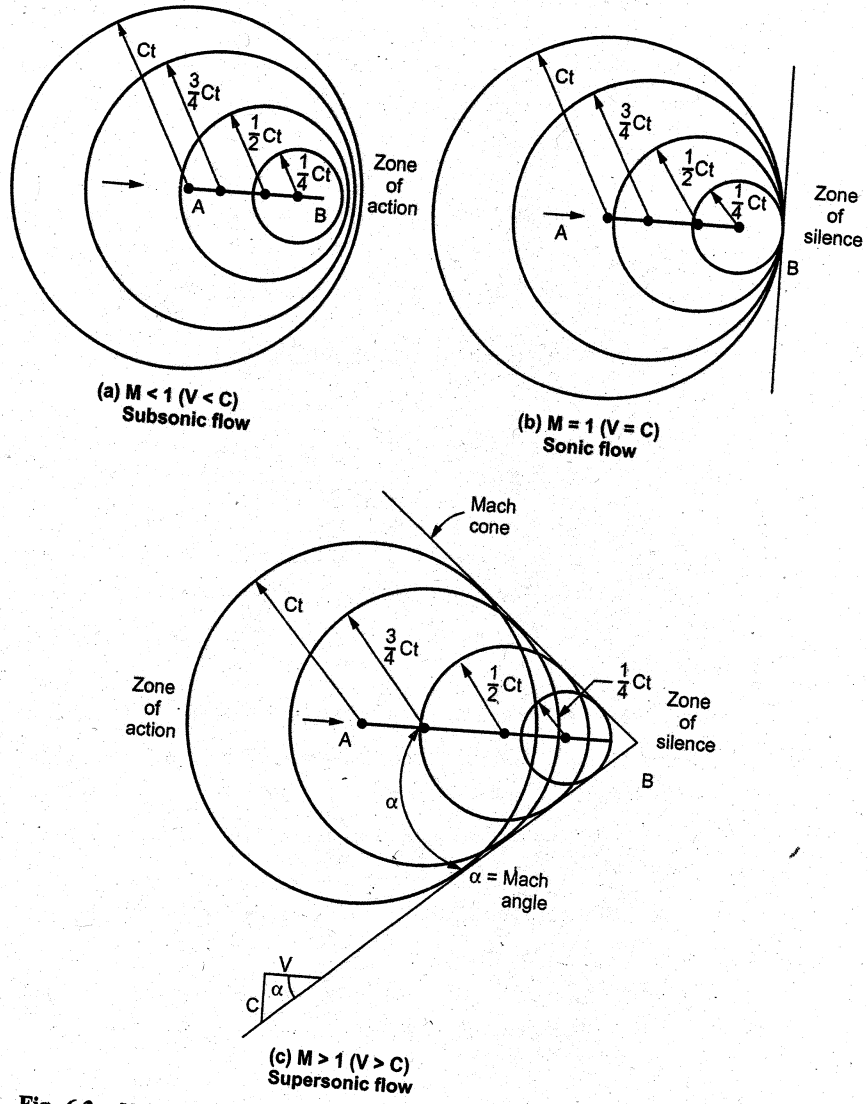


Fig. 6.3 : Nature of propagation of disturbances in compressible flow

Case 2 : Sonic flow, M = 1 and V = C

- In this case, as the V is equal to C, the disturbance will move along with projectile. Refer Fig. 6.3 (b).
- The projectile at point B will lie on the radius Ct on the circumference of sphere.

Case 3 : Supersonic flow, M > 1 and V > C

- In this case as the V is greater than C, the disturbance will lag behind the projectile. Thus, the distance AB is more than the radius Ct.
- The projectile at point B will lie outside the sphere. Refer Fig. 6.3 (c).
- If the tangents are drawn from point B to the circles, the spherical pressure vessel forms the cone with vertex at B. This is known as **Mach cone.**
- The semicone angle (α) of the Mach cone is known as **Mach angle** and it is given by,

$$\sin \alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M} \qquad \text{... (6.30)}$$

- In case of supersonic flow (i.e. M > 1), the effect of disturbance is felt only in the Mach cone. Hecne this region is known as **zone of action.**
- The disturbance is not felt outside the Mach cone. Hence the region outside the Mach cone is known as **zone of silence.**

6.8 Stagnation Properties

☞ [GTU : Summer-18, Winter-17]

- Stagnation point is a point in the fluid stream where the velocity of the flow is reduced to zero and the kinetic energy is converted into pressure energy.

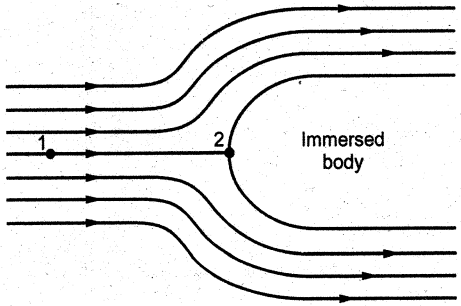


Fig. 6.4

- The values of pressure, density and temperature at this point are called as stagnation properties i.e. stagnation pressure, stagnation density and stagnation temperature. They are denoted as p_0, ρ_0 and T_0.

6.8.1 Equation for Stagnation Pressure

- Consider a compressible fluid flowing past an immersed body under frictionless adiabatic (isentropic) condition with steady flow as shown in Fig. 6.4.

Let, p_1 and p_2 = Pressure of fluid at points 1 and 2 respectively

ρ_1 and ρ_2 = Density of fluid at points 1 and 2 respectively

T_1 and T_2 = Temperature of fluid at points 1 and 2 respectively

- Now applying Bernoulli's equation for compressible flow under adiabatic process between the points 1 and 2.

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g}+Z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}+Z_2$$

- In this case, both the points lie on the same streamline hence $Z_1 = Z_2 = 0$

With point 2 as the stagnation point, velocity of flow becomes zero and kinetic energy (K.E.) is converted to pressure energy hence $V_2 = 0$, $\rho_2 = \rho_0$, $p_2 = p_0$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0 g}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_1}{\rho_1}-\frac{p_0}{\rho_0}\right) = \frac{-V_1^2}{2}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\cdot\left(\frac{p_1}{\rho_1}\right)\left[1-\left(\frac{\rho_1}{p_1}\right)\cdot\left(\frac{p_0}{\rho_0}\right)\right] = \frac{-V_1^2}{2}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\cdot\left(\frac{p_1}{\rho_1}\right)\left[1-\left(\frac{p_0}{p_1}\right)\cdot\left(\frac{\rho_1}{\rho_0}\right)\right] = \frac{-V_1^2}{2} \qquad \ldots (i)$$

For an adiabatic process,

$$\frac{p}{\rho^\gamma} = \text{Constant} \qquad \therefore \quad \frac{p_1}{\rho_1^\gamma} = \frac{p_0}{\rho_0^\gamma}$$

$$\therefore \quad \frac{\rho_1}{\rho_0} = \left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} \qquad \ldots (ii)$$

Substituting equation (ii) in equation (i),

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\cdot\left(\frac{p_1}{\rho_1}\right)\left[1-\left(\frac{p_0}{p_1}\right)\cdot\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}}\right] = \frac{-V_1^2}{2}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\cdot\left(\frac{p_1}{\rho_1}\right)\left[1-\left(\frac{p_0}{p_1}\right)\cdot\left(\frac{p_0}{p_1}\right)^{-\frac{1}{\gamma}}\right] = \frac{-V_1^2}{2}$$

$$\therefore \quad \left(\frac{\gamma}{\gamma-1}\right)\cdot\left(\frac{p_1}{\rho_1}\right)\left[1-\left(\frac{p_0}{p_1}\right)^{1-\frac{1}{\gamma}}\right] = \frac{-V_1^2}{2}$$

$$\therefore \quad 1-\left(\frac{p_0}{p_1}\right)^{1-\frac{1}{\gamma}} = \frac{-V_1^2}{2}\cdot\left(\frac{\rho_1}{p_1}\right)\cdot\left(\frac{\gamma-1}{\gamma}\right) \qquad \ldots (iii)$$

- For an adiabatic process, the velocity of sound is,

$$C = \sqrt{\gamma RT} = \sqrt{\gamma\cdot\left(\frac{p}{\rho}\right)} \qquad \ldots \left(\because RT = \frac{p}{\rho}\right)$$

At point 1,

$$C_1 = \sqrt{\gamma\cdot\left(\frac{p_1}{\rho_1}\right)} \qquad \therefore \quad C_1^2 = \gamma\cdot\left(\frac{p_1}{\rho_1}\right)$$

$$\therefore \quad \frac{\rho_1}{p_1} = \frac{\gamma}{C_1^2} \qquad \ldots (iv)$$

Substituting equation (iv) in equation (iii),

$$\therefore \quad 1-\left(\frac{p_0}{p_1}\right)^{1-\frac{1}{\gamma}} = \frac{-V_1^2}{2}\cdot\left(\frac{\gamma}{C_1^2}\right)\cdot\left(\frac{\gamma-1}{\gamma}\right)$$

$$\left(\frac{p_0}{p_1}\right)^{1-\frac{1}{\gamma}} = 1+\left[\left(\frac{V_1}{C_1}\right)^2 \cdot \left(\frac{\gamma-1}{2}\right)\right]$$

As Mach Number, $M = \frac{V_1}{C_1}$

$$\frac{p_0}{p_1} = \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$p_0 = p_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \ldots (v)$$

6.8.2 Equation for Stagnation Density

- From equation (ii) for adiabatic process,

$$\frac{\rho_1}{\rho_0} = \left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} \quad \therefore \quad \rho_0 = \rho_1 \cdot \left(\frac{p_0}{p_1}\right)^{\frac{1}{\gamma}} \quad \ldots (vi)$$

Substitute equation (v) in equation (vi),

$$\rho_0 = \rho_1 \left[\frac{p_1 \cdot \left(1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right)^{\frac{\gamma}{\gamma-1}}}{p_1}\right]^{\frac{1}{\gamma}}$$

$$\rho_0 = \rho_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{1}{\gamma-1}} \quad \ldots (vii)$$

6.8.3 Equation for Stagnation Temperature

- For point 2 i.e. stagnation point,

$$p_0 = \rho_0 R T_0 \quad \therefore \quad T_0 = \frac{p_0}{\rho_0 R} \quad \ldots (viii)$$

Substitute equations (v) and (vii) in equation (viii),

$$\therefore \quad T_0 = \frac{1}{R} \cdot \frac{p_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}}}{\rho_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{1}{\gamma-1}}}$$

$$\therefore \quad T_0 = \frac{p_1}{\rho_1 R} \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}-\frac{1}{\gamma-1}}$$

$$\therefore \quad T_0 = T_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma-1}{\gamma-1}} \quad \ldots \left[\because T = \frac{P}{\rho R}\right]$$

$$\therefore T_0 = T_1 \cdot \left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]$$

Note : As per ideal gas equation

$$pv = mRT \quad \therefore p = \frac{m}{v}RT = \rho RT \quad \ldots \left(\rho = \frac{m}{v}\right)$$

6.9 Area Velocity Relationship for Compressible Flow

☞ **[GTU : Summer-15, Winter-16]**

- For compressible flow, the continuity equation is,

$$\rho AV = \text{Constant}$$

As density ρ, area A and velocity V are changed, differentiating the above equation,

$$d(\rho AV) = 0$$

$$\therefore \quad \rho\, d(AV) + AV d\rho = 0$$

$$\therefore \quad \rho[AdV + VdA] + AV d\rho = 0$$

$$\therefore \quad \rho\, AdV + \rho VdA + AV d\rho = 0$$

Dividing the above equation by ρAV,

$$\therefore \quad \frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \quad \ldots (i)$$

Also, Euler's equation for compressible flow is,

$$\frac{dp}{\rho} + VdV + gdZ = 0 \quad \ldots (ii)$$

Neglecting, elevation difference i.e. $dZ = 0$

Sonic velocity is,

$$C = \sqrt{\frac{dp}{d\rho}} \qquad \therefore \quad C^2 = \frac{dp}{d\rho}$$

$$\therefore \quad dp = C^2 \cdot d\rho$$

- Substituting the above equation in equation (ii),

$$\frac{C^2 \cdot d\rho}{\rho} + VdV = 0$$

$$\frac{d\rho}{\rho} = \frac{-VdV}{C^2}$$

Substituting the above equation in equation (i),

$$\therefore \quad \frac{dV}{V} + \frac{dA}{A} + \frac{-VdV}{C^2} = 0$$

$$\therefore \quad \frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V}\left[\frac{V^2}{C^2} - 1\right]$$

$$= \frac{dV}{V}[M^2 - 1] \qquad \ldots \left(\because M = \frac{V}{C}\right)$$

$$\therefore \quad \frac{dA}{A} = \frac{dV}{V}[M^2 - 1] \qquad \ldots (6.31)$$

- From the above equation, the following conclusions can be drawn.

Case - I : For subsonic flow i.e. M < 1

Here, the term $[M^2 - 1]$ is negative.

i) If $\frac{dV}{V} > 0$, then $\frac{dA}{A} < 0$.

Thus, decrease of area leads to increase in velocity.

Example *Subsonic-converging nozzle. Refer Fig. 6.5 (a).*

ii) If $\frac{dV}{V} < 0$, then $\frac{dA}{A} > 0$.

Thus, increase of area leads to decrease in velocity.

Example : *Subsonic-diverging diffuser. Refer Fig. 6.5 (b).*

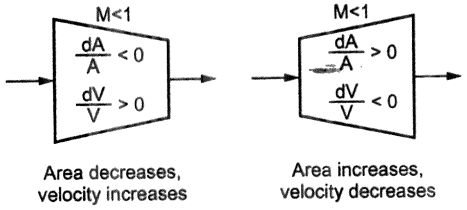


Fig. 6.5

Case-II : For sonic flow i.e. M = 1

Here, the term $[M^2 - 1]$ is zero.

$$[M^2 - 1] = 0 \qquad \therefore \quad \frac{dA}{A} = 0$$

Thus, area is constant which leads to no change in velocity. Refer Fig. 6.6.

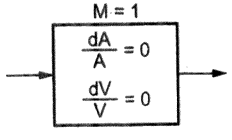


Fig. 6.6 Constant area duct

Case-III : For supersonic flow i.e. M > 1

Here, the term $[M^2 - 1]$ is positive.

i) If $\frac{dV}{V} > 0$, then $\frac{dA}{A} > 0$.

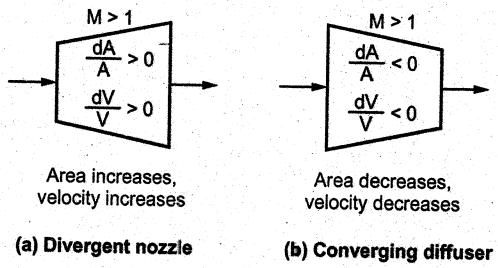
Thus, increase of area leads to increase in velocity.

Example : *Supersonic divergent nozzle, Refer Fig. 6.7 (a).*

ii) If $\frac{dV}{V} < 0$, then $\frac{dA}{A} < 0$.

Thus, decrease of area leads to increase in velocity.

Example : *Supersonic convergent nozzle, Refer Fig. 6.7 (b).*



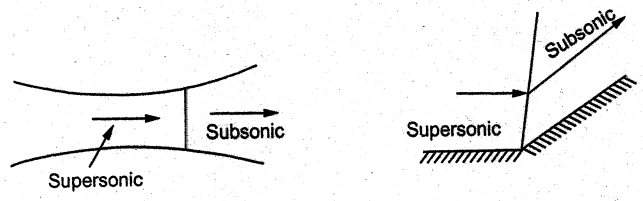
(a) Divergent nozzle (b) Converging diffuser

Fig. 6.7

- From the above conclusions, the increase or decrease in velocity of the fluid depends not only on change in area but also on the Mach number.

6.10 Normal and Oblique Shocks

- For compressible flows with little or small flow turning, the flow process is **reversible** and the entropy is constant.
- The change in flow properties is then given by the isentropic relations (isentropic means "constant entropy"). But when an object moves faster than the speed of sound, and there is an abrupt decrease in the flow area, the flow process is **irreversible** and the entropy increases.
- **Shock waves** are generated which are very small regions in the gas where the gas properties change by a large amount.
- Across a shock wave, the static pressure, temperature, and gas density increases almost instantaneously. The Mach number and speed of the flow decrease across a shock wave.
- If the upstream Mach number is in the low supersonic regime, the specific heat ratio of air remains a constant value (1.4) and air is said to be **calorically perfect**. But under low hypersonic flow conditions or high total temperature conditions, the specific heat ratio changes and air is then said to be **calorically imperfect.**
- Because a shock wave does no work, and there is no heat addition, the total enthalpy and the total temperature are constant through the shock. But because the flow is non-isentropic, the total pressure downstream of the shock is always less than the total pressure upstream of the shock; there is a loss of total pressure associated with a shock wave.
- If the shock wave is perpendicular to the flow direction it is called a **normal** shock.



(a) Normal shock (b) Oblique shock

Fig. 6.8 Normal and oblique shocks

- Depending on the shape of the object and the speed of the flow, the shock wave may be inclined to the flow direction. When a shock wave is inclined to the flow direction it is called an **oblique** shock.
- Oblique shocks are generated by the nose and by the leading edge of the wing and tail of a supersonic aircraft.
- Oblique shocks are also generated at the trailing edges of the aircraft as the flow is brought back to free stream conditions. Oblique shocks also occur downstream of a nozzle if the expanded pressure is different from free stream conditions.
- In high speed inlets, oblique shocks are used to compress the air going into the engine. The air pressure is increased without using any rotating machinery.
- Similar to a normal shock wave, the oblique shock wave consists of a very thin region across which nearly discontinuous changes in the thermodynamic properties of a gas occur. While the upstream and downstream flow directions are unchanged across a normal shock, they are different for flow across an oblique shock wave.

6.11 Solved Examples

Example 6.1 : *Find the velocity of bullet fired in standard air if Mach angle is 30°. Take R = 287.14 J/kg K and γ = 1.4 for air and temperature of air is 15 °C.*

Solution : Given data :

$\alpha = 30°$, R = 287.14 J/kgK, T = 15 °C = 273 + 15 = 288 K, $\gamma = 1.4$

To find : V

Step - 1 : Calculate the velocity of bullet fired

Velocity of sound wave is,

$$C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.14 \times 288} = 340.257 \text{ m/s}$$

Mach angle is given by,

$$\sin\alpha = \frac{C}{V} \quad \therefore \sin 30 = \frac{340.257}{V}$$

$$\therefore \quad \mathbf{V = 680.5140\ m/s} \qquad \textbf{... Ans.}$$

Example 6.2 : *A projectile travels at speed of 1500 km/hour at 20° C temperature and 0.1 MPa air pressure. Calculate the Mach number and Mach angle. Take γ = 1.4 for air and R = 287 J/kgK*

☞ **[GTU : Summer-15, Winter-16]**

Solution : Given data :

$V = 1500$ km/hour $= \frac{1500\times1000}{60\times60} = 416.6667$ m/s, $T = 20°\ C = 273 + 20 = 293$ K,

$p = 0.1$ MPa $= 0.1\times10^6$ N/m^2, $\gamma = 1.4$, $R = 287$ J/kgK.

To find : i) M ii) α

Step - 1 : Calculate the Mach number for moving projectile

Velocity of sound wave is,

$$C = \sqrt{\gamma RT} = \sqrt{1.4\times287\times293} = 343.1142 \text{ m/s}$$

Mach number is given by,

$$\mathbf{M = \frac{C}{V} = \frac{416.6667}{343.1142} = 1.2143} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the mach cone angle

Mach angle is given by,

$$\sin\alpha = \frac{C}{V} = \frac{1}{M} = \frac{1}{1.2143} = 0.8234$$

$$\therefore \quad \alpha = \sin^{-1}(0.8235) = \mathbf{55.4367°} \qquad \textbf{... Ans.}$$

Example 6.3 : *An air plane is flying at an altitude of 15 km where the temperature is − 50 °C. The speed of plane corresponds to Mach number 1.6. Assume γ = 1.4 and R = 287 J/kgK for air. Find the speed of plane and Mach angle.*

Solution : Given data :

$h = 15$ km $= 15\times10^3$ m, $T = -50$ °C $= 273 - 50 = 223$ K, $M = 1.6$, $\gamma = 1.4$,

$R = 287$ J/kgK

To find : i) V ii) α

Step - 1 : Calculate the velocity of air plane

Velocity of sound wave is,

$$C = \sqrt{\gamma RT} = \sqrt{1.4\times287\times223} = 299.3349 \text{ m/s}$$

Velocity of plane is given by,

$$M = \frac{V}{C} \quad \therefore V = M\times C$$

$$\therefore \quad \mathbf{V = 1.6\times299.3349 = 478.9358\ m/s} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the mach cone angle

Mach angle is given by,

$$\sin\alpha = \frac{1}{M}$$

$$\therefore \quad \alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.6}\right)$$

$$\therefore \quad \mathbf{\alpha = 38.6821°} \qquad \textbf{... Ans.}$$

Example 6.4 : *A projectile travels in air of pressure 15 N/cm^2 at 10 °C, at speed of 1500 km/hr. Find the Mach number and Mach angle. Assume γ = 1.4. and R = 287 J/kg.K.*

☞ **[GTU : Summer-16, Winter-17]**

Solution : Given data :

$T = 10$ °C $= 273 + 10 = 283$ K, $V = 1500$ km/hr $= \frac{1500\times1000}{3600} = 416.667$ m/s,

$\gamma = 1.4$, $R = 287$ J/kgK

To find : i) M ii) α

Step - 1 : Calculate the Mach number

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4\times287\times283}$

$$\therefore \quad C = 337.208 \text{ m/s}$$

Mach number $M = \frac{V}{C}$

$\therefore \quad M = \frac{416.667}{337.208} = 1.235$

$\therefore \quad \mathbf{M = 1.235}$ **... Ans.**

Step - 2 : Calculate the Mach angle

$\therefore \quad \sin\alpha = \frac{1}{M} = \frac{1}{1.235}$

$\therefore \quad \alpha = \sin^{-1}\left(\frac{1}{1.235}\right)$

$\therefore \quad \mathbf{\alpha = 54.068^\circ}$ **... Ans.**

Example 6.5 : *Calculate the Mach number at a point on a jet propelled air craft, which is flying at 1100 km/hour at sea-level where air temperature is 20 °C. Assume $\gamma = 1.4$ and $R = 287$ J/kg K.*

Solution : Given data :

$V = 1100$ km/hr $= \frac{1100 \times 1000}{3600} = 305.555$ m/s, $T = 20$ °C $= 20 + 273 = 293$ K,

$\gamma = 1.4$ and $R = 287$ J/kg K

To find : M

Step - 1 : Calculate the Mach number

Sonic velocity, $C = \sqrt{\gamma RT}$

$\therefore \quad C = \sqrt{1.4 \times 287 \times 293}$

$\therefore \quad C = 343.114$ m/s

Mach Number $M = \frac{V}{C} = \frac{305.555}{343.114}$

$\therefore \quad \mathbf{M = 0.8905}$ **... Ans.**

Example 6.6 : *Calculate the velocity and Mach number of a supersonic aircraft flying at an altitude of 1000 m where temperature is 280 °K. Sound of the air craft is heard 2.15 seconds after the passage of air craft on the head of an observer. Take $\gamma = 1.41$ and $R = 287$ J/kgK.*

Solution : Given data :

Altitued H = 1000 m, T = 280 °K, t = 2.15 sec, $\gamma = 1.41$, R = 287 J/kg K

To find : i) V ii) M

Step - 1 : Calculate the Mach number (M)

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.41 \times 287 \times 280}$

$= 336.6119$ m/s

In the given Fig. 6.9, O denotes the position of observer and A denotes position of air craft just above the observer.

The aircraft reaches position A' after 2.15 seconds. Here α denotes the Mach angle and OA' denotes the wave front.

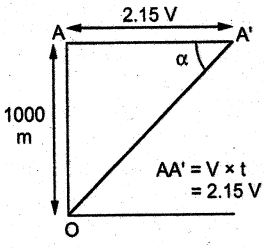


Fig. 6.9

From Fig. 6.9,

$\tan\alpha = \frac{1000}{2.15V} = \frac{465.116}{V}$ ... (i)

As Mach number, $M = \frac{V}{C} = \frac{1}{\sin\alpha}$

$\therefore \quad V = \frac{C}{\sin\alpha} = \frac{336.6119}{\sin\alpha}$ ... (ii)

Substituting above equation in equation (i),

$\therefore \quad \tan\alpha = \frac{465.116}{\left(\frac{336.6119}{\sin\alpha}\right)}$

$\therefore \quad \frac{\sin\alpha}{\cos\alpha} = 1.3817 \sin\alpha$

$\therefore \quad \cos\alpha = \frac{1}{1.3817} = 0.7237$

$\therefore \quad \alpha = \cos^{-1}(0.7237)$

$\therefore \quad \mathbf{\alpha = 43.6391^\circ}$ **... Ans.**

Step - 2 : Calculate the velocity of aircraft

$\therefore \quad M = \frac{V}{C}$

$$\therefore \quad M = \frac{1}{\sin(43.6391)} = \frac{V}{336.6119}$$

$$\therefore \quad \mathbf{V = 487.763\ m/s} \qquad \textbf{... Ans.}$$

Example 6.7 : *A gas is flowing through a horizontal pipe having cross section area 45 cm^2 and pressure 50 N/cm^2 (absolute) and temperature 12°C. AT another section the cross section is reduced to 25 cm^2 and pressure is 40 N/cm^2 (absolute). Assuming isothermal process, find the velocities at these sections if the mass rate of flow gas = 0.52 kg/s. Take gas constant R = 292 N-m/Kg°K.*

☞ **[GTU : Winter-16, Summer-15]**

Solution : Given data :

$A_1 = 45\ cm^2 = 45 \times 10^{-4}\ m^2$, $p_1 = 50\ N/cm^2 = 50 \times 10^4\ N/m^2$,

$T_1 = 12\ °C = 273 + 12 = 285\ K$, $A_2 = 25\ cm^2 = 25 \times 10^{-4}\ m^2$,

$p_2 = 40\ N/cm^2 = 40 \times 10^4\ N/m^2$, Type of Process : Isothermal

$\dot{m} = 0.52$ kg/sec, $R = 292$ N-m/kg-K

To find : V_1 and V_2

Step 1 : Calculate velocities at sections 1 and 2

Ideal Gas Equation,

$$pV = mRT \qquad \therefore \quad p = \rho RT$$

For section 1, $p_1 = \rho_1 R T_1 \qquad \therefore \quad 50 \times 10^4 = \rho_1 \times 292 \times 285$

$$\rho_1 = 6.0081\ kg/m^3$$

For isothermal process,

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} \qquad \therefore \quad \frac{50 \times 10^4}{6.0081} = \frac{40 \times 10^4}{\rho_2}$$

$$\rho_2 = 4.8064\ kg/m^3$$

Mass flow rate, $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$\therefore \quad \dot{m} = \rho_1 A_1 V_1$

$\therefore \quad 0.52 = 6.0081 \times 45 \times 10^{-4} \times V_1$

$$\mathbf{V_1 = 19.2332\ m/sec} \qquad \textbf{... Ans.}$$

Also, $\dot{m} = \rho_2 A_2 V_2$

$\therefore \quad 0.52 = 4.8064 \times 25 \times 10^{-4} \times V_2$

$$\therefore \quad \mathbf{V_2 = 43.2756\ m/sec} \qquad \textbf{... Ans.}$$

Example 6.8 : *An aeroplane is flying at 950 km/hour through still air having an absolute pressure of 80 kN/m^2 and temperature –7° C. Calculate stagnation pressure, stagnation temperature and stagnation density on the stagnation point on the nose of the plane. Take R = 287 J / Kg K and γ = 1.4 for air.*

Solution : Given data :

$V = 950\ km/hr = \frac{950 \times 1000}{3600} = 263.8888\ m/sec$, $p = 80 \times 10^3\ N/m^2$,

$T = -7\ °C = 273 - 7 = 266\ K$, $R = 287\ J/kgK$, $\gamma = 1.4$

To find : p_0, T_0 and ρ_0

Step 1 : Calculate the stagnation pressure

Sonic velocity, $C = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 266}$

$$C = 326.9232\ m/sec$$

Mach number, $M = \frac{V}{C} = \frac{263.8888}{326.9232} = 0.8071$

Stagnation pressure, $p_0 = p \times \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma - 1}}$

$$= 80 \times 10^3 \times \left[1 + \left(\frac{1.4 - 1}{2}\right) \times (0.8071)^2\right]^{\frac{1.4}{1.4 - 1}}$$

$$\mathbf{p_0 = 122.8128 \times 10^3\ N/m^2} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the stagnation temperature and stagnation density

Stagnation temperature, $T_0 = T \cdot \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]$

$$= 266 \times \left[1 + \left(\frac{1.4-1}{2}\right) \times (0.8071)^2\right]$$

$\therefore$ $\mathbf{T_0 = 300.655\ K}$ **... Ans.**

Ideal Gas Equation,

$$pV = mRT \qquad \therefore \quad p = \rho RT$$

$$\therefore \quad 80 \times 10^3 = \rho \times 287 \times 266 \qquad \therefore \quad \rho = 1.0479\ kg/m^3$$

Stagnation density, $\rho_0 = \rho \cdot \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{\frac{1}{\gamma-1}}$

$$= 1.4079 \times \left[1 + \left(\frac{1.4-1}{2}\right) \times (0.8071)^2\right]^{\frac{1}{1.4-1}}$$

$\rho_0 = \mathbf{1.9122\ kg/m^3}$ **... Ans.**

6.12 Computational Fluid Dynamics (CFD)

- **Computational fluid dynamics**, usually abbreviated as **CFD**, is a branch of fluid mechanics which uses numerical methods and algorithms to solve and analyze problems that involve fluid flows.
- In this technique, computers are used to perform the calculations required to simulate the interaction of liquids and gases with surfaces defined by boundary conditions. Refer Fig. 6.10.

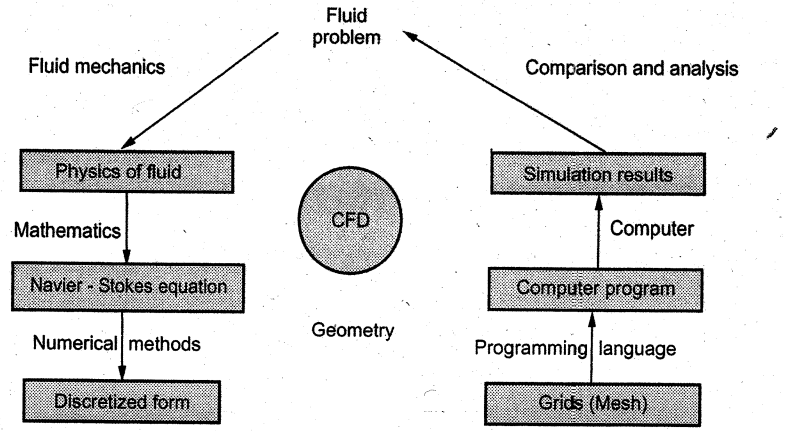


Fig. 6.10 (a)

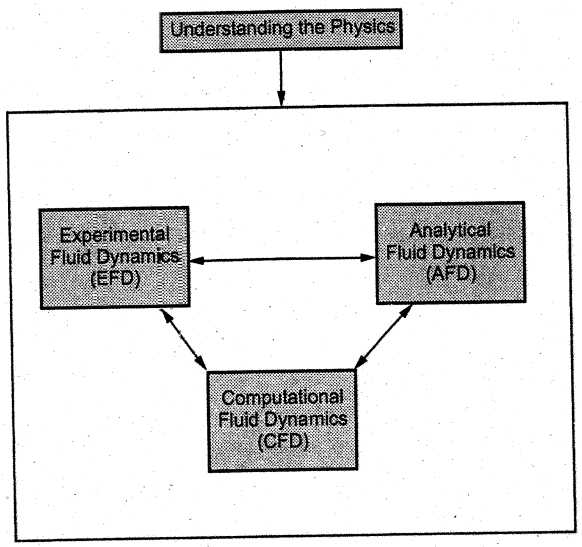


Fig. 6.10 (b)

- With high-speed supercomputers, better solutions can be achieved. Ongoing research yields software that improves the accuracy and speed of complex simulation scenarios like transonic or turbulent flows.
- In case of engineering problems the equations become more difficult. For example : Problems involving chemical reactions, heat transfer, multiphase flows, etc. In all these problems, CFD method is an effective tool to get the solutions.
- The fundamental basis of almost all CFD problems is the Navier-Stokes equations, which define any single-phase fluid flow. These equations can be simplified by removing terms describing viscosity to yield the Euler equations.

In this approach the following procedure is used :

i) During **preprocessing**
   - o The geometry (physical bounds) of the problem is defined.
   - o The volume occupied by the fluid is divided into discrete cells (the mesh). The mesh may be uniform or non uniform.
   - o The physical modeling is defined - for example, the equations of motions, enthalpy, radiation, etc.
   - o Boundary conditions are defined. This involves specifying the fluid behaviour and properties at the boundaries of the problem. For transient problems, the initial conditions are also defined.

ii) The simulation is started and the equations are solved iteratively as a steady-state or transient.

iii) Finally a **postprocessor** is used for the analysis and the results are interpreted in the visual forms like graphs, images, animations, etc.

The following methods are commonly used :

i) Finite volume method ii) Finite element method iii) Finite difference method

iv) Spectral element method v) Boundary element method

Advantages of CFD

- Relatively low cost.
  - Using physical experiments and tests to get essential engineering data for design can be expensive.
  - CFD simulations are relatively inexpensive, and costs are likely to decrease as computers becomes more powerful.
- Speed
  - CFD simulations can be executed in a short period of time.
  - Quick turnaround means engineering data can be introduced early in the design process.
- Ability to simulate real conditions.
  - Many flow and heat transfer processes can not be (easily) tested, e.g. hypersonic flow.
  - CFD provides the ability to theoretically simulate any physical condition.

Applications of CFD

- Design of vehicles to improve aerodynamic characteristics.
- It is widely used in design of system like pipes, nozzles, turbo machinery, aerodynamic objects, etc.
- It is used in medical field to cure diseases.
- It can be used effectively for simulation in various fields like flows of gases and liquids, heat and mass transfer, bodies in motion, chemical reactions, etc.

Limitations of CFD

- It requires fair amount of knowledge and experience with both CFD and the phenomena under investigation.
- The cost of tools or software require for CFD is very high.
- The solution of CFD problem is not fully reliable.
- To solve the CFD problem, it requires large number of input data.

Review Questions

1. *Define compressible and incompressible flow.*
2. *Show the relation between pressure and density for adiabatic and isothermal process.*
3. *Derive the expression for Bernoulli's equation for compressible fluid undergoing*
   i) Isothermal process ii) Adibatic process
4. *Obtain an expression for velocity of sound in a compressible fluid in terms of change of pressure and change of density.*
5. *Obtain an expression for velocity of sound in terms of bulk modulus for a compressible fluid.*
6. *Obtain an expression for velocity of sound for compressible fluid undergoing isothermal and adiabatic process.*
7. *Define Mach number. Explain its significance in compressible flow of fluid.*

Examples for Practice

Example 6.9 : *Find the velocity of a bullet fired in standard air if its Mach angle is 38°.* **[Ans. : 1989.1231 kmph]**

Example 6.10 : *Determine the velocity of a bullet fired in the air if Mach angle observed to be 30°. Given that temperature of air is 22 °C, density* $1.2\,kg/m^3$. *Take* $\gamma = 1.4$ *and* $R = 287.4$ *J/kg-K.*

[Ans. : 2480.5663 kmph]

Example 6.11 : *An aeroplane is flying at a height of 15 km where the temperature is – 50 °C. The speed of the plane is corresponding to M = 2.0. Assuming K = 1.4 and R = 287 J/kgK. find the speed of the plane.*

[Ans. : 2155.2113 km/hr]

Example 6.12 : *An air plane is flying at an altitude of 15 km where the temperature is – 50 °C. The speed of the plane corresponds to Mach number of 1.6. Assuming* $\gamma = 1.4$ *and* $R = 287$ *J/kgK for air, find the speed of the plane and Mach angle* α. **[Ans. :** α **= 38.6822°, 1724.169 kmph]**

Fundamentals of Compressible Flow ends ...

Notes

CHAPTER - 7

RECIPROCATING COMPRESSORS

Syllabus : Construction and working, Multistage conditions for minimum work, Intercooling, Efficiency and control of air compressors

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7.2	Classification of Compressors	7 - 5
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7.1 Introduction to Compressor

- Following diagram is shown the general or basic layout of prime mover and compressor.
- The compressors which are used for compressing the air called as air compressors.
- For increasing the pressure of fluid compressor machine is used. It is a work absorbing device.
- In the compressor the air is taken from atmosphere and at high pressure it is compressed. This pressurized air is stored in receiver and used as per the requirement.

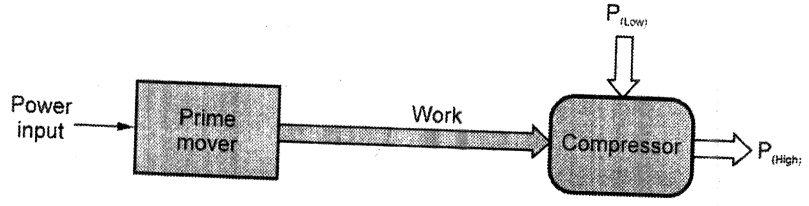
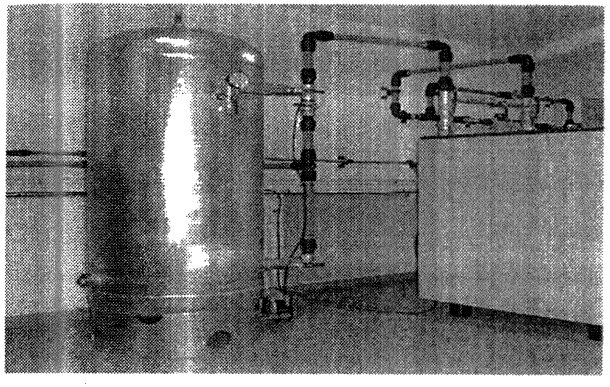


Fig. 7.1 : General layout of compressor

- During this process of compressing the air it requires some work is to be done on air. Therefore prime mover is required to run the compressor.
- For operating the compressors electric motors, internal combustion engines, gas turbines are used.
- From total work supplied to the compressor the part of work is lost in friction, heat and remaining work is used for compressing the air.
- At the time of designing the compressors, they are designed to give maximum work and minimum losses.

7.1.1 Applications of Compressed Air





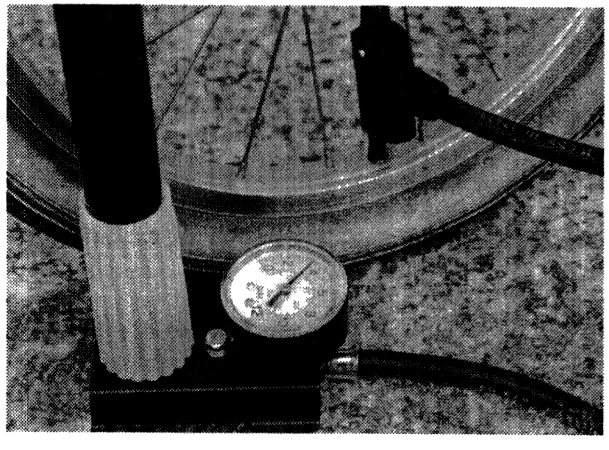


Fig. 7.2

The compressors are widely used in compressing the air. The functions of compressed air are as follows :

- General industrial cleaning purpose
- Pneumatic comparators
- Filling of air in types of automobiles
- Spray painting

- For job and surface cleaning in foundry
- In air conditioning system, refrigeration system and dryers.
- Supercharging of IC engine.
- Pharmaceutical and food processing industry.
- For operating rivetting tools in industry.

7.1.2 Important Terms Used in Compressor ☞ [GTU : Summer-16, Winter-17]

(i) Free air delivery : If the volume of air compressor is reduced to atmospheric temperature and pressure, this volume of air is called FAD (m^3/min).

$$\frac{P_1 V_a}{T_1} = \frac{P_1 (V_1 - V_4)}{T_1} = \frac{P_2 (V_2 - V_3)}{T_2}$$

If clearance volume is neglected,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

(ii) Capacity of compressor : The capacity of compressor is full rated volume of flow of gas compressed and delivered at conditions of total temperature, total pressure and composition prevailing at the compressor inlet. It sometimes means actual flow rate, rather than rated volume of flow.

(iii) Volumetric efficiency :

- Ratio of free air delivered to the displacement of the compressor.
- Ratio of effective swept volume to swept volume.
- Volumetric efficiency is in between (60 – 80%)

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} ; \qquad \eta_{vol} = 1 - C\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

where C = Clearance ratio = $\frac{V_C}{V_S}$

7.2 Classification of Compressors

- The tree diagram of classification of compressor is as follows :

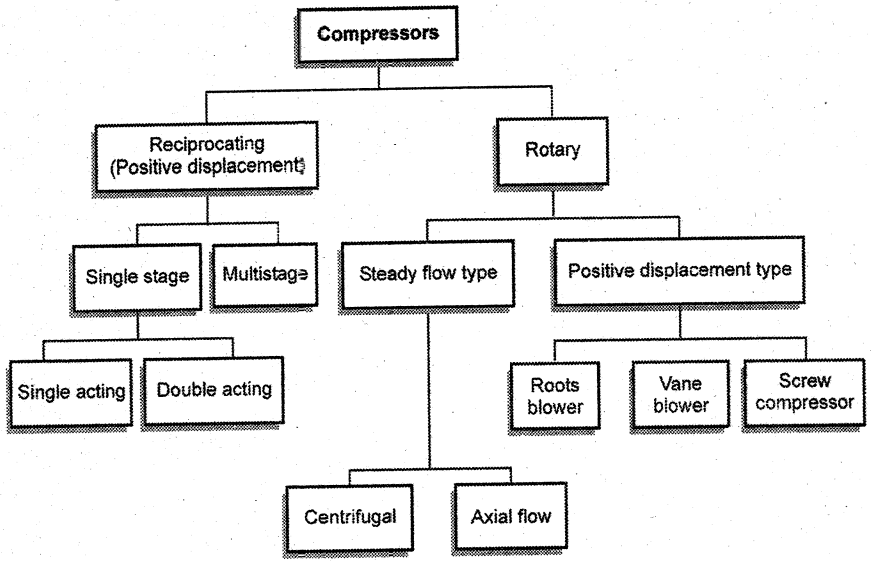


Fig. 7.2(a) : Classification of Compressors

- The compressors are classified into two types i.e., Reciprocating (positive displacement) and Rotary.
- In reciprocating compressor they are classified into single stage and multistage and in multistage they are classified as single acting and double acting.
- Now, in Rotary compressors they are classified as steady flow type and positive displacement type compressor. In steady flow type they are classified into centrifugal and flow type and in positive displacement type they are three types i.e., Roots blower, vane blower and screw compressor.

❑ 1) Based on principle of operation

On the basis of principle of operation, the compressors are classified as follow.

a) Positive displacement type : In positive displacement type they are classified into two types i.e., single stage and multistage. In that fixed amount of air is definitely entrapped between clearance volume available in piston-cylinder arrangement. This fixed volume of air is compressed and delivered at output as pressurized air. Hence definite quantity of compressed air is always available at the end of each stroke so these compressor are called as "positive" displacement compressors.

b) Steady flow compressors : It is the type of a non-positive displacement type of compressors. It's the rotary compressors. The air is entrapped between the impeller cavities and compressor casing in the non-positive displacement compressors. It contains the axial flow compressors and centrifugal compressors. There is no surely of delivery of air. Hence these compressors are called as non-positive displacement compressors.

❑ 2) Based on acting

On the basis of acting, the compressors are classified as follows. :

a) Single acting compressors : In this type of compressors the air is compressed in one side of compressor.

b) Double acting compressors : In this type of compressor air is compressed both side of piston.

❑ 3) Base on number of stages

On the basis of number of stages the compressors are classified as follow :

a) Single stage compressors : Air is compressed in one cylinder.

b) Multistage compressors : The air is compressed in more than one cylinder.

Reciprocating air compressor	Maximum delivery pressure ratio
Single stage	5
Two stage	35
Three stage	80

❑ 4) Based on delivery pressure

On the basis of delivery pressure, the compressors are classified as following.

a. Low pressure compressor : In that maximum pressure is upto 1.1 bar.

b. Medium pressure compressor : Maximum pressure upto 7 bar.

c. High press compressor : In this pressure is more than 7 bar.

❑ 5) Based on capacity

On the basis of capacity, the compressors are classified as follows.

a. Low capacity compressor : In this volume flow rate less than .

b. Medium capacity compressor : Volume flow rate less than .

c. High capacity compressor : Volume flow rate more than .

7.2.1 Fan, Blower and Compressor

As per ASME (American society of mechanical engineers) norms the pressure ratios to identify that the equipment it is fan or blower or compressor are given in following table :

Equipment	Pressure ratio (p_2/p_1)	Pressure rise (mm of water gauge)
Fans	upto 1.11	11.36
Blowers	1.12 to 1.20	1137 to 2066
Compressors	more than 1.20	very high

7.3 p-V and T-S Diagram for Compressors

[GTU : Summer-16, Winter-17]

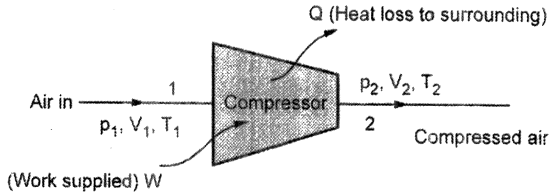


Fig. 7.2 : Block diagram of compressor

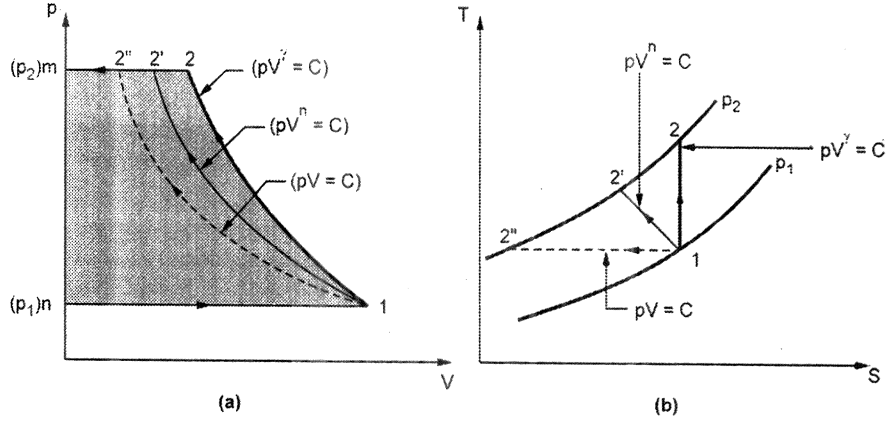


Fig. 7.3 : p-V and T-S diagram for compressor

- Fig. 7.2 shows the block diagram of compressor and Fig. 7.3 shows the p-V and T-S diagram of compressor.
- For detail explanation of Fig. 7.3 refer section 7.4.1.

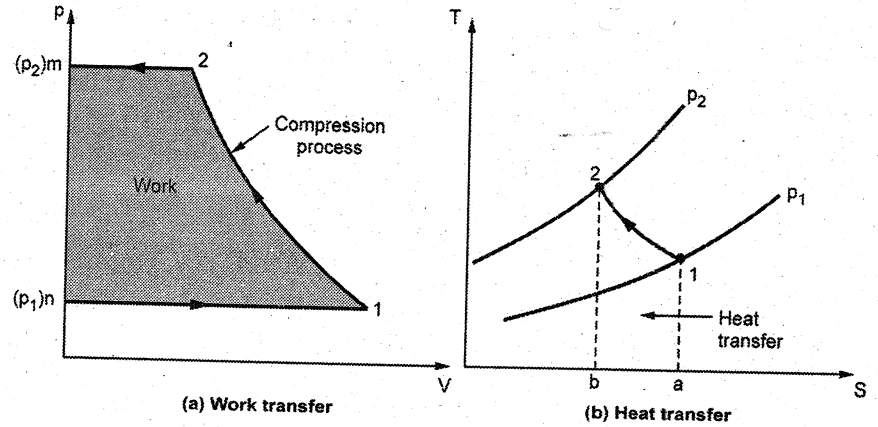


Fig. 7.4 : Work transfer and heat transferred by compressor

7.3.1 Pressure Ratio

- The pressure ratio is the important thing which is used to control the working of compressor as well as to get a good work done.
- The pressure ratio is defined as the ratio of outlet pressure to inlet pressure.
- If p_1 is the suction pressure (which is usually atmospheric pressure) and p_2 is delivery pressure then, pressure ratio becomes,

$$\text{Pressure ratio} = r_p = \frac{p_2}{p_1}$$

- Multistage compressor is used for getting the higher pressure ratio. For multi - stage compressor the condition for maximum efficiency and minimum power requirement is,

$$r_p = \frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3}$$

7.3.2 Free Air Delivered (FAD)

- Volume of air delivered by air compressor to the atmosphere at discharge pressure and atmospheric temperature is called as the free air delivered.
- It's measured in m^3/min.
- With the help of ideal gas equation (pV = mRT) and continuity equation (Mass flow rate at inlet of compressor = Mass flow rate at outlet of compressor) FAD can be calculated.

7.4 Reciprocating Compressors

[GTU : Summer-16, Winter-17]

Construction and working

- Following Fig. 7.5 shows details of single stage conventional single acting positive displacement reciprocating air compressor.

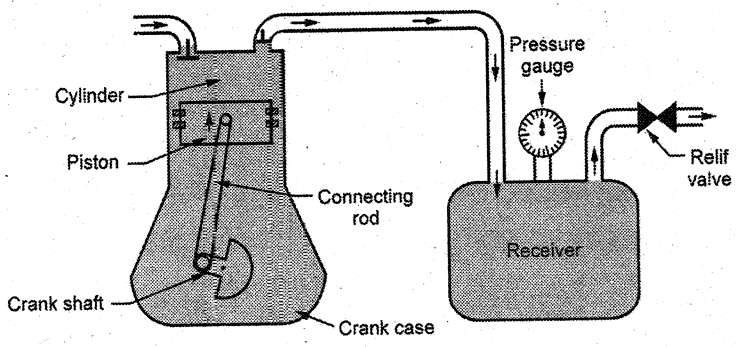


Fig. 7.5 : Conventional reciprocating compressor

- This type of compressors are also called as the positive displacement compressors, because air is positively entrapped between piston and cylinder and then compressed to high pressure and delivery and stored into storage tank.
- It consists of piston, cylinder, crank shaft, connecting rod, valves, flywheel and bearing. The low strength material can be used to manufacture reciprocating air compressors.
- The suction and discharge valves are spring loaded and they operate automatically (i.e. the valve open or close due to air pressure difference). And it's the type of slider crank mechanism. No cam shaft with cam and follower arrangement is needed to operate the valve.
- The reciprocating air compressor is power absorbing device. In that air is admitted only one side of piston. Hence this type of compressor is called single acting cylinder. And here crank shaft is driven by prime mover (usually single phase or three base AC electric motor or sometimes diesel engine.
- At high pressure delivered air is usually stored in tank/receiver from where it can be supplied continuously to desired application.
- During the compression stroke the smooth continuous discharge is obtained from storage tank.
- Air is admitted during suction stroke and pressure inside cylinder is drops when piston moves from TDC (Top Dead Center) to BDC (Bottom Dead Center).

- At high pressure entrapped air is compressed and taken out as exhaust valve opens near the end of compression stroke.
- Pressure ratio is varies in single stage. Pressure ratio for single stage reciprocating air compressor pressure ratio varies from 9 to 10 and input speed varies from 100 to 1440 r.p.m.
- Here pressure ratio is discharge air pressure to suction air pressure. And where input speed is obtained from electric motor and belt drives.

7.4.1 Thermodynamic Analysis of Reciprocating Air Compressors

1) Assumption :

Following are the assumptions are made for thermodynamic analysis of reciprocating air compressor are

a) Frictional losses are neglected.

b) Working fluid is assumed as air.

c) The clearance volume is assumed as zero.

d) There is no flow resistance during inlet and exhaust valve.

e) Leakages are neglected.

2) Working :

Following Fig. 7.6 shows the details of p-V and T-S diagrams of single acting single stage reciprocating air compressor.

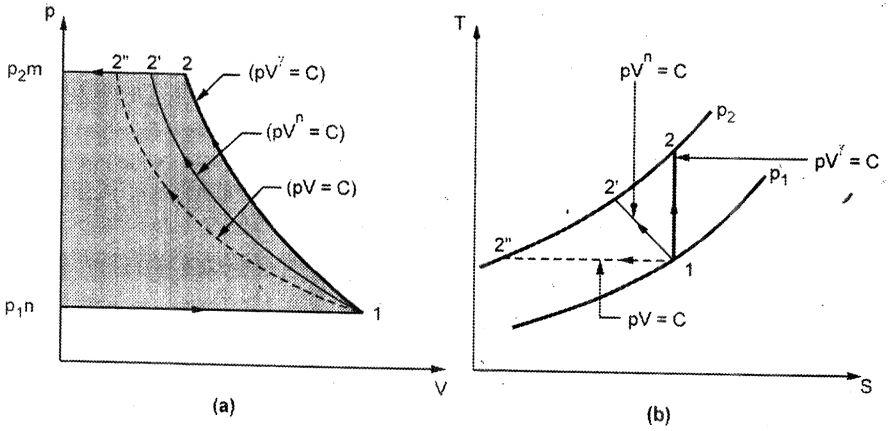


Fig. 7.6 : Ideal p-V and ideal T-S diagram for single stage reciprocating air compressor

- At first piston is at TDC and suction as well as discharge valves are closed.

Where p_1 V_1 ⇒ Pressured volume before compression.

p_2 V_2 ⇒ Pressure and volume after compression.

- Vacuum is created inside the cylinder, when piston moves from TDC to BDC. At that position suction valve opens and outside is admitted inside the cylinder.
- Here suction valve is open due to pressure difference.
- The air is drawn into the cylinder at pressure and the process 4-1 (on P-V and T-S diagrams).
- The piston is reaches BDC at point 1 and suction valve is closed. At that point piston start moving up.
- After that both the valves are closed, entrapped air is compressed (process 1-2) during compression stroke (piston moving from BDC to TDC)
- If the press of inside compressed air is more than the air pressure of receiver tank, exhaust valve opens and the pressurized air is delivered to storage tank.
- Discharge valve is open at point 2 so that air can be received into the tank.
- Process during 1-2 is reversible adiabatic process and it is denoted by law of

$$pV^\gamma = \text{Constant.}$$

- Area covered by 1-2-3-4 indicates the work absorbed by the compressor (to compress air from inlet/suction pressure p_1 to exhaust/delivery pressure)
- Process during 1-2" is isothermal compression govorned by the law of pV = constant. This indicates that temperature of working fluid remains constant throughout the cycle (which is assumption and not true in reality) process 1-2-3"-1 shows work saved.
- Polytropic process is followed in 1-2" and governed by the law of pV^n = constant.

3) Work done :

a) Reversible polytropic compression (a-b'-c-d) process :

- In Fig.7.7 the line d-a represents the suction stroke. The mass in the cylinder increases from zero at d to that required to fill the cylinder at a. In the ideal case the temperature is constant at T_1 for this process and there is no heat exchange with the surroundings.
- Line abc represent the compression and delivery stroke. It may be assumed that the general form of compression is the reversible polytropic (i.e., pV^n = Constant)
- The net work done in the cycle is given by the area of the p-V diagram and is the work done on the gas.

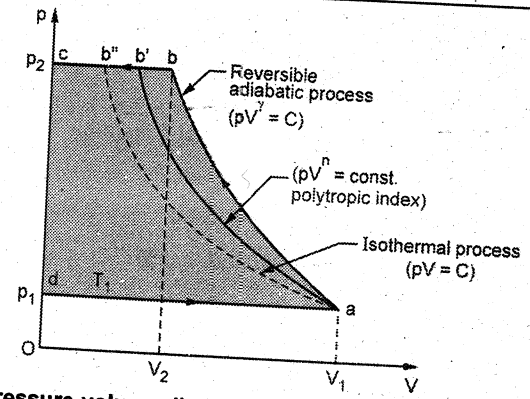


Fig. 7.7 : Pressure-volume diagram for a reciprocating compressor with clearance neglected (p-V diagram)

- Indicated work done on the gas per cycle is,

$$= \text{Area (abcd)} = \text{Area abef} + \text{Area coe} - \text{Area adf}$$

$$\text{Work input} = \frac{p_2V_b - p_1V_a}{n-1} + p_2V_b - p_1V_a = (p_2V_b - p_1V_a)\left(\frac{1}{n-1}+1\right)$$

$$\text{i.e. Work input} = (p_2V_b - p_1V_a)\left(\frac{1+n-1}{n-1}\right) = \frac{n}{n-1}(p_2V_b - p_1V_a)$$

$$p_1V_a = mRT_1 \text{ and } p_2V_b = mRT_2$$

Where m is the mass induced and delivered per cycle. Then

$$\textbf{Work input per cycle} = \frac{n}{n-1}\,mR\,(T_2 - T_1)$$

The working fluid changes state between a and b in Fig.7.8 from p_1 and T_1 to p_2 and T_2. The change being shown in Fig. 7.8 which is a diagram of properties (i.e. p against V)

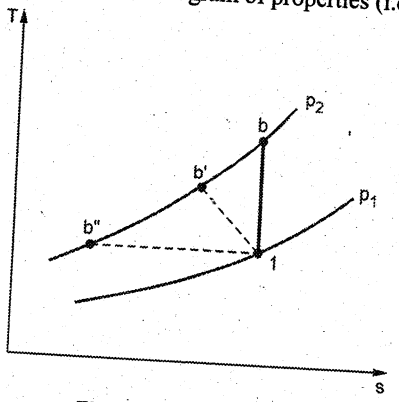


Fig. 7.8 : T-s diagram

The delivery temperature is given by the equation

i.e.
$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}}$$

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}\,R(T_2 - T_1) = \frac{n}{n-1}\dot{m}\,R\,T_1\left(\frac{T_2}{T_1} - 1\right)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}}$$

Therefore,

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}\,R\,T_1\left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

Or
$$\textbf{Indicated power} = \frac{n}{n-1}p_1V\left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

where V is the volume indicated per unit time.

b) Reversible adiabatic compression (a-b-c-d) process.

We know,

For process a-b

$$pV^\gamma = \text{Constant}$$

$$p_aV_a^\gamma = p_2V_b^\gamma$$

$$\frac{V_b}{V_a} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

We know
$$pV = mRT$$

$$\mathbf{WD_a} = \mathbf{mR\,T_1}\left[\frac{\gamma}{\gamma-1}\right]\left\{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}$$

c) Isothermal process of compression (a-b"-c-d)

The work done by isothermal compression (pV = Constant) is given by.

$$WD_a = mR\,T_1\,ln\left(\frac{p_2}{p_1}\right)$$

$$= p_1V_a\,ln\left(\frac{p_2}{p_1}\right) = p_1V_a\,ln\left(\frac{V_1}{V_2}\right)$$

❑ 4) Efficiency

a) Polytropic efficiency

It is the ratio of polytropic work supplied to the actual work supplied.

$$\eta_p = \frac{\textbf{Polytorpic work supplied}}{\textbf{Actual work supplied}}$$

b) Isothermal efficiency

It is the ratio of isothermal work done to the actual work done.

$$\eta_I = \frac{\textbf{Isothermal work done}}{\textbf{Actual work done}}$$

- Isothermal process follows $pV = C$ whereas actual process follows law $pV^n = C$

$$\eta_I = \frac{W_1}{W_P} = \frac{p_1 V_a \, ln\left(\frac{p_2}{p_1}\right)}{mR\,T_1\left(\frac{n}{n-1}\right)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}}$$

$$\eta_I = \frac{n-1}{n} \times \frac{ln\left(\frac{p_2}{p_1}\right)}{\left[\left(\frac{p_2}{p_1}\right)^{\frac{n}{n-1}} - 1\right]}$$

c) Adiabatic efficiency

It is ratio of adiabatic power to the break power.

$$\eta_a = \frac{\textbf{Adiabatic power}}{\textbf{Brake power}}$$

d) Mechanical efficiency

- Indicated power is also called as "air power" i.e., power available in the compressed air.
- In the compressors power is supplied with the help of prime move (electric motor).
- Air compressor is power absorbing device. The thing which required the output i.e., work or power is supplied as an input to compressor.
- Hence, the mechanical efficiency is defined as the ratio of indicated power to the break power.
- There is friction creaked due to that some part of brake power is lost and heat loss is also done.

- Hence, the indicated power available in the compressed air will always be less than the amount supplied by prime mover. Ratio of this available power to the supplied power is called mechanical efficiency.
- Mechanical efficiency of compressors is in the range of 70 % to 95 %.

$$\text{Mechanical efficiency} = \eta_m = \frac{\text{Indicated power}}{\text{Brake power}}$$

$$\eta_m = \frac{\textbf{IP}}{\textbf{BP}}$$

Or

$$\eta_m = \frac{\textbf{Power available in compressed air}}{\textbf{Power supplied by motor}}$$

5) Power of compressor :

- The power required to run the compressor is given by.

$$P = \frac{WD \times N}{60} \text{ watt}$$

$$\text{Adiabatic power} = \frac{WD_a \times N}{60} = \frac{WD_a \times n}{60}$$

$$\text{Isothermal power} = \frac{WD_i \times N}{60} = \frac{WD_i \times n}{60}$$

$$\text{Indicated power} = \frac{WD_p \times N}{60} = \frac{WD_p \times n}{60}$$

Where, n = N (for single acting compressor)

n = 2N (for double acting compressor)

N = Speed of compressor (r.p.m)

WD_a = Adiabatic work done

WD_i = Isothermal work done

WD_p = Polytropic work done

📖 7.5 Methods to Improve Isothermal Efficiency of Reciprocating Air Compressor

- Isothermal efficiency of air compressor is increases, the final temperature T_2 should be reduced, so that work supplied does to the work required for isothermal compression.
- At the time of compression, in the compressor the temperature of air increases.

- Area under pV diagram indicated work required by compressor to compress air from p_1 to p_2.
- Ideal compression is isothermal compression (pV = Constant) with minimum area.
- Adiabatic compression (pV^p = constant) indicated highest area.
- The polytropic compression (pV^n = constant) lies in between these two extreme valves.
- Hence, as increase isothermal efficiency the polytropic index n should be kept as close to unity (one) as possible. This is shown in following Fig. 7.9.

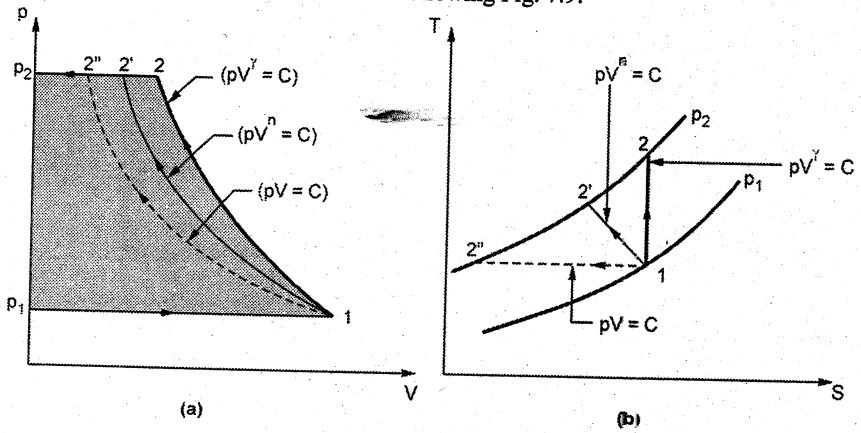


Fig. 7.9 : p-V and T-S diagram showing reduction in work to improve isothermal efficiency

- To increased the isothermal efficiency of compressor are discussed below.
- **External fins :** Effective cooling can be achieved by providing fins on external surface of cylinder. Fins increases the area of convective heat transfer.
  $(Q = h \cdot A \cdot \Delta T)$ reduces the rise in temperature of delivered air.
- **Water jacketing :** Air cooling fins is not sufficient for medium compressors. In that cases water cooling is provided. By using water jacket water is circulated around the cylinder.
- **Inter-cooling :** If the running speed of compressor is high or if the pressure ratio is large than air cooling by fins or water jacketing methods are not useful.
- This type of method is ineffective for multistage high pressure compressors.
- In that conditions in addition to air /water cooling, the delivery air from one stage is cooled in a heat exchanger and then admitted to next stage.
- This all process is use to increases the isothermal efficiency by reducing the temperature.

- **Spray injection :** This is used to reduces air temperature after compression. In this method water spray or jet is injected into compressor cylinder at the end of compression stroke.

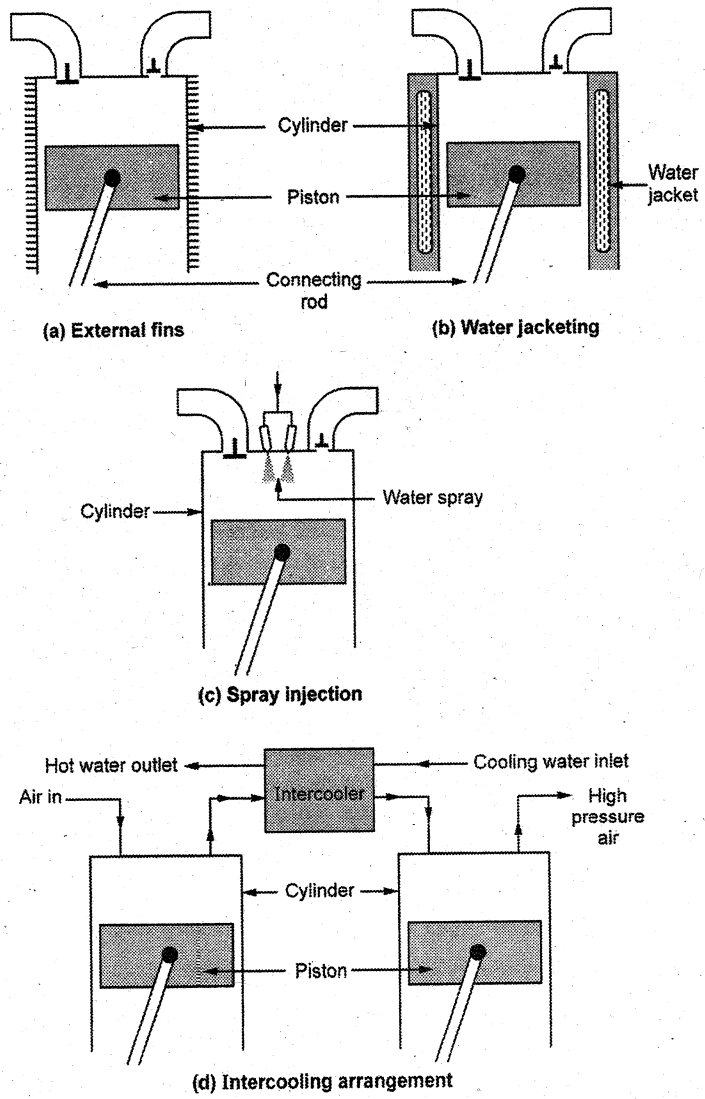


Fig. 7.10 : Methods for improving isothermal efficiency

❑ Limitations

a) Water mixed with air is required to be removed before using air in applications where dry pressurized is desired.

b) Water particles are injected into air creates corrosion problems for piston, cylinder, etc.

c) For water spray injection special gearing arrangement is needed.

7.6 Effect of Clearance Volume

- Refer Fig. 7.11 p-V diagram of single stage compressor with clearance volume (V_C).

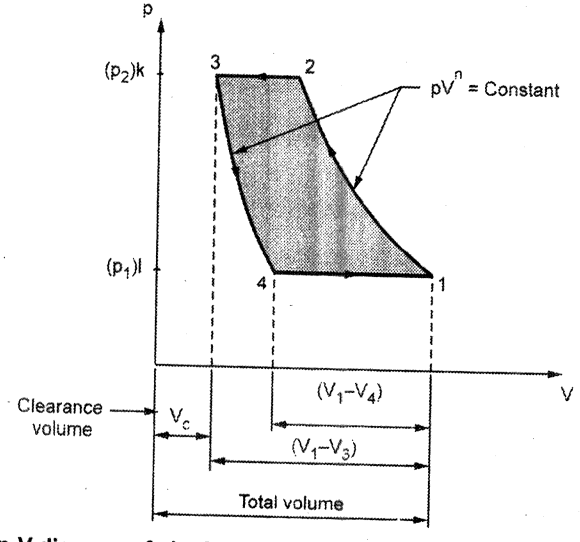


Fig. 7.11 : p-V diagram of single stage compressor with clearance volume V_C

- p-V diagram of single stage compressor with clearance volume V_C is shown in Fig. 7.11.
- We are assumed in previous analysis the zero or no clearance between piston head and cylinder.
- A small amount of clearance is madatory between piston top and cylinder head through practically so that piston will not hit or strike with cylinder head as it approaches TDC.
- For accommodate suction and discharge valves the clearance volume is most useful.
- Polytropic compression (pV^n = constant) process 1-2 and small quantity of air in clearance volume V_C expands by polytropic process (pV^n = constant) to suction pressure p_1.

- Inlet valve is opens and air is sucked into cylinder at point 4.
- Volume of air admitted, i.e., effective volume ($V_1 - V_4$) is less than the stroke volume ($V_1 - V_3$). This indicates that air handling capacity is reduced due to presence of clearance volume. Hence, V_C is kept as small as practically possible.
- By using volumetric efficiency the measure of handling capacity of compressor is defined.

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3}$$

- Volumetric efficiency is defined, as the ratio of volume of free air delivered to the displacement volume of the cylinder.
- Generally is in the range 60 % to 80 %.
- Clearance ratio 'C' is defined as the ratio of clearance volume to stroke volume and is usually kept as 5 % to 10 % of the stroke volume (C = 0.05 to 0.10).

Mathematically,

$$C = \frac{\text{Clearance volume}}{\text{Stroke volume}} = \frac{V_3}{V_1 - V_3}$$

$$C = \frac{V_C}{V_S}$$

7.6.1 Work Done of Compressor with Clearance

By using Fig. 7.11 the work done per cycle is given by

$$WD = \text{Area (1-2-3-4)} = \text{Area (1-2-k-l)} - \text{Area (3-4-l-m)}$$

$$WD = \left\{\left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]\right\} - \left\{\left(\frac{n}{n-1}\right) p_4 V_4 \left[\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right]\right\}$$

We have $p_3 = p_2$ and $p_4 = p_1$

$$\mathbf{WD} = \left(\frac{n}{n-1}\right) p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

7.6.2 Volumetric Efficiency

By using volumetric efficiency

i.e., $$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3}$$

where $V_1 - V_3$ = Stroke volume = V_S ...[From Fig. 7.11]

$$V_1 = V_S + V_C$$

where V_C = Clearance volume

$$V_{vol} = \frac{(V_s + V_C) - V_4}{V_S}$$

For process 3-4 in the Fig. 7.11

We get; $pV = n$

$$p_3 V_3^n = p_4 V_4^n$$

$$\frac{p_3}{p_2} = \left(\frac{V_4}{V_3}\right)^n \quad \because \frac{V_4}{V_3} = \left(\frac{p_3}{p_2}\right)^{1/n}$$

By using Fig. 7.11

We have, $p_3 = p_2$; $p_4 = p_1$ and $V_3 = V_C$

$$\left(\frac{V_4}{V_3}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$V_4 = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \times V_C$$

By using substitution

We get

$$\eta_{vol} = \frac{(V_1 + V_C) - \left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \times V_C\right]}{V_S}$$

$$= 1 + \frac{V_C}{V_S} - \left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \times \frac{V_C}{V_s}\right] = 1 + \frac{V_C}{V_s}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$$

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right] \qquad \because \frac{V_C}{V_S} = C \text{ (Clearance ratio)}$$

7.6.3 Factors Affecting Volumetric Efficiency of Compressor

- Fig.7.12 shows the graph of clearance ratio, pressure ratio and polytropic index n volumetric efficiency.

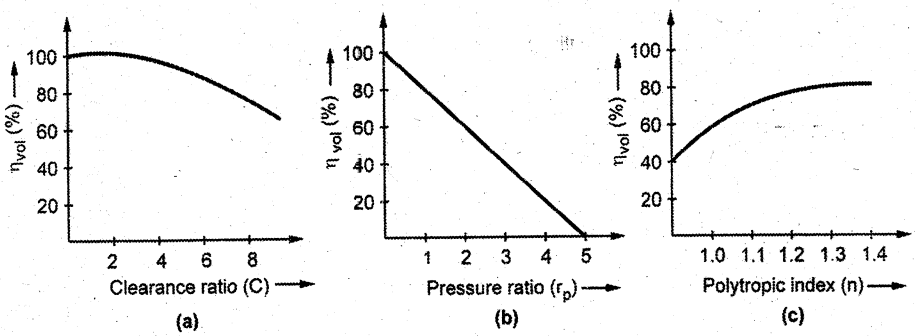


Fig. 7.12 : Effect of clearance ratio (C), pressure ratio and polytropic index (n) on volumetric efficiency

- For increases the volumetric efficiency, following points should be taken care.
  - Use fins and water cooling to control air temperature at delivery.
  - Kept small pressure ratio.
  - Also clearance volume should be kept as small as possible its around 5 % of stroke volume.
  - Polytropic index n should be kept high, say around 1.3 or more. This however will drastically reduce isothermal efficiency.
  - By using valves made up of high strength low weight material so as reduce inertia effects.
  - Use dynamic and static seals to minimize air leakages.
  - By reducing the friction, for this we are use lubrication.

7.7 Actual p-V Diagram for Reciprocating Compressor

[GTU : Summer-16, Winter-17]

- Details of actual p-V diagram is as shown in following Fig. 7.13.
- At point d inlet valve is open. However in reality the valve opens when pressure inside cylinder drops below atmospheric pressure, so that air enters in due to pressure difference.
- Valve bouncing with some vibrations this is reason of sudden opening of valve. Hence actual curve is wavy during suction process .
- The pressure difference is called intake depression.
- There polytropic process is begining beyond point b. i.e., beyond delivery line pressure and then discharge valve opens.

- Due to vibrations and inertia effects at discharge valve, discharge process b-c is wavy in nature.
- In Fig. 7.13 shows the shaded area on actual p-V diagram. It is indicated extra work required for compression.
- This is due to friction, vibrations and inertia effects the compression process requires extra work, thereby reducing overall efficiency of compressor.

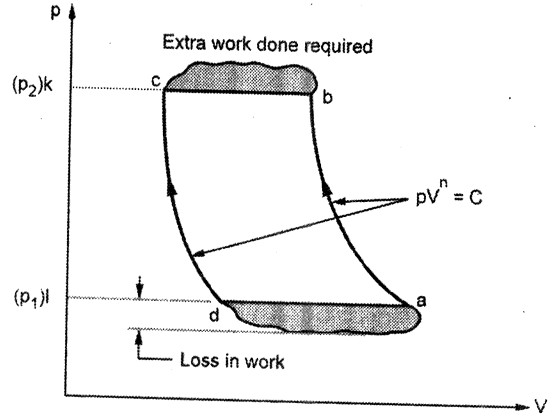


Fig. 7.13 : Actual p-V diagram

7.8 Multi-Stage Compressors

- When two or more cylinders are use to compress air one after another, then such compressors are called as multi-stage compressor.
- Generally multi-stage compressors are seen in the industrial application.
- Following Fig. 7.14 shows the block diagram of multistage compressor for two stages.

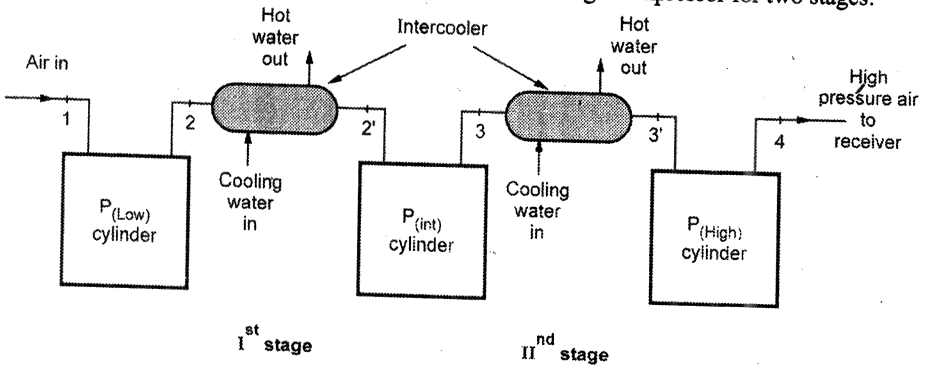


Fig. 7.14 : Block diagram of multistage compressor (two stages)

7.8.1 Need of Multistage

- Pressure ratio increases and volumetric efficiency reduces. Therefore higher pressure ratio, it's better to compress air in stages.
- If in single stage pressure ratio of air is increased to very high temperature also rises high thereby reducing isothermal efficiency.
- This is the reasons multi-stage compression is used with inter-cooling so that temperature rise is limited to small valve.
- Free Air Delivery (FAD) is one of the most important thing. It is reduces as pressure ratio increases. It is performed to have higher FAD valve and hence multi-staging is preferred.

7.8.2 Advantages of Multi-Stage Compressors

- For getting T - θ diagram is in the form of more uniform by using smaller flywheel.
- It get higher volumetric and higher isothermal efficiency by using inter-cooling system work saving is there and due to this increasing efficiency.
- Discharge temperature is low and compressed air can be directly used without further cooling.
- Good cylinder design with compact arrangement to get better efficiency.
- Less leakage problems due to reduced pressure difference.

7.8.3 p-V and T-S Diagram for Multi-Stage Compressor

[GTU : Winter-15, Summer-16]

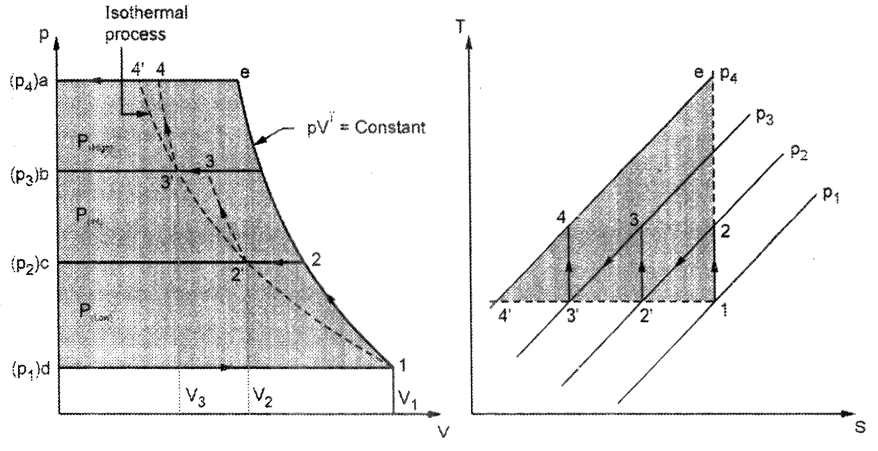


Fig. 7.15 : p-V and T-S diagram for three stage compressor

The following Fig. 7.15 shows the details of p-V and T-S diagram of three stage compressor.

7.8.4 Work Done for Multi-Stage Compressor

- The Fig.7.15 shows the details block diagram of three stage compressor containing LP (Low Pressure) cylinder stage 1, IP (Intermediate Pressure) cylinder of stage 2 and HP (High Pressure) cylinder of stage 3 (which is final stage.)
- Work done in total compression from LP and HP (by neglecting clearance)

i.e.,

$$WD = [WD \text{ in LP cylinder}] + [WD \text{ in IP cylinder}] + [WD \text{ in HP cylinder}]$$

$$WD = \left[p_1 V_1 \left(\frac{n}{n-1}\right)\left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right)\right] + \left[p_2 V_2' \left(\frac{n}{n-1}\right)\left(\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right)\right]$$
$$+ \left[p_3 V_3' \left(\frac{n}{n-1}\right)\left(\left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} - 1\right)\right]$$

For perfect cooling

$$P_1 V_1 = p_2 V_2' = p_3 V_3' = mRT_1$$

$$WD = mRT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} + \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} - 3\right]$$

$$WD = mRT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

$$WD = mRT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} + \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} + \left(\frac{p_{x+1}}{p_x}\right)^{\frac{n-1}{n}} - X\right]$$

where, X = 'n' number of stages

7.8.5 Condition for Maximum Efficiency

- The following Fig.7.16 shows the details of p-V diagram for two stage compressor.
- If there is suction and discharge pressure are firmed then the intermediated pressure required for minimum work can be determined as follows.
- Consider

$$\frac{n-1}{n} = Z$$

Work done can be written as,

$$WD = \frac{1}{Z} \times mRT_1 \times \left[\left(\frac{p_2}{p_1}\right)^Z + \left(\frac{p_3}{p_2}\right)^Z - 2\right]$$

Differentiating the above equation with respect to and equate it to zero

We get, $\frac{d}{dp_2}(WD) = 0$

$$\frac{1}{Z} \times mRT_1 \left[(Z \times (p_2)^{Z-1} \times (p_1)^{-Z}) + (-Z) \times (p_3)^Z \times (p_2)^{-Z-1}\right] = 0$$

$$\frac{(p_2)^{Z-1}}{(p_2)^{-Z-1}} = \frac{(p_3)^Z}{(p_1)^{-Z}}$$

$$(p_2)^2 = (p_1 \times p_3)$$

By removing 'Z' from both sides

$$(p_2)^2 = p_1 \cdot p_3$$

- By taking square root

$$p_2 = \sqrt{p_1 \cdot p_3}$$

- If multi-stage compressors having 'x' stage with perfect intercooling then for minimum work and maximum efficiency of compressor

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \ldots + \frac{p(x+1)}{p_x} = Y \text{ (contant)}$$

$$Y = \sqrt{\frac{p(x+1)}{p_1}}$$

- **The minimum work of compression is given by (For two stage compressor)**

$$W_{win} = 2\left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

- **For three stage compression, the condition for maximum efficiency is**

$$p_2 = \sqrt{p_1 \cdot p_3} \text{ and } p_3 = \sqrt{p_2 \cdot p_4}$$

$$\therefore \quad W_{min} = X\left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

where, X = number of stages

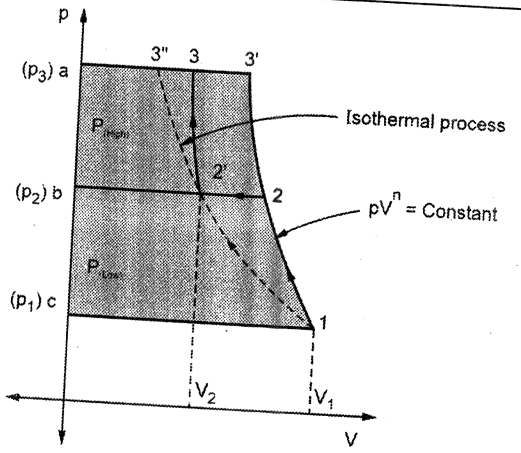


Fig. 7.16 : Two stage compression

7.8.6 Intercooling and After Cooling

☞ **[GTU : Winter-16, Summer-17]**

- Compressed air from High Pressure (HP) Cylinder in a multi-stage compressor is stored in a receiver tank. The stored air is then used as per the application needs.
- The temperature of air coming out of HP Cylinder is high due to polytropic compression. This temperature needs to be reduced.
- A heat exchanger called after-cooler is used between HP cylinder discharge and receiver tank inlet so as to reduce the temperature of pressurized air.
- Typical aftercoolers used are counter-flow heat exchangers with hot air as working medium and water as cooling medium. The Fig. 7.17 shows the block diagram of aftercooler.
- The compressed air is cooled to atmospheric temperature using after-cooler and then stored into the receiver tank.

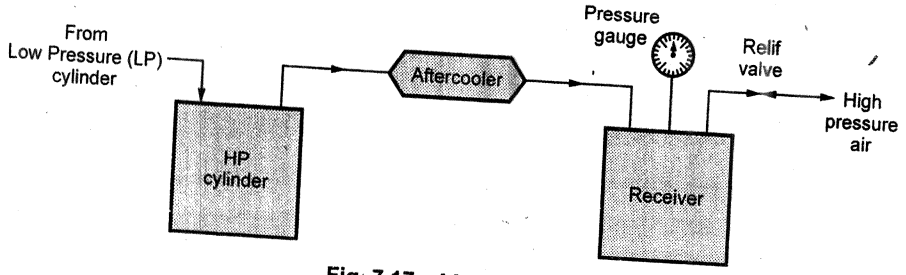


Fig. 7.17 : After cooler

❑ Advantages of After-coolers :

- Increased isothermal efficiency
- Reduction is size of receiver tank needed
- Reduction in specific volume of compressed air to be stored in receiver

❑ Disadvantages of After-coolers :

- Additional cost overhead
- Additional maintenance requirements
- Slight reduction in delivery air pressure (due to drop in heat exchanger, i.e. aftercooler)

❑ Applications of After-coolers :

- In multi-stage high capacity compressors (with 3 or more stages)
- In applications where pressurized air is needed at atmospheric temperature only

7.9 Capacity Control of Air Compressors

- Depending on the requirement of compressors should be run.
- In daily industrial usage, compressors are not run to their full rated capacity continuously at all the time.
- According to application demands, control arrangement is needed in order to control the amount of air supplied.
- The most commonly used capacity control methods for air compressors are described below.

7.9.1 Clearance Control Method

- If the volumetric efficiency increases, the clearance ratio decreases.
- If the receiver tank pressure exceeds the desire value the clearance pockets are used for controlling.
- The clearance is controlled by providing air packets adjacent to the cylinder.
- As the clearance ratio decreases the mechanical efficiency of compressor increases.

7.9.2 Throttle Control Method

- Opening of inlet valve is controlled by the built - up pressure of the air storage tank or receiver. The following Fig.7.18 shows the arrangement of throttle control.

- Desired valve, suction valve is partly close when receiver pressure exceeds. So the quantity of air succeed by the compressor is reduced.
- Limitations of this method is increase air temperature at delivery and reduced volumetric efficiency.
- If demand exceeds supply and receiver tank pressure falls below the desired value then reverse action take place.

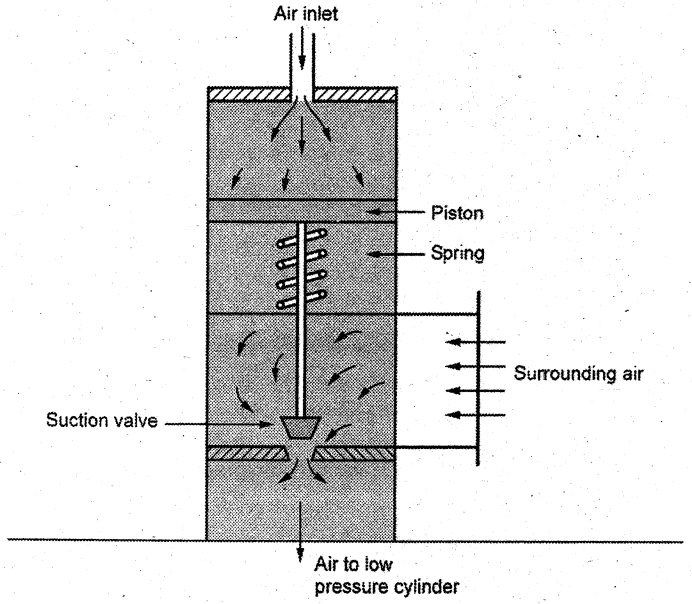


Fig. 7.18 : Throttle control arrangement

7.9.3 Blow-Off Control Method

- Following Fig. 7.19 shows details of Blow off control method, in HP cylinder, a by - pass valve is used.
- The by - pass valve is closed when the receiver pressure drops, so that all the compressed air is sent to storage tank.
- When receiver pressure exceeds set valve, by - pass valve released the compressed air instead of sending it to receiver tank.
- By loss of compressed air with same power comsumption use limitations of this method.

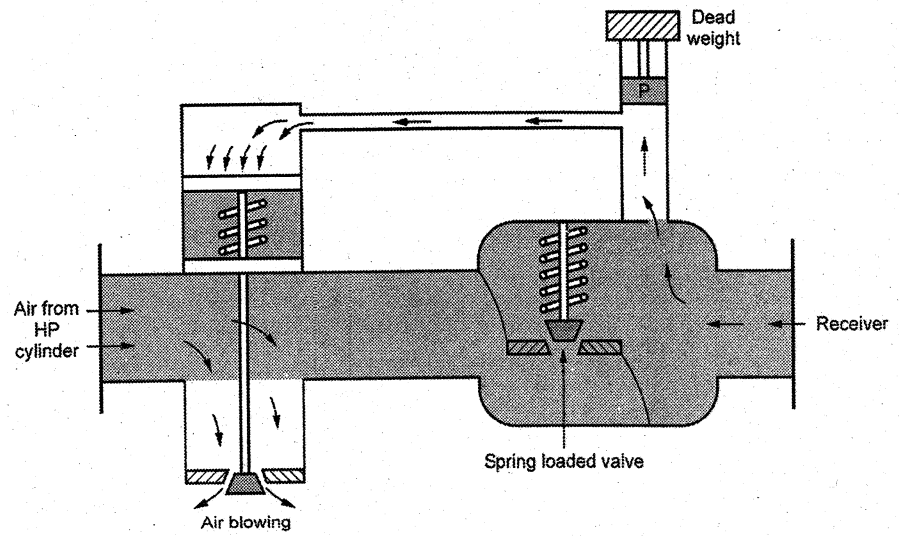


Fig. 7.19 : Blow-off control method

7.10 Solved Examples

Ex. 7.1 : A single cylinder, single acting air compressor delivers 10 kg of air per minute. The air is compressed from 1 bar and 27 deg. C to 6 bars. The compression process follows the law $pV^{1.26} = C$. Find :

i) Work done,

ii) Brake power required if mechanical efficiency is 80 %.

Sol. : Given data

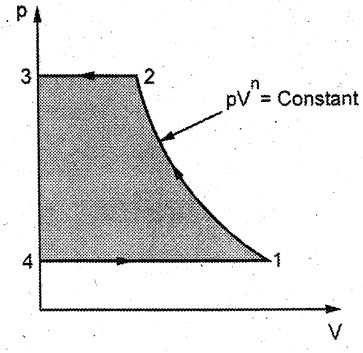
$$\dot{m} = 10 \text{ kg/min}$$

$$p_1 = 1 \times 10^5 \text{ N/m}^2$$

$$T_1 = 27 + 273 = 300 \text{ K}$$

$$p_3 = p_2 = 6 \times 10^5 \text{ N/m}^2$$

$$pV^{1.25} = C$$



To find :

i) Work done ii) B.P if $\eta_{mech} = 80\ \%$

i) Work done

$$WD = \dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$WD = \frac{10}{60} \times 0.287 \times 300 \left(\frac{1.25}{1.25-1}\right)\left[\left(\frac{6}{1}\right)^{\frac{0.25}{1.25}} - 1\right]$$

$$\mathbf{WD} = 31 \text{ kJ/sec} = \mathbf{31 \text{ kW}} \quad \text{... Ans.}$$

ii) B.P

$$\mathbf{B.P} = \frac{W.D.}{\eta_{mech}} = \frac{31}{0.8} = \mathbf{38.75 \text{ kW}} \quad \text{... Ans.}$$

Ex. 7.2 : A single stage reciprocating compressor takes 1 m^3 of air per minute at 1.013 bar at 15 °C and delivers it at 7 bar according to law $PV^{1.35}$ = Constant and the clearance is negligible. Calculate :

i) Mass of the air delivered per minute ii) Delivery temperature iii) Indicated power.

Take individual gas constant R = 287 J/kgK. **[GTU : Summer-16, Winter-17]**

Sol. :

$$\dot{V}_1 = 1 \text{ m}^3/\text{min}, \quad p_1 = 1.013 \times 10^5 \text{ N/m}^2$$

$$T_1 = 15 + 273 = 288 \text{ K}$$

$$p_2 = 7 \times 10^5 \text{ N/m}^2, \quad pV^{1.35} = C$$

i) Mass of air delivered per minute

$$p_1 \dot{V}_1 = \dot{m} R T_1 \quad \therefore \dot{m} = \frac{p_1 \dot{V}_1}{R T_1} = \frac{1.013 \times 10^5 \times 1}{287 \times 288}$$

$$\dot{\mathbf{m}} = \mathbf{1.225 \text{ kg/min}} \quad \text{... Ans.}$$

ii) Delivery temperature (T_2)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 288\left(\frac{7}{1.013}\right)^{\frac{0.35}{1.35}}$$

$$\mathbf{T_2} = \mathbf{475.37 \text{ K}} \quad \text{... Ans.}$$

iii) Indicated power (IP)

$$IP = p_1 V_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$IP = 1.013 \times 10^5 \times 1 \left(\frac{1.35}{1.35-1}\right)\left[\left(\frac{7}{1}\right)^{\frac{0.35}{1.35}} - 1\right]$$

$$\therefore \quad \mathbf{IP} = \mathbf{4.272 \text{ K}} \quad \text{... Ans.}$$

Ex. 7.3 : A single stage single acting reciprocating compressor has air entering at 1 bar, 20 °C and compression occurs following polytropic compression with index 1.2 upto delivery pressure of 12 bar. The compressor runs at speed of 240 r.p.m. and has L/D ratio of 1.8. The compressor has mechanical efficiency of 0.88. Determine the isothermal efficiency and cylinder dimensions. Also find out the rating of drive required to run the compressor which admits 1 m^3 of air per minute. **[GTU : Summer-15, Winter-16]**

Sol. : Given data :

Type : Single stage

$p_1 = 1$ bar, $T_1 = 20 + 273 = 293$ K, $N = 1.2$, $p_2 = 12$ bar, $N = 240$ rpm, $L/D = 1.8$,

$\eta_{mech} = 0.88$, $V_1 = 1 \text{ m}^3/\text{min}$.

To find

i) η_{iso} ii) Cylinder dimensions

iii) Actual power supplied

i) To calculate isothermal efficiency (η_{iso})

$$W_{iso} = p_1 V_1 \log_e [p_2 / p_1] = 1 \times 10^5 \times \log\left[\frac{12}{1}\right]$$

$$W_{iso} = 4.14 \text{ kW}$$

$$W_{poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{1.2}{1.2-1} \times 1 \times 10^5 \times 1 \left[\left(\frac{12}{1}\right)^{\frac{1.2-1}{1.2}} - 1\right]$$

$$W_{poly} = 5.130 \text{ kW}$$

$$\eta_{iso} = \frac{W_{iso}}{W_{poly}} = \frac{4.14}{5.130} = 0.807 = \mathbf{80.70 \%} \quad \text{... Ans.}$$

ii) Cylinder Dimensions

$$V_s = \frac{\pi D^2}{4} \times L_s \times N \quad \therefore \quad 1 = \frac{\pi D^2}{4} \times 1.8 D \times 240$$

$$\therefore \quad \mathbf{D} = \mathbf{0.1433 \text{ m}}$$

$$\mathbf{L_s} = 1.8 \times D = 1.8 \times 0.1433 = \mathbf{0.258 \text{ m}} \quad \text{... Ans.}$$

iii) Actual power supplied

$$\eta_{mech} = \frac{W_{poly}}{W_{actual}} \quad \therefore 0.88 = \frac{5.13}{W_{actual}}$$

$$\mathbf{W_{actual}} = \frac{5.13}{0.88} = \mathbf{5.829 \text{ kW}} \quad \text{... Ans.}$$

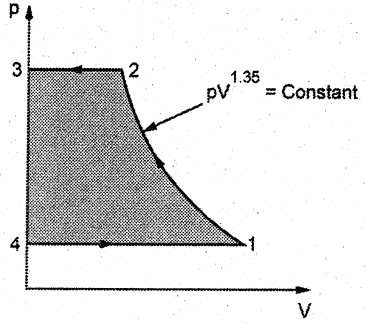
Ex. 7.4 : A single stage reciprocating compressor takes 1m^3 of air per minute at 1.013 bar at 15 °C and delivers it at 7 bar according to law $pV^{1.35}$ = Constant. Calculate power required for compression neglecting clearance.

Sol. : Given data

$V_1 = 1\ m^3/min$, $p_1 = 1.013 \times 10^5\ N/m^2$,

$T_1 = 15 + 273 = 288\ K$, $p_2 = 7 \times 10^5\ N/m^2$ $pV^{1.35} = C$,

$n = 1.35$



To find : i) Power

i) To calculate power required (P)

$$P = p_1 V_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 1.013 \times 10^5 \times 1\left[\frac{1.35}{1.35-1}\right]\left[\left(\frac{7}{1}\right)^{\frac{1.35-1}{1.35}} - 1\right]$$

$$\mathbf{P = 4.272\ kW}$$ **... Ans.**

Ex. 7.5 : Determine the size of the cylinders for a single acting single stage compressor consuming 35 kW. Also calculate mean effective pressure. Intake conditions are 1 bar and 15 °C and polytropic index is 1.3 speed is 100 rpm and mean piston speed is 152 m/min, delivery pressure is 6 bar. Also calculate isothermal power. Neglect clearance.

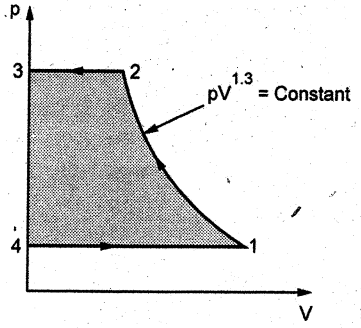
Soln. : Given data

$W = 35\ kW = 35 \times 10^3\ W$,

$p_1 = 1\ bar = 1 \times 10^5\ Pa$

$T_1 = 15°C = 15 + 273 = 288\ K$,

$N = 100\ rpm$



Mean piston speed, $2LN = 152$ m/min,

$p_2 = 6\ bar = 6 \times 10^5\ Pa$, $n = 1.3$

✦ Step 1 : Calculate the size of cylinder

It is given that, $2LN = 152$

$\therefore$ Stroke, $L_s = \frac{152}{2N} = \frac{152}{2 \times 100} = \mathbf{0.76\ m}$ **... Ans.**

$$W_{poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\therefore \quad 35 \times 10^3 = \left(\frac{1.3}{1.3-1}\right) \times 1 \times 10^5 \times V_1 \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$\therefore \quad V_1 = 0.1577\ m^3/sec$$

But $$V_1 = \frac{\pi D^2}{4} \times L_s \times N$$

$$0.1557 = \frac{\pi D^2}{4} \times 0.76 \times \frac{100}{60}$$

$$\therefore \quad \mathbf{D = 0.3972\ m}$$ **... Ans.**

Workdone/stroke,

$$\dot{W} = \frac{W}{\text{No. of strokes/sec}} = \frac{35 \times 10^3}{\left(\frac{100}{60}\right)} = 21000\ \text{Nm/stroke}$$

Stroke volume,

$$\dot{V}_s = \frac{\pi}{4} D^2 \times L_s = \frac{\pi}{4} \times (0.3972)^2 \times 0.76 = \mathbf{0.09417\ m^3/stroke}$$

Mean effective pressure,

$$p_m = \frac{\dot{W}}{\dot{V}_s}$$

$$\therefore \quad p_m = \frac{21000}{0.09417} = 2.23 \times 10^5\ Pa = \mathbf{2.23}$$ **Ans.**

Ex. 7.6 : A single acting, single stage reciprocating air compressor takes air at 1 bar and 27 °C and delivers it at 8 bar. The volume of air entering into compressor is 5 m^3/min. The compression index is 1.3. Calculate isothermal efficiency, Power required to drive compressor.

Sol : Given data :

$p_1 = 1\ bar$, $T_1 = 27 + 273 = 300\ K$, $p_2 = 8\ bar$,

$V_1 = 5\ m^3/min$, $n = 1.3$

To find

i) Isothermal efficiency ii) Power

i) To calculate isothermal efficiency (η_{iso})

$$\eta_{iso} = \frac{W_{iso}}{W_{actual}} \quad \therefore \quad W_{iso} = p_1 V_1 \log_e \left[\frac{p_2}{p_1}\right]$$

$$W_{iso} = 1 \times 10^5 \times 5 \times \log_e \left[\frac{8}{1}\right] = 17.32 \text{ kW}$$

$$\eta_{iso} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1.3}{1.3-1} \times 1 \times 10^5 \times 5 \left[\left(\frac{8}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$W_{actual} = 22.24 \text{ kW}$$

$$\eta_{iso} = \frac{W_{iso}}{W_{actual}} = \frac{17.32}{22.24} = 0.778 = \mathbf{77.8\ \%} \quad \text{... Ans.}$$

ii) Power

Power required **P = WD_{act} = 22.24 kW** ... Ans.

Ex. 7.7 : A single stage compressor with double acting draws in 17 m^3/min of air at 1 bar and 27 °C. The pressure and temperature at the end of suction are 0.98 bar and 40 °C. Delivery pressure is maintained at 6.5 bar compression to follow the law $pV^{1.35}$ = Constant. Calculate V_s of compressor.

Sol. : Given data :

Type : Single stage double acting

$V_0 = 17 \text{ m}^3/\text{min}$, $p_0 = 1 \times 10^5 \text{ N/m}^2$, $T = 27 + 273 = 300$ K, $p_1 = 0.98$ bar,

$T_1 = 313$ K, $p_2 = 6.5$ bar, $pV^{1.35} = C$

To find : i) V_s

According to ideal gas equation $\frac{p_0 V_0}{T_0} = \frac{p_1 V_1}{T_1}$

$$V_1 = \frac{p_0 V_0}{T_0} \times \frac{T_1}{p_1}$$

$$= \frac{1 \times 10^5 \times 17}{300} \times \frac{313}{0.98 \times 10^5}$$

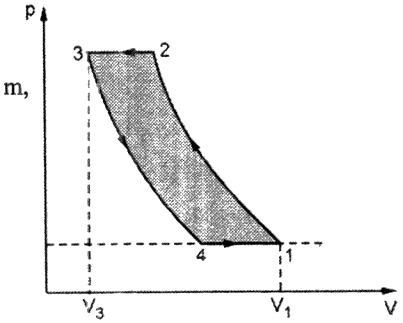
$$V_1 = 18.098 \text{ m}^3/\text{min}$$

Stroke volume $V_s = \frac{V_1}{2N} = \frac{18.098}{2N} = \mathbf{\frac{9.049}{N}\ \frac{m^3}{min}}$ ... Ans.

Ex. 7.8 : A single stage acting compressor delivers 0.6 kg of air per minute at 6 bar. The temperature and pressure at the end of suction are 30 °C and 1bar respectively. The bore and stroke of compressor are 100 mm and 150 mm respectively. The clearance is 3 % of swept volume. Assuming the index of compression and expansion to be 1.3 Find i) volume (Volumetric efficiency)
ii) Power required if mechanical efficiency is 85 %.
iii) Speed of compressor. **[GTU : Winter-16, Summer-17]**

Sol. : Given data

Type : Single stage single acting



$\dot{m} = 0.6$ kg/min, $p_2 = 6$ bar $= 6 \times 10^5 \text{ N/m}^2$,

$T_1 = 30 + 273 = 303$ K, $p_1 = 1 \times 10^5 \text{ N/m}^2$, D = 0.1 m,

$L_s = 0.15$ m, C = 0.03, n = 1.3

To find :

i) η_v ii) 'p' if $\eta_{mech} = 85\ \%$ iii) N

i) To calculate volumetric efficiency

$$\eta_{vol} = 1 - \left[\left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right)\right]$$

$$= 1 - 0.03\left[\left(\frac{6 \times 10^5}{1 \times 10^5}\right) - 1\right]$$

$$\eta_{vol} = \mathbf{0.85 = 85\ \%} \quad \text{... Ans.}$$

ii) Power required to drive the compressor (BP)

$$\eta_{mech} = \frac{IP}{BP} \text{ or } BP = \frac{IP}{\eta_{mech}}$$

$$IP = \dot{m} RT \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{0.6}{60} \times (2.087) \times 303 \left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$\mathbf{IP = 1.92\ kW} \quad \text{... Ans.}$$

$$\mathbf{BP} = \frac{IP}{\eta_{mech}} = \frac{1.9297}{0.85} = \mathbf{2.27\ kW} \quad \text{... Ans.}$$

iii) Compressor speed (N)

$$p_1 V_a = \dot{m} RT_1 \quad \therefore \quad p_1(V_1 - V_4) = \dot{m} RT_1$$

$$(V_1 - V_4) = \frac{\dot{m}RT_1}{p_1} = \frac{0.6 \times 287 \times 303}{1 \times 10^5}$$

$$= 0.5217 \text{ m}^3/\text{min}$$

$$\eta_V = \frac{V_a}{V_s} = \frac{\dot{V}_1 - \dot{V}_4}{\dot{V}_s} \quad \therefore \dot{V}_s = \frac{V_1 - V_4}{\eta_V} = \frac{0.5218}{0.919}$$

$$\dot{V}_s = 0.5678 \text{ m}^3/\text{min}$$

$$\dot{V}_s = V_s \times N = \frac{\pi}{4} D^2 \times L_s \times N$$

$$0.5678 = 0.15 \times N \times \frac{\pi \times (0.1)^2}{4}$$

$$\mathbf{N = 481.96 \text{ rpm}} \quad \text{... Ans.}$$

Ex. 7.9 : A single stage reciprocating air compressor takes in 7.5 m^3/min of air at 1 bar and 30°C and delivers it at 5 bar. The clearance is 5% of the stroke. The expansion and compression are polytropic, n = 1.3.Calculate :

(i) The temperature of delivered air

(ii) Volumetric efficiency

(iii) Power of the compressor

Sol. : Given :

$V_1 = 7.5 \text{ m}^3/\text{min}$, $p_1 = 1$ bar,

$T_i = 30 \text{ °C} = 303 \text{ K}$, $P_2 = 5$ bar

$V_c = 0.05 V_S$, $n = 1.3$

To find : 1) T_2 2) η_{vol} 3) P

1) To find T_2

From process 1 → 2

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} - 1)$$

$$T_2 = 1.449 \times 303$$

$$\mathbf{T_2 = 439.281 \text{ K}} \quad \text{... Ans.}$$

2) To find η_{vol}

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right] = 1 - 0.05\left[\left(\frac{5}{1}\right)^{\frac{1}{1.3}} - 1\right]$$

$$\eta_{vol} = \mathbf{87.75 \%} \quad \text{... Ans.}$$

$$W_{actual} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right] = \frac{1.3}{1.3-1} \frac{1 \times 10^5 \times 7 \left[\left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]}{1000 \times 60}$$

$$\mathbf{W_{actual} = 22.738 \text{ kW}} \quad \text{... Ans.}$$

$$\mathbf{P = W_{actual} = 22.738 \text{ kW}} \quad \text{... Ans.}$$

Ex. 7.10 : A single stage, single acting reciprocating air compressor delivers 0.7 kg of air per min at 6 bar. The suction, temperature and pressure are 25 °C and 1 bar. The bore and stroke of the compressor are 100 mm and 150 mm respectively. The clearance is 3 % of swept volume. Assuming index of compression and expansion to be 1.3. Find :

i) Volumetric efficiency of the compressor.

ii) Power supplied to drive the compressor if mechanical efficiency is 85 %

iii) Speed of the compressor (R.P.M.). **[GTU : Summer-15, Winter-16]**

Sol. : Given data

Type : Single stage single acting

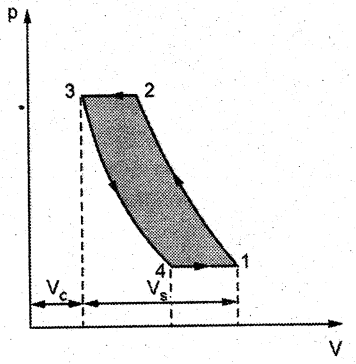
$\dot{m} = 0.7$ kg/min, $p_1 = p_a = 1 \times 10^5 \text{ N/m}^2$,

$P_2 = p_3 = 6 \text{ bar} = 6 \times 10^5 \text{ N/m}^2$, $T_1 = 25 + 273 = 298$ K,

$D = 0.1$ m, $L_s = 0.15$ m, $V_C = 0.03 V_S$, $n = 1.3$

To find

i) η_{vol} ii) BP if $\eta_{mech} = 85$ %, iii) N



i) To calculate η_{vol} = ?

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

$$\eta_{vol} = 1 - 0.03\left[\left(\frac{6}{1}\right)^{\frac{1}{1.3}} - 1\right]$$

$$= 0.92 = \mathbf{92 \%}$$

ii) Power required to drive compressor (BP)

$$\eta_{mech} = \frac{IP}{BP}$$

$$IP = \dot{m} RT \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$IP = \frac{0.7}{60} \times 2.087 \times 298 \left(\frac{1.3}{1.3-1}\right)\left[(6)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$\mathbf{IP = 2.21\ kW}$$

$$B.P = \frac{IP}{\eta_{mech}} = \frac{2.21}{0.85} = \mathbf{2.6\ kW}$$ **... Ans.**

iii) Speed of compressor (N)

$$V_s = \frac{\pi D^2}{4} \times L_s = \frac{\pi \times (0.1)^2}{4} \times 0.15$$

$$V_s = 1.178 \times 10^{-3}\ m^3$$

$$p_1 V_a = \dot{m} R T_1 \quad \therefore \quad p_1(V_1 - V_4) = 0.7 \times 287 \times 298$$

$$(V_1 - V_4) = \frac{0.7 \times 287 \times 298}{1 \times 10^5} = 0.598\ m^3/min$$

$$\therefore \quad \eta_V = \frac{V_a}{\dot{V}_s} \quad \because \quad \dot{V}_s = \frac{V_a}{\eta_V} = \frac{0.598}{0.92} = 0.65\ m^3/min$$

$$\dot{V}_s = V_s \times N$$

$$N = \frac{\dot{V}_s}{V_s} = \frac{0.65}{1.178 \times 10^{-3}}$$

$$\mathbf{N = 551.78\ rpm}$$ **... Ans.**

Ex. 7.11 : A single stage reciprocating air compressor has swept volume of 2000 m^3 runs at 800 rpm and takes in air 1.013 bar and 15 °C. It operates on a pressure ratio of 8, with clearance of 5 % of the swept volume. Assume polytropic compression and expansion with n = 1.25. Calculate a) Volumetric efficiency, b) Indicate power, c) Isothermal efficiency, d) Actual power needed to drive the compressor, if the mechanical efficiency is 85 %.

Sol. : Given Data :

$$V_s = 2000\ cm^3,\ N = 800\ rpm,\ p_1 = 1.013 \times 10^5\ N/m^2$$

$$T_1 = 15 + 273 = 288\ K,\ r_p = \frac{p_2}{p_1} = 8,\ C = \frac{V_c}{V_s} = 5\ \%$$

$$n = 1.25,\ V_s = V_1 - V_3,\ V_s = V_1 - 0.05,\ V_1 = 1.05\ V_s$$

To find : a) η_{vol} b) IP c) η_{iso} d) BP

a) Volumetric efficiency (η_{vol})

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right] = 1 - 0.05\left[\left(\frac{8}{1}\right)^{\frac{1}{1.25}} - 1\right]$$

$$\mathbf{\eta_{vol} = 78.60\ \%}$$ **... Ans.**

b) Indicated power (IP)

$$IP = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \times \frac{N}{60}$$

$$IP = \frac{1.25}{1.25-1} \times 1.013 \times 10^5 \times 1.05 \times \frac{2000}{10^6}\left[\left(\frac{8}{1}\right)^{\frac{0.25}{1.25}} - 1\right] \times \frac{800}{60}$$

$$\mathbf{IP = 7.313\ kW}$$ **... Ans.**

c) η_{iso}

$$W_{iso} = p_1 V_1 \log_e\left[\frac{p_2}{p_1}\right] \times \frac{N}{60}$$

$$W_{iso} = \left[1.013 \times 1.05 \times \frac{2000}{10^6} \times \log_e(8/1)\right] \times \frac{800}{60}$$

$$\mathbf{W_{iso} = 5.89\ kW}$$

$$\eta_{iso} = \frac{W_{iso}}{IP} = \frac{5.89}{7.313} = \mathbf{80.54\ \%}$$ **... Ans.**

d) BP

$$BP = \frac{IP}{\eta_{mech}} = \frac{7.313}{0.85}$$

$$\mathbf{BP = 8.6035\ kW}$$ **... Ans.**

Ex. 7.12 : A single stage single acting reciprocating compressor sucks air at 1 bar and 30 °C and delivers 0.6 kg of air per minute at 6 bar. The clearance volume is 3% of stroke volume. Taking the index for compression and expansion as 1.3, calculate :

i) The volumetric efficiency of the compressor,

ii) Indicated power.

iii) Power required if the mechanical efficiency is 85%.

Sol. : Given data

$$n = N,\quad p_1 = 1 \times 10^5\ N/m^2,\quad T_1 = 30 + 273 = 303\ K$$

$$\dot{m} = 0.6\ kg/min,\quad p_2 = 6 \times 10^5\ N/m^2,\quad C = 0.03,\quad n = 1.3$$

i) The volumetric efficiency of the compressor (η_{vol})

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

$$\eta_{vol} = 1 - 0.03\left[\left(\frac{6}{1}\right)^{\frac{1}{1.3}} - 1\right]$$

$$\eta_{vol} = \mathbf{91.09\ \%} \qquad \textbf{... Ans.}$$

ii) Indicated Power (IP)

$$IP = \dot{m}\, RT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$IP = \frac{0.6}{60} \times 2.087 \times 303 \left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{6}{1}\right)^{\frac{0.3}{1.3}} - 1\right] \qquad \textbf{... Ans.}$$

$$\mathbf{IP = 1.929\ kW}$$

iii) Power required BP

$$\eta_m = \frac{IP}{BP} \qquad \mathbf{BP} = \frac{IP}{\eta_m} = \frac{1.929}{0.85} = \mathbf{2.269\ kW} \qquad \textbf{... Ans.}$$

Ex. 7.13 : A single stage, single acting air compressor has a bore of 200 mm and stroke 300 mm. It runs at 420 r.p.m. and has a clearance ratio of 0.065. The index of compression can be assumed as 1.3. The intake pressure is 1 bar and intake temperature is 27 °C. Delivery pressure is 500 kPa. Determine :

i) FAD at NTP in m^3/hr ii) Volumetric efficienc

iii) Delivery air temperature iv) Power required and

v) Isothermal efficiency. **[GTU : Summer-15, Winter-16]**

Sol. : Given data

Type : Single stage single acting

$D = 0.2$ m, $L_s = 0.3$ m, $N = 420$ r.p.m, $C = \frac{V_C}{\eta_s} = 0.065$,

$n = 1.3$, $p_1 = 1 \times 10^5$ N/m^2, $T_1 = 27 + 273 = 300$ K,

$p_2 = 5 \times 10^5$ N / m^2, $p_0 = 1.013 \times 10^5$ N/m^2.

Determine

i) FAD at NTP in

ii) η_{vol} iii) T_2 iv) p v) η_{iso}

i) To calculate F.A.D at NTP in m^3/hr

$$\frac{p_0 V_0}{T_0} = \frac{p_1 V_a}{T_1} \quad \text{and} \quad V_a = \eta_{vol} \times \dot{V}_s$$

But, $$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

$$= 1 - 0.065\left[\left(\frac{5}{1}\right)^{\frac{1}{1.3}} - 1\right] = \mathbf{84\ \%}$$

$$\dot{V}_s = \frac{\pi D^2}{4} \times N \times L_s = \frac{\pi \times (0.2)^3}{4} \times 420 \times 0.3$$

$$\dot{V}_s = \mathbf{3.95\ m^3/min}$$

$$V_a = \eta_{vol} \times \dot{V}_s = 0.84 \times 3.95$$

$$= \mathbf{3.318\ m^3/min}$$

$$V_0 = \frac{p_1 V_a}{T_1} \times \frac{T_0}{p_0} = \frac{1 \times 10^5 \times 3.318}{300} \times \frac{273}{1.013 \times 10^5}$$

$$V_0 = 2.98\ m^3/min \qquad \mathbf{V_0 = 179.83\ m^3/hr} \qquad \textbf{... Ans.}$$

iii) Delivery air temperature (T_2)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \because T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300\left(\frac{5}{1}\right)^{\frac{0.3}{1.3}} = \mathbf{434.93\ K} \textbf{ ... Ans.}$$

iv) Power required (P) (BP)

$$P = \frac{n}{(n-1)}\, p_1 V_a \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$P = \frac{1.2}{(1.3-1)} \times 1 \times 10^5 \times 3.318\left[\left(\frac{5}{1}\right)^{\frac{0.3}{1.3}} - 1\right] = \mathbf{10.67\ kW} \qquad \textbf{... Ans.}$$

v) Isothermal efficiency

$$\eta_{iso} = \frac{W_{iso}}{\eta_{actual}} \qquad \therefore W_{iso} = p_1 V_a \log_e\left[\frac{p_2}{p_1}\right]$$

$$W_{iso} = 1 \times 10^5 \times \left(\frac{3.318}{60}\right) \times \log_e\left[\frac{5}{1}\right] = 8.9\ kW$$

$$\eta_{iso} = \frac{8.9}{10.67} = \mathbf{83.41\ \%} \qquad \textbf{... Ans.}$$

Ex. 7.14 : A single cylinder, double acting air compressor sucks in air at the rate of 5 m^3/min at a pressure of 100 kPa and 25 °C. It delivers air to the receiver at a pressure of 6 bar. The speed of the compressor is 250 rpm and stroke is 1.5 times cylinder diameter. Neglect the effect of clearance and if law of compression is $pV^{1.28} = C$, find :

i) Cylinder dimensions

ii) Indicated power of the compressor

iii) The shaft power if the mechanical efficiency is 90 %.

Sol. : Given data :

$$\dot{V}_a = 5 \text{ m}^3/\text{min}, \quad p_1 = 1 \times 10^5 \text{ N/m}^2$$

$$T_1 = 25 + 273 = 298 \text{ K}, \quad p_2 = 6 \times 10^5 \text{ N/m}^2$$

$$\eta_{mech} = 90\ \%, \quad N = 250 \text{ rpm}, \quad L = 1.5\ D$$

i) Cylinder dimensions

$$V_S = \frac{\pi}{4} D^2 \times \frac{L\,(2N)}{60} \quad \text{... (double acting)}$$

Assume $V_S = 1 \text{ m}^3/\text{min}$

$$1 = \frac{\pi}{4} D^2 \times 1.5\ D \times \frac{250\,(2)}{60}$$

$\therefore$ **D = 0.46 m** **... Ans.**

$$L = 1.5\ D = 1.5 \times 0.46$$

L = 0.69 m **... Ans.**

ii) Indicated power of compressor

$$\text{I.P.} = \left(\frac{n}{n-1}\right) p_1 \dot{V}_a \left[\left(\frac{p_2}{p}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\text{IP} = \left(\frac{1.28}{1.28-1}\right) \times 1 \times 10^5 \times \frac{5}{60} \left[\left(\frac{6 \times 10^5}{1 \times 10^5}\right)^{\frac{1.28-1}{1.28}} - 1\right] = 18280.37 \text{ W}$$

$\therefore$ **I.P. = 18.28 kW** **... Ans.**

iii) The shaft power (B.P.) :

$$\eta_{mech} = \frac{\text{I.P.}}{\text{B.P.}}$$

$$0.9 = \frac{18.28}{\text{B.P.}}$$

$\therefore$ **B.P. = 20.31 kW** **... Ans.**

Ex. 7.15 : A single stage, single acting compressor delivers air from 1 bar to 7 bar. Assuming compression follow the law $pV^{1.35}$ = Constant and clearance 5 % of the swept volume, find volumetric efficiency of a compressor.

Sol. : Given data

$p_1 = 1 \times 10^5 \text{ N/m}^2$, $p_2 = 7 \times 10^5 \text{ N/m}^2$, $C = 0.05$

To find : η_{vol}

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right]$$

$$= 1 - 0.05\left[\left(\frac{7}{1}\right)^{\frac{1}{1.35}} - 1\right]$$

η_{vol} = **0.8396 or 83.96 %** **... Ans.**

Ex. 7.16 : The following data relate to a performance test of a single acting reciprocating acting compressor :
Suction pressure - 1 bar ; Discharge pressure - 6 bar
Suction temperature - 20 °C ;
Discharge temperature - 180 °C
Speed of compressor - 1200 r.p.m. ; Shaft power - 6.25 kW.
Mass of air delivered - 1.7 kg/min ; D = 14 cm and L = 10 cm.
Calculate :
Volumetric efficiency , Indicated power,
Isothermal efficiency , Mechanical efficiency. **[GTU : Summer-14, Winter-15]**

Sol. : Given data :

$$P_1 = 1 \times 10^5 \text{ N/m}^2, \quad p_2 = 6 \times 10^5 \text{ N/m}^2.$$

$$T_1 = 20 + 273 = 293 \text{ K},$$

$$T_2 = 180 + 273 = 453 \text{ K}, \quad N = 1200 \text{ rpm}, \quad \text{B.P} = 6.25 \text{ kW},$$

$$\dot{m}_a = 1.7 \text{ kg/min} = \frac{1.7}{60} = 0.0283 \text{ kg/sec}$$

To find

i) η_{vol} ii) IP iii) η_{iso} iv) η_{mech}

i) To calculate η_{vol}

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} \quad \text{and} \quad \dot{V}_s = V_s \times \frac{N}{60}$$

But $$V_s = \frac{\pi D^2}{4} \times L_s$$

$$V_s = \frac{\pi (0.14)^2}{4} \times 0.1 = 1.539 \times 10^{-3} \text{ m}^3/\text{sec.}$$

$$\dot{V}_s = 1.539 \times 10^{-3} \times \frac{1200}{60} = 0.0307 \text{ m}^3/\text{sec.}$$

We have,

$$p_1 \dot{V}_a = \dot{m} R T_1 \quad \therefore \dot{m} = \frac{p_1 \dot{V}_a}{R T_1} = \frac{1 \times 10^5 \times 0.0307}{287 \times 293}$$

$$\dot{m} = \mathbf{0.0366 \text{ kg/sec}}$$

$$\eta_{vol} = \frac{\dot{m}_a}{\dot{m}} = \frac{0.0283}{0.0366} = \mathbf{77.32\ \%} \qquad \text{... Ans.}$$

ii) Indicated power (P)

$$Q = m \times c \times \Delta t$$

or $$P = \dot{m} \times C_p \times [T_2 - T_1] = 0.0366 \times 1.005 \times (453-293)$$

$$\mathbf{P = 5.8 \text{ kW}}$$

iii) Isothermal efficiency (η_{iso}) ... Ans.

$$W_{iso} = \dot{m} R T_1 \log_e \left[\frac{p_2}{p_1}\right]$$

$$= 0.0366 \times 0.287 \times 293 \times \log_e \left[\frac{6}{1}\right]$$

$$\mathbf{W_{iso} = 5.514 \text{ kW}}$$

$$\eta_{iso} = \frac{W_{iso}}{B.P.} = \frac{5.5.14}{6.25} = \mathbf{88.22\ \%} \qquad \text{... Ans.}$$

Ex. 7.17 : For a single stage single acting reciprocating air compressor, actual volume of air taken in is 10 m^3/min. Initial intake pressure is 1.013 bar and initial temperature is 27 °C. Final pressure is 900 kPa, clearance is 6 % of stroke. Compressor runs at 400 r.p.m. Assume : L/D = 1.25 and index of compression = 1.3

Determine :

1) Volumetric efficiency 2) Cylinder dimensions

3) Indicated power

Sol. : Given data

Type : single stage single acting

$V_a = 10^3 \text{ m}^3/\text{min}$, $p_1 = 1.013 \times 10^5 \text{ N/m}^2$

$T_1 = 27 + 273 = 300$ K, $p_2 = 9 \times 10^5 \text{ N/m}^2$, C = 0.06,

N = 400 rpm, L/D = 1.25, n = 1.3

To find : i) η_{vol} ii) Cylinder dimensions iii) Indicated power (IP)

i) Volumetric efficiency (η_{vol})

$$\eta_{vol} = 1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} - 1\right] = 1 - 0.06\left[\left(\frac{9}{1.013}\right)^{\frac{1}{1.35}} - 1\right]$$

$$\eta_{vol} = \mathbf{73.8\ \%}$$

ii) Cylinder dimensions ... Ans.

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s}$$

$$\dot{V}_s = \frac{\dot{V}_a}{\eta_{vol}} = \frac{10}{0.738} = 13.88 \text{ m}^3/\text{min}$$

$$\dot{V}_s = \frac{\pi D^2}{4} \times L_s \times N \quad \therefore 13.55 = \frac{\pi \times D^2}{4} \times 1.25\, D \times 400$$

$$\mathbf{D = 0.32 \text{ m}}$$

and $$L = 1.25 \times D = 1.25 \times 0.325 = \mathbf{0.406 \text{ m}} \qquad \text{... Ans.}$$

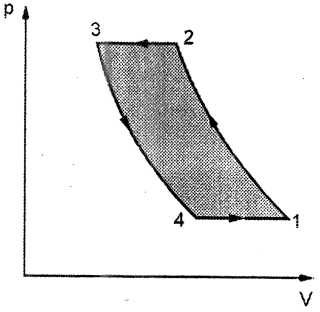
iv) Indicated power (IP)

$$IP = \frac{n}{(n-1)} p_1 V_a \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$IP = \left(\frac{1.3}{1.3-1}\right) \times 1 \times 10^5 \times \frac{10}{60}\left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] = \mathbf{47.69 \text{ kW}} \qquad \text{... Ans.}$$

Ex. 7.18 : A reciprocating compressor of single stage, double acting type delivers 20 m^3/min when measured at free air condition of 1 bar, 27 °C. The compressor has pressure ratio of 7 and the conditions at the end of suction are 0.97 bar, 35 °C. Compressor runs at 240 r.p.m. with clearance volume of 5 % of swept volume. The L / D ratio is 1.2. Determine the volumetric efficiency and dimensions of cylinder and isothermal efficiency taking the index of compression and expansion as 1.25. Also show the cycle on p-V diagram.

Sol. : Given data



$\dot{V}_a = 20\ m^3/min$, $\dot{V}_a = \frac{20}{60} = 0.34\ m^3/sec.$,

$p_0 = 1 \times 10^5\ N/m^2$,

$T_1 = 27 + 273 = 300\ K$, $r_p = 7$, $p_1 = 0.97 \times 10^5\ N/m^2$,

$T_1 = 308\ K$, $C = 0.05$, $= 1.2$, $n = 1.25$

To find : i) η_{vol} ii) L and D

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \therefore\ T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 308\ (7)^{\frac{0.25}{1.25}} = \mathbf{454.53\ K}$$

$$p_1 V_a = mRT_1$$

$$m = \frac{p_1 V_a}{RT_1} = \frac{1 \times 10^5 \times 20}{287 \times 300} = \mathbf{23.23\ kg/min}$$

$$\frac{p_3}{p_4} = \left(\frac{V_4}{V_3}\right)^n$$

$$\therefore\ V_4 = V_4\left(\frac{p_3}{p_4}\right)^{\frac{1}{n}}$$

$$V_4 = 0.05\ V_s\ (7)^{\frac{1}{1.25}} = 0.273\ V_s$$

$$V_1 - V_4 = 1.05\ V_s - 0.237\ V_s = 0.813\ V_s$$

$$= \frac{0.97 \times 300}{1(308)} \times (V_1 - V_4)$$

$$= \frac{0.97\ (300)}{1 \times 308}\ (0.813\ V_s)$$

$$= 0.768\ V_s$$

$$\text{Volumetric efficiency} = \frac{V_1 - V_4}{V_4} = \frac{0.768\ V_s}{V_s} = 0.768$$

$$= \mathbf{76.8\ \%}$$

Dimension of cylinder = Volume of air saved in a cycle

$$= \frac{V_1}{2 \times N \times 2} = \frac{20}{2 \times 240 \times 2}$$

$$= \mathbf{0.0208\ m^3/cycle}$$

$$0.0208 = \frac{\pi D^2}{4} \times L_s = \frac{\pi D^2}{4} \times 1.2\ D$$

$$\mathbf{D = 0.2805\ m\ or\ 28.05\ cm}$$

and $$\mathbf{L_s = 1.2\ D = 33.67\ cm}$$ **... Ans.**

Reciprocating compressor work can be determined,

$$W = \frac{n}{n-1} mR\ (T_2 - T_1)$$

$$W = \left(\frac{1.25}{1.25 - 1}\right) 23.38\ (0.287)\ [454.54 - 308]$$

$$\mathbf{W = 4884.92\ kJ/min}$$ **... Ans.**

$$W_{iso} = mR\ T_1 \log_e\left[\frac{p_2}{p_1}\right]$$

$$= 23.23 \times 0.287 \times 308 \times \log_e [7]$$

$$\mathbf{W_{iso} = 3995.81\ kJ/min}$$ **... Ans.**

$$\eta_{iso} = \frac{W_{iso}}{W} = \frac{3995.8}{4884.92} = \mathbf{81.33\ \%}$$

Ex. 7.19 : During an experiment on reciprocating air compressor the following observations are being taken Barometer reading = 75.6 cm of Hg, Manometer reading across orifice = 13 cm of Hg Atmospheric temperature = 25 °C ; Diameter of orifice = 15 cm; Cofficient of discharge across the orifice = 0.65 ; Density of Hg = 0.0135951 kg/cm^3 Determine the volume of free air handled by compressor in m^3/min.

Sol. : Given data

Barometer reading = 75.6 cm of Hg

Manometer reading = 13 cm of Hg

$T_1 = 25 + 273 = 298\ K$, $d = 15\ cm$, $C_d = 0.65$,

$\rho_{Hg} = 0.0135951\ kg/cm^3$

To find : F.A.D

$$p_{ambient} = \gamma_{Hg} \times h_{Hg} = 13.6 \times 9810 \times 0.756 \qquad \ldots (\therefore p = \gamma h)$$

$$= 100.83 \times 10^3\ Pa$$

$$p_{amb} = 100.83\ kPa$$

$$pV_a = mRT \quad \therefore V_a = \frac{RT}{P} = \frac{1 \times 0.287\,(298)}{100.83}$$

$$= 0.848\ m^3/kg$$

$$\rho_a = \frac{1}{V_a} = 1.18\ kg/m^3$$

Difference of pressure in orifice

$$p_0 = \gamma_{Hg} \times h_{Hg} = 13.6 \times 9810 \times 0.13$$

$$= 17.35 \times 10^3\ Pa$$

$$p_0 = \mathbf{17.35\ kPa}$$

$$h_a = \frac{p_0}{\rho_a \times g} = \frac{17.35}{1.18 \times 9.81} = \mathbf{1497.95\ m} \qquad \ldots (\therefore \gamma = \rho g)$$

$$FAD = C_d \times A \times \sqrt{2gh_a}$$

$$= 0.65 \times 1.77 \times 10^{-4} \sqrt{2 \times 9.81 \times 1797.95}$$

$$\mathbf{FAD = 0.01972\ m^3/\ seec.\ or\ 1.183\ m^3/min} \qquad \ldots \textbf{Ans.}$$

Ex. 7.20 : A reciprocating compressor of single stage and double acting type is running at 200 r.p.m. with mechanical efficiency of 85 %. Air flows into compresssor at the rate of 5 m^3/min measured at atmospheric condition of 1.02 bar, 27 °C. Compressor has compressed air leaving at 8 bar with compression following polytropic process with index of 1.3. Compressor has clearance volume of 5 % of stroke volume. During suction of air from atmosphere into compressor its temperature rises by 10 °C. There ocurrs pressure loss of 0.03 bar during suction and pressure loss of 0.05 bar during discharge passage through valves. Determine the dimensions of cylinder, volumetric efficiency and power input required to drive the compressor if stroke to bore ratio is 1.5. **[GTU : Summer-16, Winter-17]**

Sol. :

Type : Single stage double auting

N = 200 rpm, η_{mech} = 85 %, V_a = 5 m^3 / min, T_a = 300 K,

p_a = 1.02 bar, p_2 = 8 + 0.05 = 8.05 bar,

p_1 = 1.02 – 0.03 = 0.99 bar, C = 0.05

To find : Cylinder dimensions, η_{vol} and Pi

$$\frac{p_1 V_1}{T_1} = \frac{p_a V_a}{T_a} \quad \therefore V_1 = \frac{p_a V_a T_1}{p_1 T_a} = \frac{1.02 \times 310 \times 5}{0.99 \times 300}$$

$$V_1 = 5.23\ m^3/min$$

$$\therefore \quad W_{poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$\eta_{vol} = \left(\frac{1.3}{1.3 - 1} \right) \frac{0.99 \times 10^5 \times 5.25}{60} \left[\left(\frac{8.05}{0.99} \right)^{\frac{1.3}{1.3}} - 1 \right]$$

$$\mathbf{W_{poly} = 23.66\ kW} \qquad \ldots \textbf{Ans.}$$

$$B.P = \frac{W_{poly}}{\eta_{mech}} = \frac{23.66}{0.85} = \mathbf{27.83\ kW} \qquad \ldots \textbf{Ans.}$$

$$\text{Volumetric efficiency } \eta_{vol} = \frac{p_1 T_a}{p_a T_1} \left[1 + C - C \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \right]$$

$$\eta_{vol} = \frac{0.99 \times 300}{1.02 \times 310} \left[1 + 0.05 - 0.05 \left(\frac{8.05}{0.99} \right)^{\frac{1}{13}} \right]$$

$$\mathbf{\eta_{vol} = 75.8\ \%} \qquad \ldots \textbf{Ans.}$$

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} \qquad \dot{V}_s = \frac{0.0125}{0.75}$$

$$\dot{V}_s = 0.0167\ m^3 / cycle$$

$$\dot{V}_s = \frac{\pi D^2}{4} \times L_s = 0.0167\ m^3$$

$$0.0167 = \frac{\pi \times D^2}{4} \times 1.5\ D$$

$$\therefore \mathbf{D = 0.2420\ m} \qquad \ldots \textbf{Ans.}$$

$$L = 1.5 \times D = 1.5 \times 0.2420 = \mathbf{0.363\ m = 36.3\ cm} \qquad \ldots \textbf{Ans.}$$

Ex. 7.21 : A two stage single reciprocating air compressor, takes in air at 1 bar and 20°. It discharges air at 40 bar. The temperature rise in inter-cooler is 22°C. Assume polytropic compression and expansion with n = 1.33, perfect inter-cooling with ideal intermediate pressure and calculate a) Work of compression, b) % saving in work of compression as compared to single stage, c) Mass of cooling water required in the inter-cooler per kg of air. Assume C_p = 1.005 kJ/kg for air and 4.187 kJ/kg for water.

Sol. : Given Data :

$x = 2$, $p_1 = 1 \times 10^5\ N/m^2$,

$T_2 = 22 + 273 = 295\ K$, $T_1 = 20 + 273 = 293\ K$,

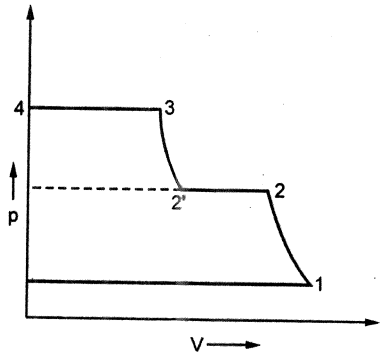
$p_3 = 40 \times 10^5\ N/m^2$, $n = 1.33$

To find :

i) Work of compression

2) % saving in work compression

3) Mass of cooling water/kg



C_{pa} = 1.005 kJ/kg,

C_{pw} = 4.187 kJ/kg

$p_2 = \sqrt{p_1 \times p_3}$

$= \sqrt{1 \times 40}$ = 6.324 bar

1) Work of compression

$$W_c = 2\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \times (\dot{m}\, RT_1)$$

$$= (2 \times 1 \times 0.287 \times 293)\left(\frac{1.33}{1.33-1}\right)\left[\left(\frac{6.324}{1}\right)^{\frac{1.33-1}{1.33}} - 1\right]$$

$$= (0.5803 \times 4.03 \times 84.091)$$

W_c = **393.32 kJ/kg** **... Ans.**

2) % of saving in work compare to single stage

$$W = mRT_1\left[\frac{n}{n-1}\right]\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 1 \times 0.287 \times 293\left[\frac{1.33}{1.33-1}\right]\left[\left(\frac{40}{1}\right)^{\frac{0.33}{1.33}} - 1\right]$$

W = 507.47 kJ/kg **... Ans.**

$$\% \text{ in work saving} = \left(\frac{W - W_c}{W}\right) \times 100 = \frac{507.47 - 393.32}{507.47} \times 100$$

% in work saving = 22.5 %

3) Mass of cooling water required

$$m_W = \frac{\dot{m}_a\, CP_a\, (T_2 - T_1)}{CP_2\, (\Delta T)_W}$$

$$= \frac{1 \times 1.005 \times (295 - 293)}{4.187 \times 2}$$

m_W = **0.24 kg/kg of air** **... Ans.**

Ex. 7.22 : A two-stage reciprocating air compressor takes in air at 1 bar and 27 °C. Air is delivered at 10 bar. The intermediate pressure is deal and intercooling is perfect. The law for compression is $pV^{1.3} = C$. The rate of discharge is 0.1 kg/s. Find :

i) Power required to drive the compressor

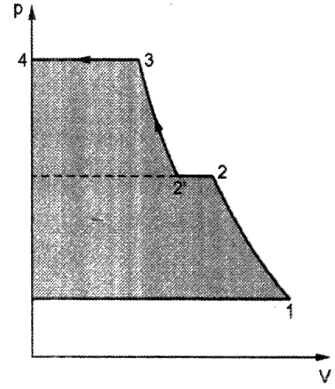
ii) Saving in work compared to single stage

iii) Isothermal efficiency for multistage

iv) Heat rejected in intercooler.

Take R = 0.287 kJ/kgK and C_p = 1 kJ/kgK.

Sol. : Given data



Type : Multistage compressor

$p_1 = 1 \times 10^5$ N/m^2, $T_1 = 27 + 273 = 300$ K,

$p_3 = 10 \times 10^5$ N / m^2, n = 1.3, $\dot{m}$ = 0.1 kg/s

Intermediate pressure p_2

$p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 10}$

p_2 = 3.1623 bar

i) Power required to drive the compressor (P)

$$P = x \cdot \dot{m}\, RT_1 \frac{n}{(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$P = 2 \times 0.1 \times 0.287 \times 300\left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{3.162}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

P = 22.78 kW **... Ans.**

ii) Saving in work compared to single stage

For single stage

$$P_1 = x \cdot \dot{m}\, RT\left(\frac{n}{n-1}\right)\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$P_1 = 1 \times (0.1) \times (0.287) \times 300\left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{10}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

P_1 = 26.16 kW

Saving in work = $P_1 - P = 26.16 - 22.7$ = **3.46 kW** **... Ans.**

iii) Isothermal efficiency for multistage

$$\eta_{iso} = \frac{P_1}{P} = \frac{19.82}{22.7} = \mathbf{87.31\ \%} \qquad \textbf{... Ans.}$$

iv) Heat rejected in intercooler (Q)

$$Q = \dot{m} \times C_p \times (T_2 - T_1)$$

But, $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300\,(3.162)^{\frac{0.3}{1.3}} = 391.3\ K$$

$$Q = 0.1 \times 1 \times (391.3 - 300) = \mathbf{9.13\ kW} \qquad \textbf{... Ans.}$$

Ex. 7.23 : A two stage reciprocating air compressor takes in air at 1 bar and 300 K. Air is delivered at 15 bar. The intermediate pressure is ideal and intercooling is perfect. The law for compression is $PV^{1.25} = C$. The rate of discharge is 10 kg/min. Find : i) Power required to drive the compressor ii) Saving in work compared to single stage iii) Heat rejected in intercooler **[GTU : Summer-15, Winter-16]**

Sol. : Given data

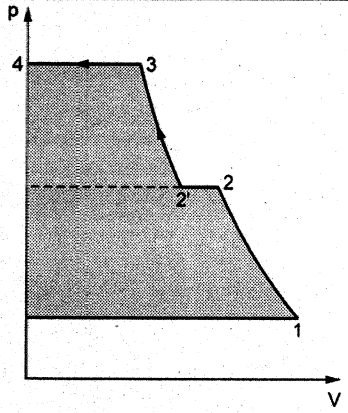
$x = 2$

$p_1 = 1 \times 10^5\ N/m^2$, $T_1 = 300\ K$,

$p_3 = 10 \times 10^5\ N/m^2$, $n = 1.25$,

$\dot{m} = \frac{10}{60} = 0.167\ kg/sec$

$$p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 15} = 3.87\ bar$$



i) Power required to drive the compressor (P)

$$P = x \cdot \dot{m} RT_1 \frac{n}{(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2 \times 0.167 \times 0.287 \times 300 \left(\frac{1.25}{1.25-1}\right)\left[\left(\frac{3.87}{1}\right)^{\frac{1.25-1}{1.25}} - 1\right]$$

$$\mathbf{P = 44.69\ kW} \qquad \textbf{...Ans.}$$

ii) Saving in work compared to single stage

For single stage

$$P_1 = x\,\dot{m}\,RT\left(\frac{n}{n-1}\right)\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 1 \times (0.167) \times (0.287) \times 300 \left(\frac{1.25}{1.25-1}\right)\left[\left(\frac{15}{1}\right)^{\frac{1.25-1}{1.25}} - 1\right]$$

$$\mathbf{P_1 = 51.67\ kW}$$

Saving in work $= P_1 - P = 51.76 - 44.69$

$\therefore$ **Saving in work = 6.98 kW** **...Ans.**

iii) Heat rejected in intercooler (Q)

$$Q = \dot{m} \times C_p \times (T_2 - T_1)$$

But, $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 300\left[\frac{3.87}{1}\right]^{\left(\frac{1.25-1}{1.25}\right)}$$

$$T_2 = 393.25\ K$$

$\therefore$ $$Q = 0.167 \times 1 \times (393.25 - 300)$$

$\therefore$ $$\mathbf{Q = 15.57\ kW} \qquad \textbf{...Ans.}$$

Ex. 7.24 : A two stage air compressor with perfect intercooling takes in air at 1 bar and 27°C. The law of compression in both the stages is $pV^{1.3}$ = Constant. The compressed air is delivered at 9 bar from the H.P. cylinder to an air receiver. Calculate, per kg of air, a) Minimum work of compression, b) Heat rejected in intercooler, c) Work required for single stage compression to the same delivery pressure.

Sol. :

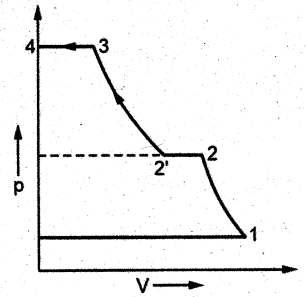
Given : $x = 2$, $p_1 = 1$ bar, $p_3 = 9$ bar, $T_1 = 27°C = 27 + 273 = 300$ k, $n = 1.3$

Take $R = 0.287$ kJ/kg and $C_p = 1$ kJ/kg k

To find : 1) Min. work of compression

2) H.R. in intercooler

3) Work required for single stage compression



Intermediate pressure p_2,

$$p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 9} = 3\ bar$$

1) Min work of compression :

$$P = x \cdot (mRT_1) \times \frac{n}{(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2\,[1 \times 0.287 \times 3000]\,\frac{1.3}{(1.3-1)}\left[\left(\frac{3}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$P = 215.324 \text{ kJ/kg}$$ ... Ans.

2) H.R. in intercooler – (Q) :

$$Q = \dot{m} \times (C_p) \times [T_2 - T_1]$$

$$T_2 = T_1 \left[\frac{p_2}{p_1}\right]^{\frac{n-1}{n}} = 300 \left[\frac{3}{1}\right]^{\frac{0.3}{1.3}}$$

$$T_2 = 386.56 \text{ k}$$

$$Q = 1 \times 1 \times [286.56 - 300]$$

$$Q = 86.56 \text{ kJ/kg}$$ ... Ans.

3) Work required for single stage compression :

$$P' = mRT_1 \frac{n}{(n-1)} \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right] = 1 \times 0.287 \times 300 \frac{1.3}{(1.3-1)} \left[\left(\frac{9}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$

$$P' = 246.391 \text{ kJ/kg}$$ ... Ans.

Ex. 7.25 : A two stage double acting reciprocating air compressor running at 200 rpm has air entering at 1 bar 25 °C. The low pressure stage discharges air at optimum intercooling pressure into intercooler after which it enter, at 2.9 bar 25 °C into high pressure stage. Compressed air leaves HP stages at 9 bar. The LP cylinder and HP cylinder have some stroke length and equal clearance volume of 5 % respective cylinder swept volumes. Bore of LP cylinder is 30 cm and stroke is 40 cm. Index of compression for both stages may be taken as 1.2. Determine

i) Heat rejected in intercooler ii) The bore of HP cylinder

iii) hp required to drive the HP cylinder

☞ **[GTU : Summer-14, Winter-15]**

Sol. : Given data :

$D_L = 0.3$ m, $D_H = 0.4$ m, $L = 0.4$ m, $p_1 = 1 \times 10^5$ N/m^2,

$T_1 = 298$ K, $N = 200$ rpm, $p_5 = 2.9 \times 10^5$ N/m^2,

$p_6 = 9 \times 10^5$ N/m^2, $T_5 = 298$ K, $C = 0.05$

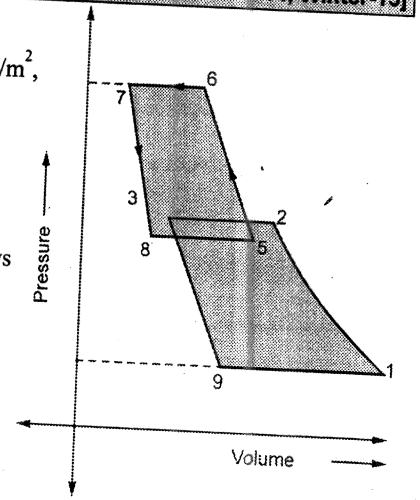
Type : Multistage with clearance

Volume of L.P cylinder can be calculated as follows

$$V_{L.P} = \frac{\pi \times D_L^2}{4} \times L_s \times N \times 2$$

$$= \frac{\pi \times (0.3)^2}{4} \times 0.4 \times 200 \times 2$$

$$V_{L.P} = 11.31 \text{ m}^3\text{/min}$$



$$\eta_{vol\ (L.P.)} = 1 + C - \left[C\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]$$

$$= 1 + 0.05 - \left[0.05\left(\frac{3}{1}\right)^{\frac{1}{1.2}}\right]$$

$$\eta_{vol} = 0.9250 = 92.50\ \%$$

$\dot{V}_a$ of L.P cylinder $= V_{L.P.} \times \eta_{vol} = 11.31 \times 0.9250$

$$= 10.47 \text{ m}^3\text{/min}$$

But, $$\dot{m} = \frac{p_1 V_1}{RV_1} = \frac{1 \times 10^5 \times 10.46}{287 \times 298} = 12.24 \text{ kg/min}$$

Temperature at end of L.P cylinder

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 298 \times \left(\frac{3}{1}\right)^{\frac{1.2-1}{1.2}}$$

$$= 357.98 \text{ K}$$

Volume of air entering HP cylinder

$$V_5 = \frac{mR\,T_5}{p_5} = \frac{12.23 \times 287 \times 298}{2.9 \times 10^5}$$

$$= 3.60 \text{ m}^3\text{/ min}$$

The clearance ratio is same for both L.P and H.P cylinder. Hence the volumetric efficiency same for both.

η_{vol} of H.P $= \eta_{vol}$ L.P $= 0.9251$

Volume of air entering H.P cylinder $V_{H.P.} = \frac{V_5}{\eta_{vol}}$

$$= \frac{3.60}{0.9251} = \text{Volume of H.P cylinder is,}$$

$$3.89 = \frac{\pi \times (D_{HP})^2}{4} \times 0.4 \times 2 \times 200$$

$$D_{H.P.} = 0.1762 \text{ m}$$ ... Ans.

Heat rejected in inter cooler $Q_R = m\,C_p\,[T_2 - T_5]$

$$Q_R = 12.23 \times 1.005\ (357.88 - 298)$$

$$= 735.99 \text{ kJ/min}$$ ... Ans.

To calculate T_6

$$\frac{T_6}{T_5} = \left(\frac{p_6}{p_5}\right)^{\frac{n-1}{n}} \qquad \therefore \; T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{n-1}{n}}$$

$$T_6 = 298 \times \left(\frac{9}{2.9}\right)^{\frac{1.2-1}{1.2}}$$

$$\mathbf{T_6 = 359.91\ K}$$

Power required for H.P stage

$$W_{H.P.} = \left(\frac{n}{n-1}\right) \times mR\,(T_6 - T_5)$$

$$W_{H.P.} = \left(\frac{1.2}{1.2-1}\right) \times 12.23 \times 0.287 \times (359.9 - 298)$$

$$\mathbf{W_{H.P.} = 1303.62\ kJ/min}$$

$$W_{H.P.} = \frac{1303.62}{60} = \mathbf{21.70\ kW}$$

or $$W_{H.P.} = \frac{21.70}{0.74} = \mathbf{29.36\ hP}$$ **... Ans.**

Ex. 7.26 : A reciprocating air compressor has four stage compression with 2 m^3/min of air being delivered at 150 bar when initial pressure and temperature are 1 bar 27 °C. Compression occur polytropically following polytropic index of 1.25 in four stages with perfect intercooling between stages. For the optimum intercooling conditions determine the intermediate pressures and the work required for driving compressor.

Sol. : Given data

$\dot{V}_1 = 2\ m^3/min$, $x = 4$, $p_5 = 150$ bar, $p_1 = 1$ bar, $T_1 = 300$ K

Type : Multistage with perfect intercooling without clearance

To find : i) Pressure p_2, p_3 and p_4 ii) W_{poly}

i) Intermediate pressure

We have :

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \frac{p_5}{p_4} = x^4$$

$$x^4 = \frac{p_5}{p_4} \qquad \therefore \; x = \left(\frac{p_5}{p_1}\right)^{\frac{1}{4}} = \left(\frac{150}{1}\right)^{\frac{1}{4}} = 3.5$$

$$p_2 = 3.5\ \text{bar} = x$$

$$\frac{p_2}{p_1} = x \qquad \therefore \; p_2 = x \times p_1 = 3.5 \times 1 = \mathbf{3.5\ bar}$$

$$\frac{p_3}{p_2} = x \qquad \therefore \; p_3 = p_2 \times x = 3.5 \times 3.5 = \mathbf{12.25\ bar}$$

$$\frac{p_4}{p_3} = x \qquad \therefore \; p_4 = p_3 \times x = 12.25 \times 3.5 = \mathbf{42.87\ bar}$$

ii) Work required for compressor ()

$$W_{poly} = X \times \left(\frac{n}{n-1}\right) p_1 \dot{V}_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_{poly} = 4 \times \left(\frac{1.25}{0.25}\right) \times 1 \times 10^5 \times \frac{2}{60}\left[\left(\frac{3.5}{1}\right)^{\frac{0.25}{1.25}} - 1\right]$$

$$W_{poly} = \mathbf{18.98 \times 10^3\ W = 18.98\ kW}$$ **... Ans.**

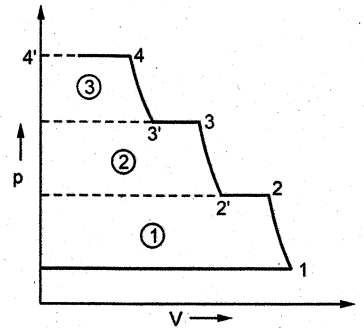
Ex. 7.27 : A three stage compressor delivers air at 70 bar from an atmospheric pressure of 1 bar and 30°C. Assuming the intercooling complete, estimate the amount of minimum work required to deal with 1 kg of air. Also find the amount of heat rejected in each intercooler. The index of compression is 1.2 throughout. Take Cp of air = 1.005 kJ/kg K.

Sol. : Given data,

$x = 3$, $P_4 = 70$ bar, $p_1 = 1$ bar,

$T_1 = 30 + 273 = 303$ K, $m = 1$ kg

$C_p = 1.005$ kJ/kg, $n = 1.2$



To find : 1) Minimum work required (W_{poly})

2) Heat rejected

1) Minimum work required (W_{poly})

$$x = \left(\frac{p_4}{p_1}\right)^{1/3} = \left(\frac{70}{1}\right)^{1/3} = 4.121$$

$$\therefore \; P_2 = 4.121\ \text{bar}$$

$$W_{poly} = x\left(\frac{n}{n-1}\right) \times p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_{poly} = x\left(\frac{n}{n-1}\right)\times mRT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]$$

$$W_{poly} = 3\left(\frac{1.2}{1.2-1}\right)\times 1\times 0.287\times 303\left[\left(\frac{4.21}{1}\right)^{\frac{1.2-1}{1.2}}-1\right]$$

$$W_{poly} = \mathbf{423.74\ kJ/kg} \quad \text{... Ans.}$$

2) Heat rejected (Q_R)

$$Q_R = m\times C_p\times[T_2-T_1]$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{4.121}{1}\right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 303\left(\frac{4.121}{1}\right)^{\frac{0.2}{1.2}}$$

$$= \mathbf{383.656\ K}$$

$$\therefore\quad Q_R = m\times C_p\times[T_2-T_1]$$

$$= 1\times 1.005\,(383.656-303)$$

$$Q_R = \mathbf{81.05\ kJ/kg} \quad \text{... Ans.}$$

Ex. 7.28 : A four stage compressor work between limits of 1 bar and 120 bar. The n = 1.32. the temperatures at the start of compression in each stage is C and intermediate pressure are so chosen that the work is equally divided amongst stages. Assume mass of air delivered = 1 kg/s. Neglecting clearance
Find : i) FAD at 1.013 bar and 15 °C

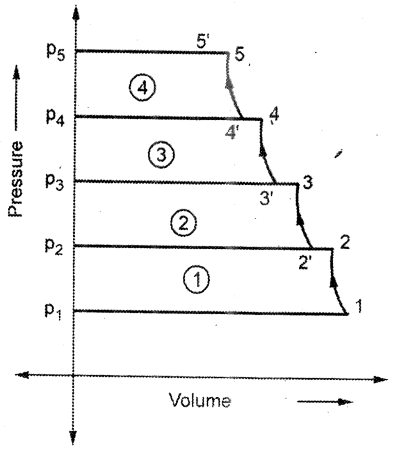
Sol. : Given data

$P_1 = 1\times 10^5\ N/m^2$,

$P_5 = 120$ bar, comdition : perfect intercoding

$pV^{1.32} = C$, $p_0 = 1.013$, $T_0 = 288$ K, $T_1 = 305$ K

To find : i) FAD



i) Free Air Delivery (FAD)

$$p_0 = 1.013\times 10^5\ N/m^2,$$

$$T_0 = 288\ K,\ m = 1\ kg$$

$$\frac{p_0V_0}{T_0} = \frac{p_1V_1}{T_1}$$

and $$p_0V_1 = mRT_1$$

$$V_1 = \frac{mRT_1}{p_1}$$

$$V_1 = \frac{1\times 287\times 305}{1\times 10^5}$$

$$= 0.8754\ m^3/kg$$

$$V_0 = \frac{p_1V_1\times T_0}{T_1\times p_0} = \frac{1\times 10^5\times 0.8754\times 288}{1.013\times 10^5\times 305} = \mathbf{0.816\ m^3/kg} \quad \text{... Ans.}$$

Ex. 7.29 : A two stage double acting air compressor runs at 120 rpm, if draws in air at 1 bar and 293 K and compressor with a total pressure ratio of 10, The intercooler used in between the stages work at pressure of 3 bar and the air is cooled in it upto a temperature of 302 K. Determine shaft power of the compressor having mechanical efficiency and volumetric efficiency of 90 % and 82 % respectively. The diameter of LP cylinder is 25 cm and is 37.5 cm. The index of compression for each stage is 1.3

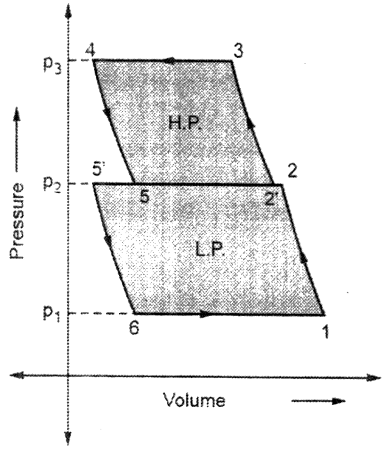
Sol. : Given data

$p_1 = 1$ bar $= 1\times 10^5\ N/m^2$, $T_1 = 293$ K,

$p_3 = 10\times 10^5\ N/m^2$,

$p_2 = 3.5\times 10^5\ N/m^2$, $\eta_{mech} = 0.9$,

$p_2 = 0.82$, D = 25 cm, L = 37.5, N = 120 rpm



Type : Multistage Double acting with clearance

$$\dot{V}_s = \frac{\pi D^2}{4}L_s\times 2N$$

$$= \frac{\pi(0.25)^2}{4}\times 0.375\times 2\times 120$$

$$\dot{V}_s = \mathbf{4.417\ m^3/min}$$

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} \qquad \therefore \dot{V}_a = \eta_{vol}\times\dot{V}_s = 0.82\times 4.417 = 3.622\ m^3$$

$$p_1\dot{V}_a = \dot{m}RT_1$$

$$\dot{m} = \frac{p_1\dot{V}_a}{RT_1} = \frac{1\times 10^5\times 3.622}{287\,(293)} = \mathbf{4.30\ kg/min}$$

Now, $W_A = (W)_{LP} + (W)_{H.P}$

$$W_A = \dot{m}\, RT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \dot{m}\, RT_5 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$W_A = \frac{4.308}{60} \times 0.287 \left(\frac{1.3}{0.3}\right) \times 293 \left[\left(\frac{3.5 \times 10^5}{1 \times 10^5}\right)^{\frac{0.3}{1.3}} - 1\right]$$

$$+ \frac{4.308}{60} \times 0.287 \times 302 \times \left[\left(\frac{10 \times 10^5}{3.5 \times 10^5}\right)^{\frac{0.3}{1.3}} - 1\right] \times \left(\frac{1.3}{0.3}\right) = \mathbf{16.18\ kW}$$

$$\text{Power P} = \frac{W_A}{\eta} = \frac{16.18}{0.9} = \mathbf{17.9\ kW} \quad \text{... Ans.}$$

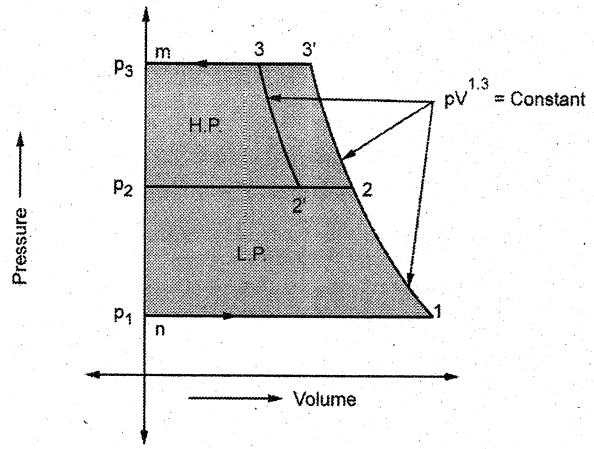
Ex.7.30 : Free air to be compressed from 1 bar to 15 bar pressure. Two alternatives are available i.e., single stage and multistage compression. Assuming polytropic index 1.3, justify which alternative should be used by calculating work. Assume perfect intercooling.

Sol. : Given data

$p_1 = 1 \times 10^5$ N/m^2, $p_3 = 15 \times 10^5$ N/m^2, Condition : perfect cooling

Type : Single and multistage with perfect cooling

$n = 1.3$, $p_2 = \sqrt{p_1 \times p_3} = \sqrt{1 \times 15} = \mathbf{3.873\ bar}$



To find

i) Workdone of compressor with perfect intercooling

ii) Workdone of single stage with perfect intercooling

i) W_m

$$W_m = mRT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \times X$$

$$W_m = mRT_1 \left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{3.873}{1}\right)^{\frac{0.3}{1.3}} - 1\right] \times 2$$

$$W_m = 3.178\, mR\, T_1$$

ii) W_s

$$W_s = mRT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right] = mRT_1 \left(\frac{1.3}{1.3-1}\right)\left[\left(\frac{1.5}{3.873}\right)^{\frac{0.3}{1.3}} - 1\right]$$

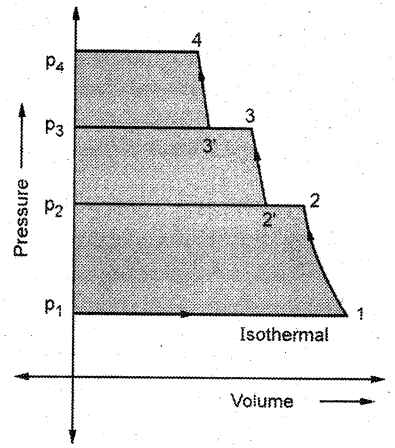
$W_s = \mathbf{3.762\, mRT_1}$ As $W_m < W_s$ $\therefore$ W_m **is best** ... Ans.

Ex. 7.31 : A three stage reciprocating air compressor compresses air from 1 bar and 17 °C to 35 bar. The law of compression is $pV^{1.25} = C$ and is same for all stages of compression. Assuming perfect intercooling, neglecting clearance, find the minimum power required to compress 15 m^3/min of free air. Also find the intermediate pressures.

Sol. : Given data

$p_1 = 1 \times 10^5$, $T_1 = 290$ K, $p_4 = 35 \times 10^5$ N/m^2,

$\dot{V}_1 = 15$ m^3 / min $= 0.25$ m^3/sec.



Type : Three stage without clearance (X = 3)

To find : i) Minimum power required

ii) Pressure p_2 and p_3

i) Minimum power required (W)

$$W = X \times \left(\frac{n}{n-1}\right) \times p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$X = \frac{p_4}{p_1} = \left(\frac{35}{1}\right)^{\frac{1}{3}} = \mathbf{3.271}$$

$$W = 3 \times \left(\frac{1.25}{1.25-1}\right) \times 1 \times 10^5 \times \frac{15}{60}\left[\left(\frac{3.271}{1}\right)^{\frac{0.25}{1.25}} - 1\right]$$

$W = \mathbf{100.3\ kW}$ ... Ans.

ii) Intermediate pressure p_2 and p_3

$$\frac{p_2}{p_1} = X \qquad \therefore p_2 = X \times p_1 = 3.271 \times 1 = \mathbf{3.271\ bar}$$

$$\frac{p_3}{p_2} = X \qquad \therefore p_3 = X \times p_2 = 3.271 \times 3.271 = \mathbf{10.69\ bar} \quad \textbf{... Ans.}$$

Ex. 7.32 : Determine the minimum number of stages required in an air compressor which admits air at 1 bar 27°C and delivers at 180 bar. The maximum discharge temperature at any stage is limited to 150°C. Consider the index for polytropic compression as 1.25 and perfect and optimum inter-cooling in between the stages. Neglect the effect of clearance.

Sol. : Given data

$p_1 = 1 \times 10^5\ N/m^2$, $T_1 = 300\ K$, $p_x = 180\ bar$, $T_2 = 150°C = 423\ K$, $n = 1.25$

i) To calculate number of stages (X)

$$\frac{p_x}{p_0} = \frac{p_1}{p_0} \times \frac{p_2}{p_1} \times \frac{p_3}{p_2} \times \frac{p_4}{p_3} \cdots \frac{p_x}{p_{x-1}} \qquad \ldots (1)$$

$\therefore$ For perfect intercooling, the pressure ratio is same

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \cdots \frac{p_x}{p_{x-1}}$$

From equation (1) we get

$$\frac{p_x}{p_0} = (X)^x$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$\therefore \frac{p_x}{p_0} = X^x = \left[\left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}\right]^x$$

$$\frac{180}{1} = \left(\frac{423}{300}\right)^{\frac{1.25 \times x}{1.25-1}}$$

$$\log(180) = \frac{1.25\,x}{0.25}\left[\log\left(\frac{423}{300}\right)\right]$$

$$\mathbf{x = 3.022}$$

$x = 3.022 \approx 3$ hence three stage compressor is used **... Ans.**

REVIEW QUESTIONS

Q. 1 Explain construction and working of single stage reciprocating air compressor.

Q. 2 Explain the terms FAD and isothermal efficiency for air compressors.

Q. 3 Derive expression for volumetric efficiency for single stage compressor with clearance.

Q. 4 Compare rotary and reciprocating air compressors.

Q. 5 Why multistage compressors are preferred over single stage ?

Q. 6 Explain inter-cooling and after cooling used in compressors.

❑❑❑

Notes

CHAPTER - 8

CENTRIFUGAL COMPRESSORS

Syllabus : Essential parts, Static and total head properties, Velocity diagram, Degree of reaction, surging and choking, Losses in centrifugal compressor.

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8.1 Introduction

- In various industrial applications compressed air is required. A device or machine providing air at high pressure is called as **air compressor.**
- An air compressor takes in air at atmospheric pressure, compresses it and delivers the high pressure air to a storage vessel called as **receiver**.
- From receiver compressed air may be conveyed by the pipe-line to a place where the supply of compressed air is required.
- As the process of compressing the air requires that work should be done upon it, it will be clear that a compressor must be driven by some prime-mover.
- Out of the total energy received by the compressor from the prime-mover, some will be absorbed in work done against friction, some will be lost to radiation and the rest will be maintained within the air itself.
- The prime-mover converts only a fraction of heat it receives from the source into work and for a compressor the energy is available at the shaft of prime-mover.

8.2 Classification of Rotary (Rotodynamic) Compressors

- The rotary compressors are the machines which develop pressure and have a rotor as their primary element as compared to slider-crank mechanism of reciprocating compressor.
- Rotary compressors are used when the large and continuous quantity of air is required at relatively low pressure.
- The rotary compressors are broadly classified as follows :

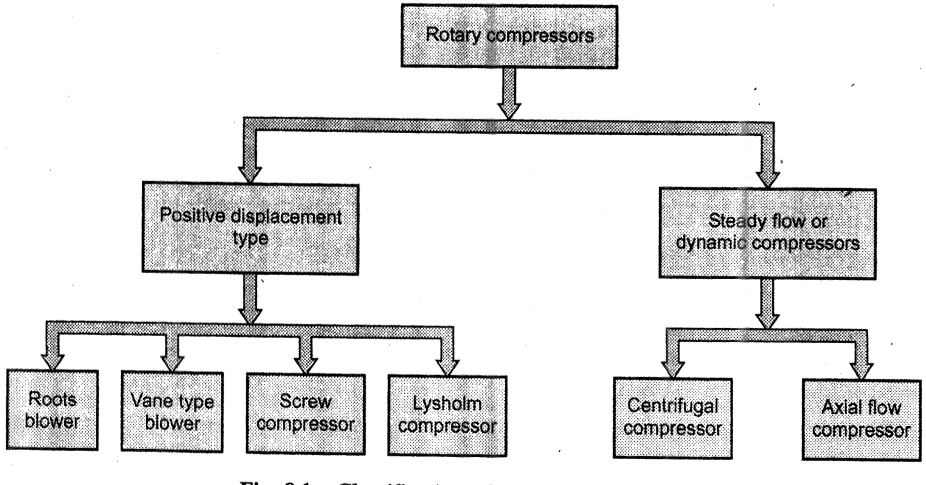


Fig. 8.1 : Classification of rotary compressors

- In case of positive displacement rotary compressor, the air is entrapped between the two sets of mutually engaging surfaces and squeezing action takes place.
- It includes roots blower, vane type blower, screw compressor, Lysholm compressor etc.
- In case of steady flow or dynamic compressor, the air is not trapped within the specified boundaries but it flows at a steady rate continuously through the compressor.
- In these compressors, the kinetic energy is converted into pressure energy by using the arrangement of impeller and fixed rings.
- It includes centrifugal compressor and axial flow compressor.
- The centrifugal compressor can produce pressure ratio upto 4 with one impeller and by using number of impellers the pressure ratio can reach upto 10.
- In axial flow compressor the pressure ratio is upto 1.5 per stage and by using number of stages it can be upto 20.

8.3 Centrifugal Compressor

☞ **[GTU : Summer-15, Winter-16]**

- The centrifugal compressors are used to supply large quantities of air but with low pressure ratio. It can develop pressure ratio upto 4 with single stage.
- The total pressure ratio developed in multistage centrifugal compressor is limited to 10. But the use of multistage centrifugal compressors in series are not preferred due to problem in designing the air passages.
- For low pressure ratios, the centrifugal compressor is lighter and is able to operate effictively over a wide range of mass flows at any one speed.
- A centrifugal compressor consists of a rotating impeller, a diffuser and a casing as shown in Fig. 8.2.

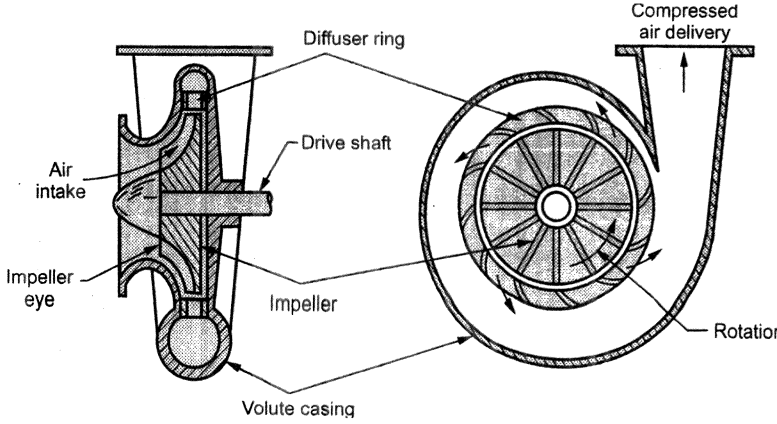


Fig. 8.2 : Centrifugal compressor

i) Impeller :

- The impeller consists of a disc on which radial blades are attached. It can run at speeds of 20,000 to 30,000 rpm.
- The air enters the eye of the compressor at low velocity and atmospheric pressure. The air moves radially outwards passing through the impeller and it is guided by the impeller vanes.
- The impeller increases the momentum of air which results in rise in temperature as well as pressure of the air.
- The main function of impeller is to impart the energy to the air by rotating the blades. The rotation of impeller causes a static pressure rise alongwith increase in kinetic energy of the gas.

ii) Diffuser :

- The diffuser is an important part of the compressor which surrounds the impeller and provides diverging passages for air flow. This increases the pressure of air.
- The air leaving the impeller blades enters the diffuser where its velocity is reduced by providing more cross-sectional area for the air flow.
- It causes to convert the kinetic energy into pressure energy by diffusion. Almost half of the total pressure rise is achieved in impeller and the remaining half in the diffuser.
- The change of pressure and velocity of air passing through the impeller and diffuser are shown in Fig. 8.3.

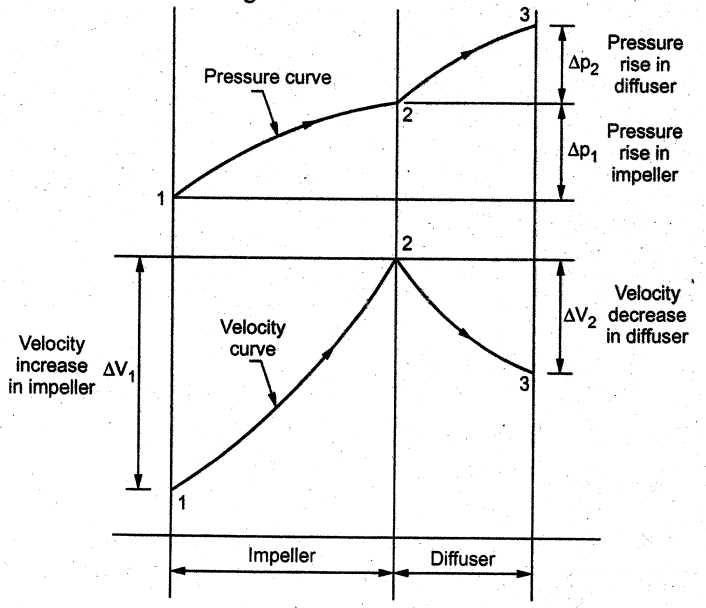
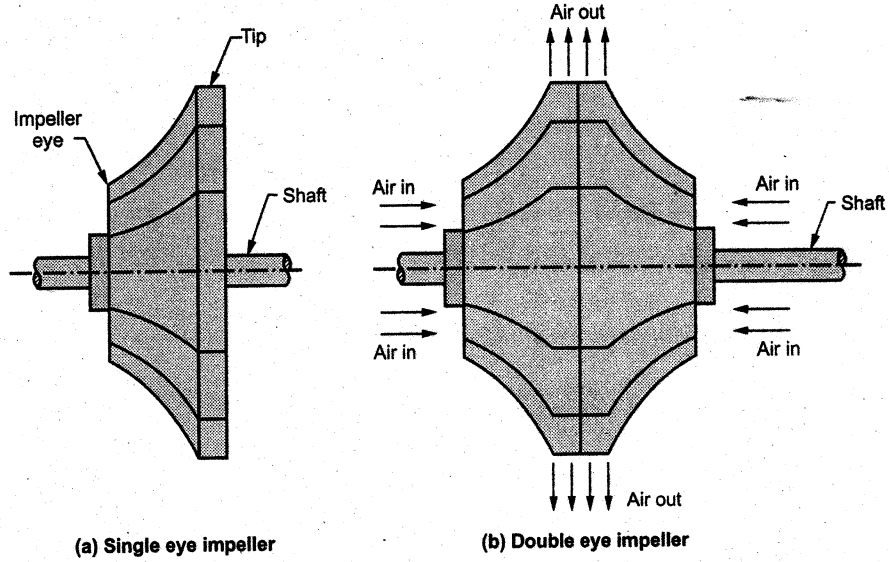


Fig. 8.3 : Pressure and velocity variation of air passing through impeller and diffuser

- The two types of impellers are commonly used namely *single eye type and double eye type.*
- In case of single eye type impeller the air enters the compressor from one side only whereas, in case of double eye type impeller the air enters from both sides. Refer Fig. 8.4.



(a) Single eye impeller **(b) Double eye impeller**

Fig. 8.4 : Types of impeller

- The main advantage of double eye impeller is that, it is subjected to equal axial forces in opposite directions.

iii) Volute casing :

- The air coming out of the diffuser is collected in the casing and taken out from the outlet of the compressor.
- The increased cross-sectional area of volute casing also causes small rise in pressure with reduction in the kinetic energy.

8.3.1 Advantages, Disadvantages and Applications of Centrifugal Compressor

Advantages

- Centrifugal compressors are simple in design and operation.
- They can operate efficiently over a wide range of mass flow at any particular speed.
- Centrifugal compressors are less expensive.

- It occupies less length as compared to axial compressors.
- Contaminated atmosphere does not affect the performance of the compressor.

Disadvantages

- It has low maximum efficiency.
- It needs large frontal area for given mass flow rates.
- It is not suitable for multistaging due to large losses in between the stages.

Applications

- These compressors are used in supercharging of I.C. engines.
- They are also used in jet engines, blowers, even furnaces, for pipeline flow and for compression of refrigerants and other industrial applications.

8.4 Static and Total Head Values

☞ [GTU : Summer-16, Winter-17]

- The velocities encountered in centrifugal compressors are very large as compared to reciprocating compressors. Hence, the total head quantities are considered in the analysis of centrifugal compressors.
- Total head quantities consider the kinetic energy of the air passing through the compressor.
- Consider a horizontal flow passage of variable area as shown in Fig. 8.5 through which air flows.

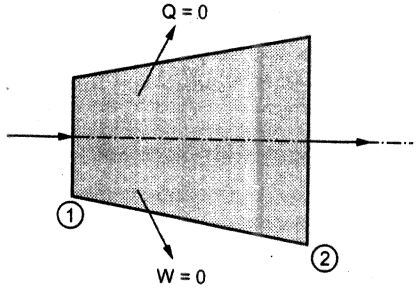


Fig. 8.5

- It is assumed that there is no work transfer (W = 0) and heat transfer (Q = 0) to the system.
- Now applying the steady flow energy equation for unit mass of air flow,

$$u_1 + p_1v_1 + \frac{V_1^2}{2} = u_2 + p_2v_2 + \frac{V_2^2}{2}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \qquad \ldots(\because h = u + pv)$$

or $$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2} \qquad \ldots(\because h = C_p T)$$

or $$C_p T + \frac{V^2}{2} = \text{Constant} \qquad \ldots(8.1)$$

- In the above equation, 'T' is called as **static temperature** of air measured by the thermometer when the thermometer is moving at the air velocity.

- If the moving air is suddenly brought to rest under reversible adiabatic conditions, the total kinetic energy of the air is converted into the heat energy. This increases the temperature and pressure of the air.
- This temperature and pressure of the air is called as **stagnation** or **total head** temperature and pressure. These values are denoted by a suffix notation '0'.

$$\therefore \quad C_pT + \frac{V^2}{2} = C_p T_0 \qquad \ldots(8.2)$$

where, T_0 = Stagnation or total head temperature

$$\therefore \quad \frac{V^2}{2} = C_p (T_0 - T) \text{ or } \frac{V^2}{2C_p} = (T_0 - T) \qquad \ldots(8.3)$$

or $$\frac{V^2}{2} = C_p T_0 - C_p T = h_0 - h$$

- To calculate the total head pressure (p_0) apply,

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}} \qquad \ldots(8.4)$$

where, p and T = Static pressure and temperature respectively

p_0 and T_0 = Total head or stagnation pressure and temperature respectively.

γ = Adiabatic index = 1.4

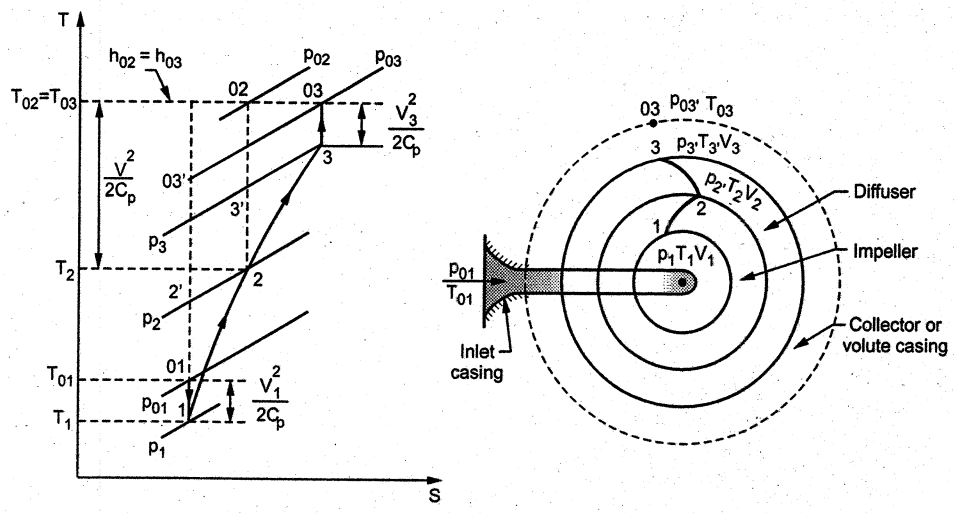
Note : The above equations (8.5) and (8.6) are applicable only for isentropic flow. The value of C_p for air is 1.005 kJ / kgK and the density of air can be found by using,

$$pv = mRT \text{ or } p = \rho RT \qquad \ldots\left(\rho = \frac{m}{v}\right)$$

$$R_{air} = 0.287 \text{ kJ / kgK}$$

8.5 Representation of Process on T-S Diagram

- During the process of centrifugal compressor, the air is drawn from surrounding air at stagnation conditions into inlet casing and then it enters the impeller eye with some velocity and compressed.
- The combined T-S diagram with total head or stagnation values and static values is shown in Fig. 8.6.



(a) Representation of process in centrifugal compressor

(b) Representation of points on centrifugal compressor

Fig. 8.6

- A point 01 indicates the condition of ambient or atmospheric air with values p_{01} and T_{01}.
- During the isentropic process 01 to 1, air is drawn by the impeller and the velocity of air increases to V_1 with the help of convergent nozzle of inlet casing.
- Point 1 represents the static condition of air at inlet to the impeller (p_1, T_1 and V_1).
- Now in the impeller air is compressed and its pressure as well as temperature increases. Process 1 to 2' represents isentropic compression whereas process 1 to 2 represents actual compression with friction.
- Point 2 indicates the condition of air at the exit of impeller (p_2, T_2 and V_2). Also p_{02} corresponds to delivery pressure if the total kinetic energy ($V_2^2/2$) is converted into pressure energy in the diffuser.
- After impeller the air enters into the fixed diffuser blades and its kinetic energy is converted into pressure energy. As diffusion process is not isentropic, the final delivery pressure corresponds to p_3.
- Point 03 represents the condition of air at discharge (p_{03}, T_{03} and negligible V_3). The pressure p_{03} is less than p_{02}.

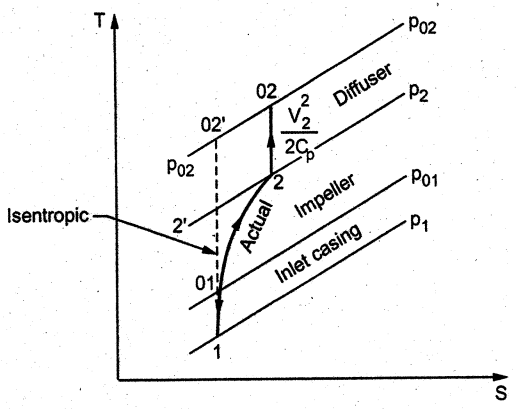


Fig. 8.7 : Ideal diffusion process

Ideal diffusion process

- Fig. 8.10 shows ideal diffusion process with negligible velocity.
- The process 01 to 1 indicates the isentropic process in inlet casing with reduction to static pressure p_1 and increase in velocity V_1.
- Process 1 to 2 represents actual compression in impeller whereas 1 to 2' represents ideal isentropic (reversible adiabatic) compression.
- At the exit of impeller, the air conditions are p_2, T_2 and V_2 which enters the diffuser.
- In diffuser the kinetic energy is converted into pressure energy isentropically (process 2 to 02).
- Thus, at the exit of compressor the state of air is p_{02}, T_{02} with negligible velocity V_2.

8.5.1 Losses and Isentropic Efficiency of Compressor

- During the flow of air through the impeller there are many internal losses. The common losses are as follows :
  - The friction between the air and flow passage. Also there is friction between the air and flow passage. Also there is friction between the air layers moving with relative velocities.
  - Turbulance caused in the air.
  - Shock at entry, etc.
- All the above losses results in increase in enthalpy of air without increasing pressure. Hence, the actual temperature of air at exit of compressor is more than the air temperature if it is compressed isentropically.

- Fig. 8.8 shows isentropic and actual compression for the same pressure ratio.

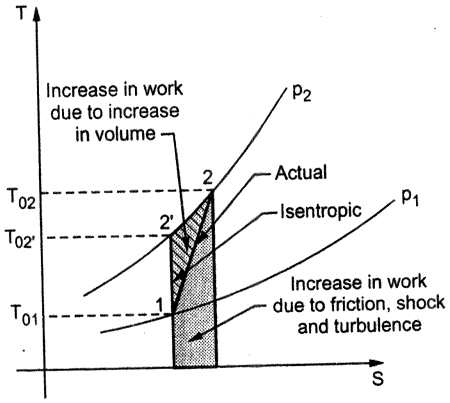


Fig. 8.8 : Actual and isentropic compression on T-S diagram

- The **isentropic efficiency** of centrifugal compressor is defined as the ratio of isentropic (ideal or therotical) work to the actual work. It is given by,

$$\eta_i = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{h_{02'} - h_{01}}{h_{02} - h_{01}} = \frac{C_p\,(T'_{02} - T_{01})}{C_p\,(T_{02} - T_{01})}$$

$$\therefore \quad \eta_i = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} \qquad \text{...(Based on total head values)...(8.5)}$$

$$\text{or} \quad \eta_i = \frac{T_{2'} - T_1}{T_2 - T_1} \qquad \text{...(Based on static values)...(8.6)}$$

$$\text{Now,} \quad \eta_i = \frac{T_1\left(\frac{T'_2}{T_1} - 1\right)}{T_1\left(\frac{T_2}{T_1} - 1\right)} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1} \qquad \text{...(8.7)}$$

- The value of 'n' is always greater than γ.
- As per steady flow energy equation for unit mass,

$$h_1 + \frac{V_1^2}{2} + Z_1 g + Q = h_2 + \frac{V_2^2}{2} + Z_2 g + W$$

As there is no heat transfer, Q = 0 and the work is done on the system, W is taken as negative.

$$\therefore \quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} - W \qquad \text{... } (\because Z_1 = Z_2)$$

$$\therefore \quad W = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

$$\therefore \quad W = C_p\,(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \qquad \text{...(8.8)}$$

- The above equation (8.10) represents work done on the impeller.

8.6 Velocity Diagram for Centrifugal Compressor

☞ **[GTU : Summer-15, Winter-16]**

- In the analysis of centrifugal compressor the following notations are used :
- Let,

u_1 and u_2 = Mean blade velocity at entry and exit

V_1 and V_2 = Absolute velocity at inlet and outlet to the rotor

V_{r1} and V_{r2} = Relative velocity of air at entry and exit of rotor

V_{w1} and V_{w2} = Velocity of whirl at inlet and outlet (x-component of absolute velocity)

V_{f1} and V_{f2} = Velocity of flow at inlet and outlet (y-component of absolute velocity)

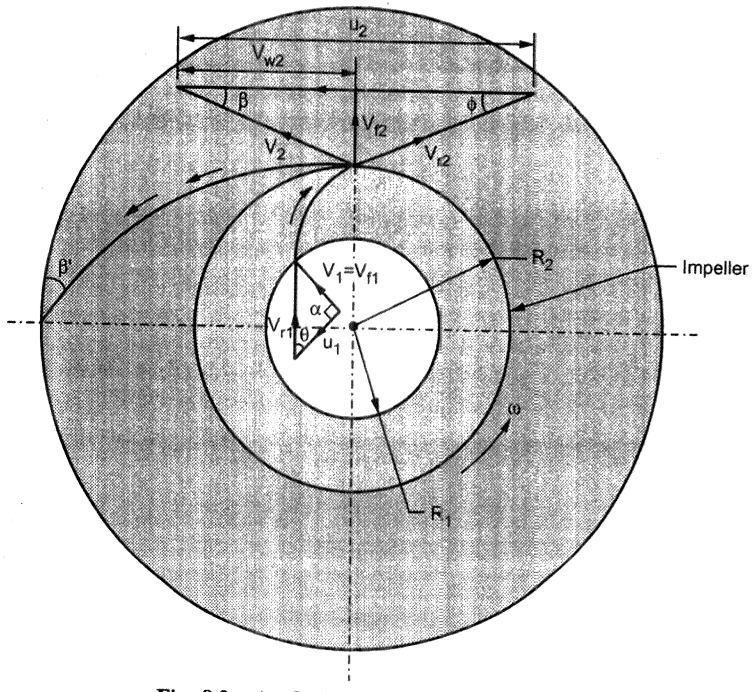


Fig. 8.9 : Analysis of centrifugal compressor

Note : $V_{f1} = V_1 \sin\alpha$, $V_{w1} = V_1 \cos\alpha$, $V_{f2} = V_2 \sin\beta$ and $V_{w2} = V_2 \cos\beta$

α = Inlet angle of guide vane

θ = Inlet angle to the rotor or impeller

ϕ = Outlet angle from the rotor or impeller

β = Inlet angle to the diffuser

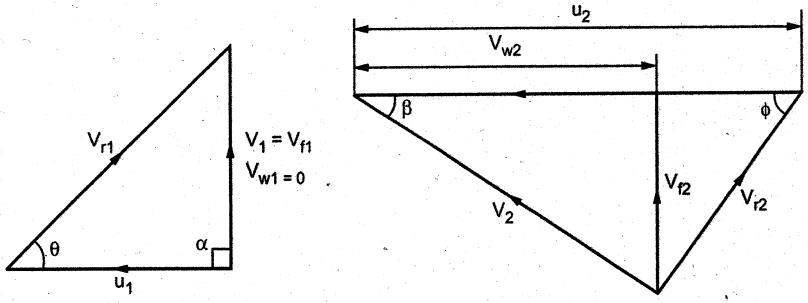
R_1 and R_2 = Inlet and outlet radius of impeller blades

N = Speed of compressor in rpm

ω = Angular velocity of compressor = $\frac{2\pi N}{60}$, rad/sec

$\dot{m}$ = Mass flow rate of air in kg/sec.

- During the analysis it is assumed that, the entry of air is axial ($\alpha = 90°$) hence $V_{w1} = V_1 \cos 90 = 0$ and $V_1 = V_1 \sin 90 = V_{f1}$. Refer Fig. 8.9. (See Fig. 8.9 on previous page.)
- The enlarged views of inlet and outlet velocity diagrams is shown in Fig. 8.10.



(a) Inlet velocity diagram (b) Outlet velocity diagram

Fig. 8.10 : Velocity diagram of centrifugal compressor

- The blade must be parallel to the relative velocity of air at entry and exit to avoid shock. Hence, angles θ and ϕ are the inlet and outlet blade angles.
- The diffuser blade angle must be parallel to the absolute velocity of air from the impeller (V_2). Thus, β is the diffuser blade angle at inlet and β' is at outlet.
- From inlet velocity triangle,

$$\tan\theta = \frac{V_1}{u_1} = \frac{V_{f1}}{u_1} \qquad \ldots(8.9)$$

From outlet velocity triangle,

$$\sin\phi = \frac{V_{f2}}{V_{r2}} \qquad \therefore V_{f2} = V_{r2} \sin\phi$$

or $$\sin\beta = \frac{V_{f2}}{V_2} \qquad \therefore V_{f2} = V_2 \sin\beta$$

$$\therefore \quad V_{f2} = V_{r2} \sin\phi = V_2 \sin\beta \qquad \ldots(8.10)$$

8.6.1 Work Done by Impeller (Euler's Work)

- The work supplied to a fluid in a stage of compressor may be found by applying Newton's second law of motion.
- As per this law, the torque in the direction of motion of blade is equal to the rate of change of angular momentum.

Therefore, $$T = \dot{m}\,(V_{w2} R_2 - V_{w1} R_1),\ \text{N-m} \qquad \ldots(8.11)$$

But, $V_{w1} = 0$

$$\therefore \quad T = \dot{m}\, V_{w2} R_2,\ \text{N-m} \qquad \ldots(8.12)$$

Now, the rate of energy transfer for angular velocity 'ω' is,

$$\frac{WD}{sec} = \text{Power } P = T \times \omega = \dot{m}\, V_{w2} R_2 \times \omega$$

$$\therefore \quad \frac{WD}{sec} = P = \dot{m}\, V_{w2} u_2,\ \text{watts} \qquad \ldots(u_2 = R_2\,\omega)\ldots(8.13)$$

For work done per kg,

$$W = V_{w2} u_2,\ \text{N-m/kg} \qquad \ldots(8.14)$$

- The above equation (8.14) is called as **Euler's equation** or **Euler's work.**
- Also the above equation (8.13) represents the indicated power (IP) required to drive the compressor.
- If the exit from the impeller is radial ($\phi = 90°$) i.e. ideal case then the velocity triangle is shown in Fig. 8.11.

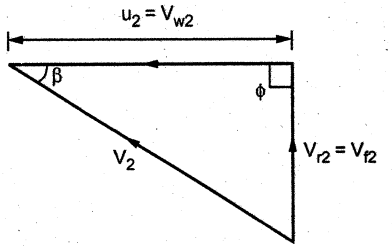


Fig. 8.11

For this condition, $u_2 = V_{w2}$

$$\therefore \quad P = \dot{m}\, u_2^2 \qquad \ldots[\text{From equation (8.13)}]$$

or $$W = u_2^2 \qquad \ldots[\text{From equation (8.14)}]$$

From Fig. 8.13 (a) we can write,

$$V_{r1}^2 = V_1^2 + u_1^2 \quad \ldots(i)$$

From Fig. 8.13 (b),

$$V_{r2}^2 = V_{f2}^2 + (u_2 - V_{w2})^2 = V_{f2}^2 + u_2^2 - 2u_2V_{w2} + V_{w2}^2$$

But, $V_2^2 = V_{f2}^2 + V_{w2}^2$

$$\therefore \quad V_{r2}^2 = V_{f2}^2 + V_{w2}^2 + u_2^2 - 2u_2V_{w2} = V_2^2 + u_2^2 - 2u_2V_{w2} \quad \ldots (ii)$$

Subtracting equation (i) and (ii),

$$V_{r1}^2 - V_{r2}^2 = (V_1^2 + u_1^2) - (V_2^2 + u_2^2 - 2u_2V_{w2})$$

$$\therefore \quad V_{r1}^2 - V_{r2}^2 = (V_1^2 + u_1^2) - V_2^2 - u_2^2 + 2u_2V_{w2}$$

$$\therefore \quad 2u_2V_{w2} = (V_{r1}^2 - V_{r2}^2) + (u_2^2 - u_1^2) + (V_2^2 - V_1^2)$$

or $$u_2V_{w2} = \frac{V_{r1}^2 - V_{r2}^2}{2} + \frac{u_2^2 - u_1^2}{2} + \frac{V_2^2 - V_1^2}{2} \quad \ldots (8.15)$$

But, Eluer's work $W = u_2V_{w2}$ ... [From equation (8.14)]

$$\therefore \quad W = u_2V_{w2} = \frac{V_{r1}^2 - V_{r2}^2}{2} + \frac{u_2^2 - u_1^2}{2} + \frac{V_2^2 - V_1^2}{2} \quad \ldots (8.16)$$

- In the above equation (8.16), the first term of RHS indicates that the pressure rise in impeller is due to diffusion action (as relative velocity reduces from entry to exit).
- The second term of RHS indicates that the pressure rise in impeller is due to centrifugal action (as air enters the lower radius and comes out at higher radius).
- The third term of RHS indicates that the increase in kinetic energy of air in impeller is converted into pressure energy in diffuser.

8.6.2 Width of Impeller Blades

- To find the width of impeller blades at inlet and outlet it is assumed that the mass of air flowing through the system per second is constant ($\dot{m}_1 = \dot{m}_2$).
- Let, $\dot{m}$ = Mass flow rate of air in kg/sec
  b_1 = Width or height of impeller blade at inlet in m
  b_2 = Width or height of impeller blade at outlet in m
  v_1 = Specific volume of air at inlet in m^3/kg

v_2 = Specific volume of air at outlet in m^3/kg

ρ = Density of air $= \frac{1}{v}$, kg/m^3

We know that, the mass flow rate of air is,

$$\dot{m} = \rho Q = \frac{1}{v} \times Q = \frac{1}{v} \times A \times V_f$$

$$\therefore \quad \dot{m} = \frac{\text{Area of flow} \times \text{Flow velocity}}{\text{Specific volume}}$$

$$\therefore \quad \dot{m} = \frac{2\pi R_1 b_1 \times V_{f1}}{v_1} = \frac{2\pi R_2 b_2 \times V_{f2}}{v_2} \quad \ldots(8.17)$$

- If 'n' be the number of blades and 't' be the thickness of blades then the mass flow rate is,

$$\dot{m} = \frac{(2\pi R_1 - n \cdot t) b_1 \times V_{f1}}{v_1} = \frac{(2\pi R_2 - n \cdot t) b_2 \times V_{f2}}{v_2} \quad \ldots(8.18)$$

8.7 Slip Factor, Power Input Factor and Pressure Coefficient

☞ **[GTU : Summer-15, Winter-16]**

1. Slip factor (ϕ_s)

- In the previous section 8.6.1 it is already discussed that, if the blade is radial at outlet ($\phi = 90°$) then $V_{w2} = u_2$.
- But, practically this condition is not satisfied due to secondary flow effects. Therefore, in actual compressor V_{w2} is always less than u_2.
- This difference between u_2 and V_{w2} is called as **slip.**

$$\therefore \quad \text{Slip} = u_2 - V_{w2} \quad \ldots(8.19)$$

- The **slip factor** or **slip coefficient** is defined as the ratio of actual whirl component (when discharge is not radial) to the ideal whirl component (when discharge is radial i.e. $V_{w2} = u_2$). It is denoted by ϕ_s.

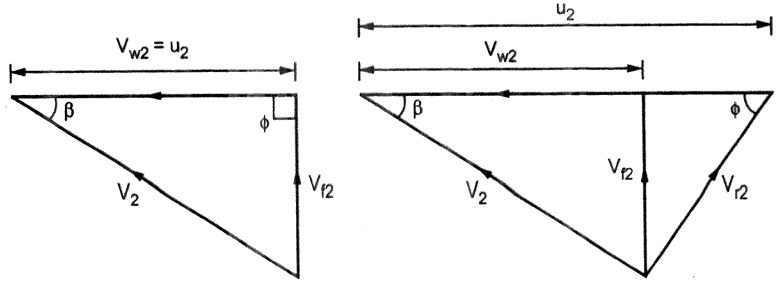


Fig. 8.12

$$\therefore \quad \phi_s = \frac{\text{Actual whirl component}}{\text{Ideal whirl component}} = \frac{V_{w2}}{u_2} \qquad \ldots(8.20)$$

- The value of slip factor depends on the number of impeller blades. For the less number of blades, the value of ϕ_s is high. Generally, its value is between 0.9 to 0.96.

2. Power input factor (ϕ_w)

- The actual work done per kg of air by the compressor $[C_p\ (T_{02} - T_{01})]$ is always greater than the theoretical work $(V_{w2} \cdot u_2)$ due to the fluid friction and turbulence losses.
- Therefore, the actual work is obtained by multiplying $(V_{w2} \cdot u_2)$ by a factor called as **power input factor** or **work factor**. It is denoted by ϕ_w.

$$\therefore \quad C_p\ (T_{02} - T_{01}) = \phi_w\ (V_{w2} \cdot u_2)$$

$$\therefore \quad \phi_w = \frac{\text{Actual work}}{\text{Theoretical work}} = \frac{C_p\ (T_{02} - T_{01})}{V_{w2}\ u_2} \qquad \ldots(8.21)$$

- Generally, its value is 1.04.

3. Pressure coefficient (ϕ_p)

- Pressure coefficient is defined as the ratio of isentropic work to the theoretical work. It is denoted by ϕ_p.

$$\therefore \quad \phi_p = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{C_p\ (T'_{02} - T_{01})}{V_{w2}\ u_2} \qquad \ldots(8.22)$$

8.7.1 Relation between Isentropic Efficiency, ϕ_p, ϕ_s and ϕ_w

- Fig. 8.13 shows the pressure rise in centrifugal compressor. The process 01 to 02′ shows isentropic compression whereas 01 to 02 shows actual compression.
- The work required to compress the air is given by,

$$W = C_p\ (T_{02} - T_{01}) \qquad \ldots(i)$$

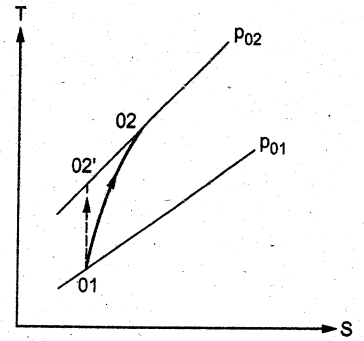


Fig. 8.13 : Pressure rise in centrifugal compressor

The stagnation or total head pressure ratio is given by,

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02'}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{T_{02'}}{T_{01}} - 1\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{p_{02}}{p_{01}} = \left(1 + \frac{T'_{02}}{T_{01}} - 1\right)^{\frac{\gamma}{\gamma-1}} \qquad \ldots \text{(ii)}$$

But, isentropic efficiency is,

$$\eta_i = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

$$\therefore \quad T_{02'} - T_{01} = (T_{02} - T_{01})\ \eta_i$$

Substituting this value in equation (ii),

$$\therefore \quad \frac{p_{02}}{p_{01}} = \left[1 + \frac{\eta_i\ (T_{02} - T_{01})}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \qquad \ldots(iii)$$

Also, $$\phi_w = \frac{C_p\ (T_{02} - T_{01})}{V_{w2}\ u_2} \qquad \ldots[\text{From equation (8.21)}]$$

$$\therefore \quad T_{02} - T_{01} = \frac{\phi_w \cdot V_{w2}\ u_2}{C_p}$$

But, slip factor is given as,

$$\phi_s = \frac{V_{w2}}{u_2} \qquad \therefore V_{w2} = \phi_s\ u_2$$

$$\therefore \quad T_{02} - T_{01} = \frac{\phi_w \times \phi_s\ u_2 \times u_2}{C_p} = \frac{\phi_w \times \phi_s\ u_2^2}{C_p} \qquad \ldots(iv)$$

Substituting this value in equation (iii),

$$\frac{p_{02}}{p_{01}} = \left[1 + \frac{\eta_i}{T_{01}} \times \frac{\phi_w\ \phi_s\ u_2^2}{C_p}\right]^{\frac{\gamma}{\gamma-1}} \qquad \ldots(8.23)$$

From equation (8.22), the pressure coefficient is,

$$\phi_p = \frac{C_p\ (T'_{02} - T_{01})}{V_{w2}\ u_2}$$

$$\phi_p = \frac{C_p (T_{02} - T_{01})}{V_{w2}\ u_2} \qquad \ldots(v)$$

Now, substituting the value of equation (iv) in equation (v),

$$\therefore \quad \phi_p = \frac{C_p\ \phi_w\ \phi_s\ u_2^2 \times \eta_i}{C_p \times V_{w2}\ u_2}$$

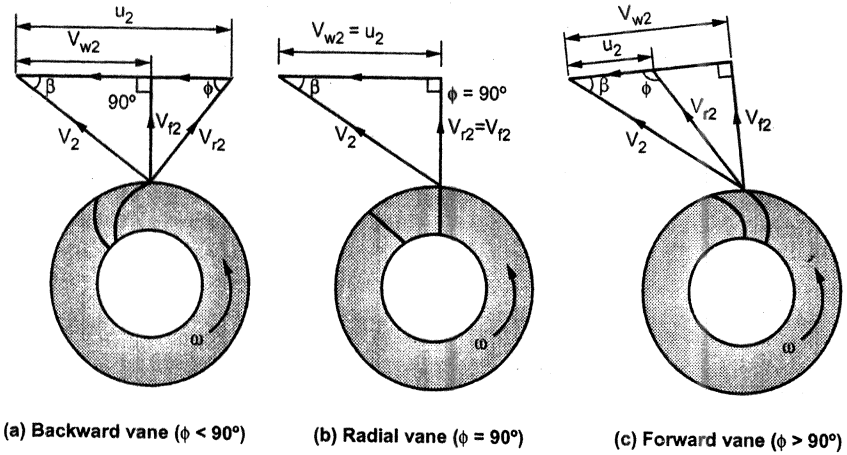
Assuming radial vanes ($V_{w2} = u_2$),

$$\therefore \quad \phi_p = \frac{\phi_w \times \phi_s \times u_2^2 \times \eta_i}{u_2^2}$$

$$\therefore \quad \phi_p = \phi_w \times \phi_s \times \eta_i \qquad \ldots(8.24)$$

8.8 Effect of Impeller Blade Shape on Performance

- In case of centrifugal compressor the vane shapes are of following three types :
  i) Backward curved vanes ($\phi < 90°$)
  ii) Radial curved vanes ($\phi = 90°$)
  iii) Forward curved vanes ($\phi > 90°$)
- The three types of vanes with their outlet velocity diagram are shown in Fig. 8.14.



(a) Backward vane ($\phi < 90°$) (b) Radial vane ($\phi = 90°$) (c) Forward vane ($\phi > 90°$)

Fig. 8.17 : Shape of vanes of a centrifugal compressor with outlet velocity diagram

- We know that, the work done per unit mass of air is,

$$W = V_{w2}\ u_2 \qquad \ldots[\text{From equation (8.14)}]$$

The above work is called as Euler's work which is responsible to develop the change in head H of the air. Therefore, Euler's head is,

$$H = V_{w2}\ u_2$$

From Fig. 8.14 (a),

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = u_2 - V_{f2}\ \cot\phi$$

or $$V_{w2} = u_2 - \frac{Q}{A}\cot\phi \qquad \ldots(\because V_f = \frac{Q}{A})$$

$$\therefore \quad H = V_{w2}\ u_2 = \left(u_2 - \frac{Q}{A}\ \cot\phi\right) u_2 \qquad \ldots(8.25)$$

where, Q = Volume flow rate of air

A = Area of flow at exit

- By assuming constant speed of compressor and constant flow area the following conclusions are made :
  - *For backward curved vanes ($\phi < 90°$),* the value of cot ϕ is positive hence the value of H keep on falling with increase in value of Q.
  - *For radial curved vanes ($\phi = 90°$),* the value of cot ϕ is zero hence the value of H remains constant irrespective of the value of Q.
  - *For forward curved vanes* ($\phi > 90°$), the value cot ϕ is negative hence the value of H keep on increasing with increase in value of Q.
- Fig. 8.15 shows the variation in value of H with respect to the volume flow rate Q.
- The centrifugal effects on the curved blade produces bending moment which results in increased stresses. This reduces the maximum speed of the impeller.
- Generally, backward vanes with angle $\phi = 20°$ to $25°$ are commonly used.

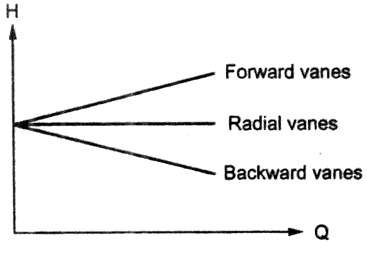


Fig. 8.15 : Characteristics of forward, backward and radial vanes

- Sometimes, compromise is made between the backward and forward vanes by using radial vanes.

Advantages of radial vanes

- Easy to manufacture.
- Free from complex bending stresses.
- Equal energy conversion in impeller and diffuser.

8.8.1 Prewhirl

- Sometimes, the relative velocity of air at inlet (Mach number M = Actual velocity/Sonic velocity) exceeds a value beyond which the shock waves are formed. These shock waves increases the loss of energy.
- At that time it is required to reduce the relative velocity by giving the air an initial pre-rotation to the initial absolute velocity V_1.
- This process which causes the air to enter the impeller blades at a reduced relative velocity is known as **prewhirl** or **pre-rotation.**
- For this purpose a set of fixed guide vanes preceding the impeller is used.

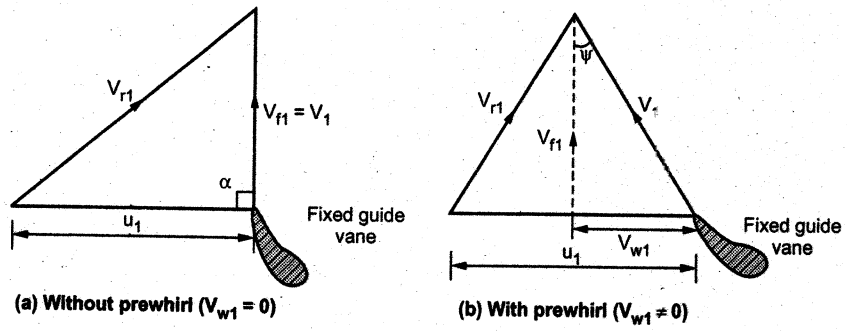


Fig. 8.19 : Effect of prewhirl

- Fig. 8.19 shows the effect of prewhirl. We know that, the work done by the impeller is,

$$W = (V_{w2}\, u_2 - V_{w1}\, u_1)$$

For Fig. 8.19 (a), $W = V_{w2}\, u_2 - 0 = V_{w2}\, u_2$ ...($V_{w1} = 0$)

For Fig. 8.19 (b), $W = V_{w2}\, u_2 - V_{w1}\, u_1$ ...($V_{w1} \neq 0$)

- It means, the prewhirl reduces the compressor work by an amount $V_{w1}\, u_1$.
- The angle ψ is called as **angle of prewhirl**. It is equal to the inlet angle of fixed guide vane.

8.9 List of Formulae

- For total head or stagnation values :

$$C_p\,(T_0 - T) = \frac{V^2}{2} \quad \text{or} \quad T_0 - T = \frac{V^2}{2\,C_p}$$

- Pressure ratio :

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

- Actual work done of centrifugal compressor :

$$WD_{act} = C_p\,(T_{02} - T_{01})$$

$$WD_{isen} = C_p\,(T'_{02} - T_{01})$$

- Isentropic efficiency : $\eta_i = \dfrac{WD_{isen}}{WD_{act}}$
- If inlet and outlet absolute velocities are given then,

$$WD_{act} = C_p\,(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

- Actual work done per kg :

$$WD = V_{w2}\, u_2$$

- For width of impeller :

$$\dot{m} = \frac{2\pi R_1\, b_1 \times V_{f1}}{v_1} = \frac{2\pi R_2\, b_2 \times V_{f2}}{v_2}$$

If thickness of blades is considered,

$$\dot{m} = \frac{(2\pi R_1 - nt)\, b_1\, V_{f1}}{v_1} = \frac{(2\pi R_2 - nt)\, b_2\, V_{f2}}{v_2}$$

- Slip = $u_2 - V_{w2}$

Slip factor : $\phi_s = \dfrac{V_{w2}}{u_2}$

- Power input factor : $\phi_w = \dfrac{C_p\,(T_{02} - T_{01})}{V_{w2}\, u_2}$
- Pressure coefficient : $\phi_p = \dfrac{C_p\,(T'_{02} - T_{01})}{V_{w2}\, u_2}$
- Relative between ϕ_p, ϕ_s, ϕ_w and η_i :

$$\phi_p = \phi_s\, \phi_w\, \eta_i$$

8.10 Solved Examples

Example 8.1 : *Air with temperature of 17°C flows into the centrifugal compressor running at 20,000 r.p.m. Using the following given data :*
Slip factor = 0.80
Work input factor = 1
Isentropic efficiency = 80%
Outer diameter of blade tip = 50 cm.
Assuming the absolute velocities of air entering and leaving the compressor are same.
Find :
a) The temperature rise of air passing through compressor.
b) The static pressure ratio.

Solution : Given data :

$T_1 = 17°C = 17 + 273 = 290$ K,

$N = 20{,}000$ rpm, $\phi_s = 0.8$, $\phi_w = 1$,

$\eta_{isen} = 80\% = 0.8$, $D_2 = 50$ cm $= 0.5$ m,

$V_1 = V_2$

To find : i) $(\Delta T)_{act}$ ii) $\frac{p_2}{p_1}$

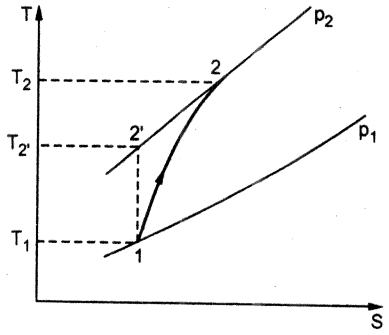


Fig. 8.17

Step - 1 : Calculate the temperature rise of air passing through compressor

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 20000}{60} = 523.5987 \text{ m/sec}$$

Slip factor, $\phi_s = \frac{V_{w2}}{u_2}$ $\quad\therefore\quad 0.8 = \frac{V_{w2}}{u_2}$

$$V_{w2} = 0.8\ u_2$$

Work done per kg of air,

$$WD_{act} = \phi_w\ V_{w2}\ u_2 = 1 \times 0.8\ u_2 \times u_2$$
$$= 0.8 \times 523.5987^2 = 219.3244 \times 10^3 \text{ J/kg}$$

Also, $$WD_{act} = \left[C_p\ (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}\right]$$

$$= [C_p\ (\Delta T)_{act} + 0] \qquad \ldots[(\Delta T)_{act} = T_2 - T_1,\ V_1 = V_2]$$

$\therefore\quad 219.3244 \times 10^3 = 1.005 \times 10^3 \times (\Delta T)_{act}$

$\therefore\quad (\Delta T)_{act} = \mathbf{218.2332\ K}$ **...Ans.**

Step - 2 : Calculate the static pressure ratio

Isentropic efficiency,

$$\eta_{isen} = \frac{T_2' - T_1}{T_2 - T_1} = \frac{(\Delta T)_{isen}}{(\Delta T)_{act}}$$

$\therefore\quad 0.8 = \frac{T_2' - 290}{218.2332}$

$\therefore\quad T_2' = 464.5865$ K

Static pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{464.5865}{290}\right)^{\frac{1.4}{1.4-1}}$$

$\therefore\quad \frac{p_2}{p_1} = \mathbf{5.204}$ **...Ans.**

Example 8.2 : *Air at a temperature of 350 K flows in a centrifugal compressor rotating at 18,000 rpm. Use following data :-*
Isentropic total head efficiency = 0.85
Outer diameter of blade tip = 550 mm
Slip factor = 0.82 Find :
a) Temperature rise of air passing through the compressor.
b) Stagnation pressure ratio.
Assume that, absolute velocity of air at inlet and outlet of compressor are same.
Take C_p = 1.005 kJ/kgK ☞ **[GTU: Summer-15, Winter-16]**

Solution : Given data :

$T_1 = 350$ K, $N = 18{,}000$ rpm, $\eta_{isen} = 85\% = 0.85$, $D_2 = 550$ mm $= 0.55$ m,

$\phi_s = 0.82$, $C_p = 1.005$ kJ/kgK, $V_1 = V_2$

To find : i) $(\Delta T)_{act}$ ii) p_2/p_1

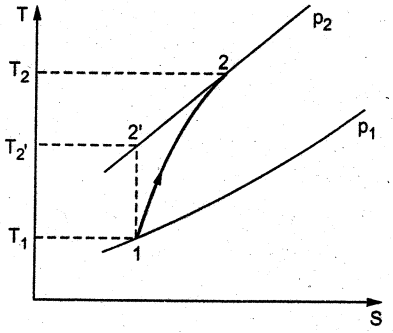


Fig. 8.18

Step - 1 : Calculate the temperature rise of air passing through the compressor

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.55 \times 18000}{60} = 518.3627 \text{ m/sec}$$

Slip factor, $\phi_s = \frac{V_{w2}}{u_2} \quad \therefore 0.82 = \frac{V_{w2}}{518.3627}$

$\therefore \quad V_{w2} = 425.0574$ m/sec

As we know that,

Work done, $WD_{act} = V_{w2}\ u_2 = 425.0574 \times 518.3627$

$= 220.3339 \times 10^3$ J/kg

Also, $WD_{act} = C_p\ (\Delta T)_{act} + \frac{V_2^2 - V_1^2}{2}$ ...[$\because T_2 - T_1 = (\Delta T)_{act}$]

$\therefore \quad 220.3339 \times 10^3 = 1.005 \times 10^3 \times (\Delta T)_{act} + 0$ ...[$\because V_1 = V_2$]

$\therefore \quad \mathbf{(\Delta T)_{act} = 219.2377\ K}$ **...Ans.**

Step - 2 : Calculate the static pressure ratio

Isentropic efficiency,

$$\eta_{isen} = \frac{(\Delta T)_{isen}}{(\Delta T)_{act}} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \quad \therefore 0.85 = \frac{T'_{02} - 350}{219.2377}$$

$\therefore \quad T'_{02} = 536.352$ K

As we know that,

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{p_{02}}{p_{01}} = \left(\frac{T'_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \quad ...[\because p_{02} = p'_{02}]$$

$\therefore \quad \mathbf{\frac{p_{02}}{p_{01}} = \left(\frac{536.352}{350}\right)^{\frac{1.4}{1.4-1}} = 4.4548}$ **...Ans.**

Example 8.3 : *The free air delivered by a centrifugal compressor is 20 kg/min. The suction condition is 1 bar and 20°C. The velocity of air at the inlet is 60 m/sec. The isentropic efficiency of the compressor is 75 %. If the total head pressure ratio of the compressor is 3, find (a) The total head temperature of air at the exit of the compressor, and (b) Indicated power required to run the compressor assuming mechanical efficiency of 90 %.*

Pressure and temperature of air at inlet are static. For air, take $C_p = 1.005$ kJ/kg K, $\gamma = 1.4$. **[GTU: Summer-16, Winter-17]**

Solution : Given data :

$\dot{m} = 20$ kg/min $= \frac{20}{60} = 0.3333$ kg/sec., $p_1 = 1$ bar, $T_1 = 20°C = 20 + 273 = 293$ K,

$V_1 = 60$ m/sec, $\eta_{isen} = 75\% = 0.75$, $r_p = \frac{p_{02}}{p_{01}} = 3$, $\eta_{mech} = 90\% = 0.9$.

To find : i) T_{02} ii) P_{req}

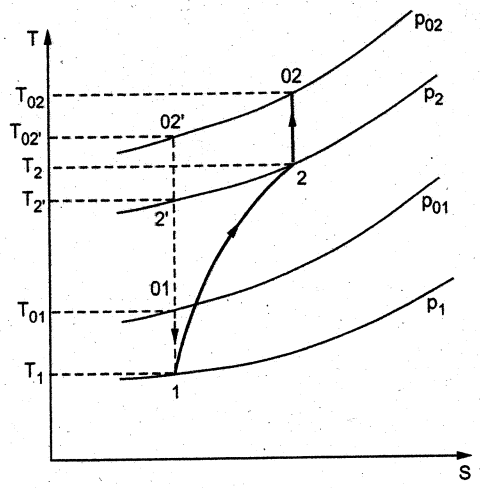


Fig. 8.19

Step - 1 : Calculate the total head temperature at the exit of compressor

As we know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2 C_p}$$

At point 01,

$$T_{01} - T_1 = \frac{V_1^2}{2 C_p} \qquad \therefore T_{01} - 293 = \frac{60^2}{2 \times 1.005 \times 10^3}$$

$$\therefore \quad T_{01} = 294.791 \text{ K}$$

We know that,

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T'_{02}}{294.791} = (3)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T'_{02} = 403.4916 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \qquad \therefore 0.75 = \frac{403.4916 - 294.791}{T_{02} - 294.791}$$

$$\therefore \quad \mathbf{T_{02} = 439.7251 \text{ K}} \qquad \textbf{...Ans.}$$

Step - 2 : Calculate the indicated power required to run the compressor

Power consumed,

$$P_{cons} = \dot{m} C_p (T_{02} - T_{01})$$
$$= 0.3333 \times 1.005 \times 10^3 \times (439.7251 - 294.791)$$
$$= 48.548 \times 10^3 \text{ W}$$

Here, indicated power for the compressor means power required.

$\therefore$ Mechanical efficiency,

$$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} \qquad \therefore 0.90 = \frac{48.548 \times 10^3}{P_{req}}$$

$$\therefore \quad \mathbf{P_{req} = 53.9422 \times 10^3 \text{ W}} \qquad \textbf{...Ans.}$$

Example 8.4 : *The ambient conditions at inlet is 1 bar at 20°C. At exit the total head temperature and pressure are 150°C and 3 bar with the static pressure at exit as 2.5 bar. Find isentropic efficiency and velocity of air at exit.*

Solution : Given data :

$p_{01} = 1$ bar, $T_{01} = 20°\text{C} = 20 + 273 = 293$ K, $p_2 = 2.5$ bar, $p_{02} = 3$ bar,

$T_{02} = 150°\text{C} = 150 + 273 = 423$ K

To find : i) η_{isen} ii) V_2

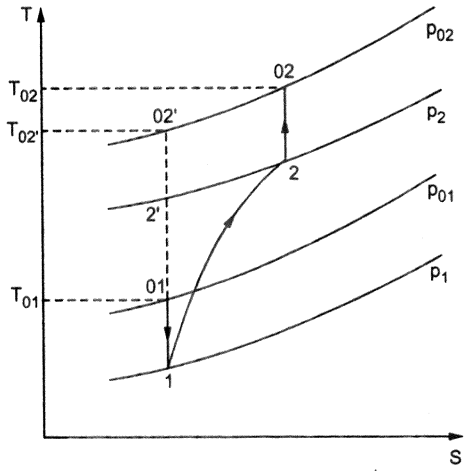


Fig. 8.20

Step - 1 : Calculate the isentropic efficiency

We know that,

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \qquad \ldots(\because p'_{02} = p_{02})$$

$$\therefore \quad \frac{T'_{02}}{293} = \left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} \quad \therefore \quad T'_{02} = 401.0402 \text{ K}$$

Isentropic efficiency, $\eta_{isen} = \dfrac{T'_{02} - T_{01}}{T_{02} - T_{01}} = \dfrac{401.0402 - 293}{423 - 293}$

$$\therefore \quad \eta_{isen} = 0.831 = 83.1\% \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the velocity of air at exit

We know that,

$$T_{02} - T_2 = \frac{V_2^2}{2C_p}$$

Also, $\frac{T_{02}}{T_2} = \left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{423}{T_2} = \left(\frac{3}{2.5}\right)^{\frac{1.4-1}{1.4}}$

$\therefore \quad T_2 = 401.5292$ K

$\therefore \quad 423 - 401.5292 = \frac{V_2^2}{2\times1.005\times10^3} \quad \therefore \quad V_2^2 = 43.1563\times10^3$

$\therefore \quad \mathbf{V_2 = 207.7409\ m/sec}$ **...Ans.**

Example 8.5 : *A centrifugal compressor supplies 16.5 kg/sec of air with total head pressure ratio of 4 : 1. The speed of compressor is 16,000 rpm. Inlet total head temperature is 20° C, slip factor is 0.9 power input factor is 1.04 and isentropic efficiency is 85%.*

Find : a) Outer diameter of impeller b) Power input.

☞ **[GTU: Summer-16, Winter-17]**

Solution : Given data :

$\dot{m} = 16.5$ kg/sec, $\frac{p_{02}}{p_{01}} = 4$, N = 16,000 rpm, $T_{01} = 20°C = 20 + 273 = 293$ K,

$\phi_s = 0.9$, $\phi_w = 1.04$, $\eta_{isen} = 85\% = 0.85$

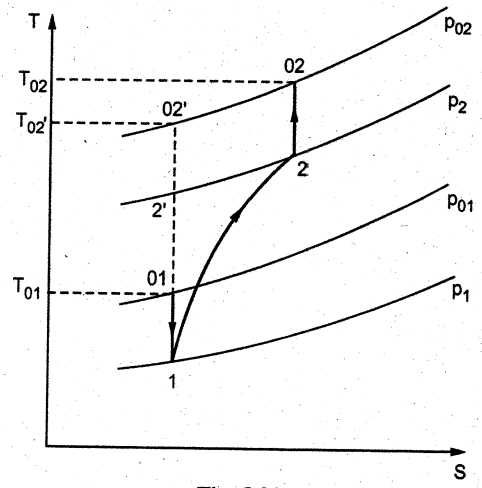


Fig. 8.21

To find : i) D_2 ii) P

Step - 1 : Calculate the overall diameter of the impeller

We know that,

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad ...(\because p'_{02} = p_{02})$$

$\therefore \quad \frac{T'_{02}}{293} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}} \quad T'_{02} = 435.3963$ K

As isentropic efficiency,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \quad \therefore \ 0.85 = \frac{435.3963 - 293}{T_{02} - 293}$$

$\therefore \quad T_{02} = 460.525$ K

As actual work required,

$$WD_{act} = C_p(T_{02} - T_{01})$$
$$= C_p(T_{02} - T_{01})$$
$$= 1.005\times10^3 \times (460.525 - 293) = 168.3626\times10^3 \ \text{J/kg}$$

Slip factor, $\phi_s = \frac{V_{w2}}{u_2} \quad \therefore 0.9 = \frac{V_{w2}}{u_2} \quad \therefore V_{w2} = 0.9u_2$

Also $WD_{act} = \phi_w V_{w2} u_2 \quad \therefore 168.3626\times10^3 = 1.04\times0.9u_2 \times u_2$

$u_2^2 = 179.8745\times10^3 \quad \therefore u_2 = 424.1161$ m/sec

As blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} \quad \therefore 424.1161 = \frac{\pi\times D_2 \times 16000}{60}$$

$\therefore \quad \mathbf{D_2 = 0.5062\ m}$ **... Ans.**

Step - 2 : Calculate the power input

Assuming mechanical efficiency $\eta_{mech} = 100\%$,

$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$

As power input means power required,

$$P_{req} = \dot{m} WD_{act} = 16.5 \times 168.3626 \times 10^3$$

$$\therefore \quad \mathbf{P = 2777.9829 \times 10^3 \ W} \qquad \textbf{... Ans.}$$

Example 8.6 : *A centrifugal compressor is used for supercharging in aircraft. It draws air with inlet conditions of 0.8 bar, 7°C and 200 m/sec. The air is compressed in impeller upto 1.5 bar and 70°C with velocity 500 m/sec. The mass flow rate of air 3 kg/sec.*

Find : a) Impeller power.

b) Isentropic efficiency of impeller based on - i) Static conditon ii) Stagnation condition.

☞ **[GTU: Summer-17, Winter-18]**

Solution : Given data :

$p_1 = 0.8$ bar, $T_1 = 7°C = 273 + 7 = 280$ K, $V_1 = 200$ m/sec, $p_2 = 1.5$ bar,

$T_2 = 70°C = 273 + 70 = 343$ K, $V_2 = 500$ m/sec, $\dot{m} = 3$ kg/sec.

To find : i) P ii) η_{isen} based on static conditions

iii) η_{isen} based on stagnation conditions

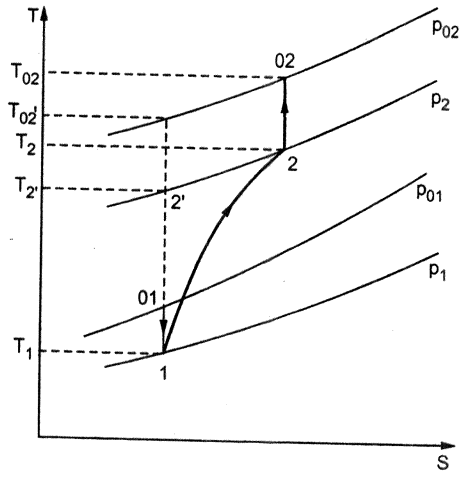


Fig. 8.22

Step - 1 : Calculate the isentropic work and actual work

Isentropic work, $WD_{isen} = C_p(T_2' - T_1) + \dfrac{V_2^2 - V_1^2}{2}$

We know that,

$$\frac{T_2'}{T_1} = \left(\frac{p_2'}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{T_2'}{280} = \left(\frac{1.5}{0.8}\right)^{\frac{1.4-1}{1.4}} \qquad \therefore T_2' = 335.0876 \text{ K}$$

$$\therefore \quad WD_{isen} = 1.005 \times 10^3 \times (335.0876 - 280) + \frac{500^2 - 200^2}{2}$$

$$= 160.363 \times 10^3 \text{ J/kg}$$

Actual work, $WD_{act} = C_p(T_2 - T_1) + \dfrac{V_2^2 - V_1^2}{2}$

$$= 1.005 \times 10^3 \times (343 - 280) + \frac{500^2 - 200^2}{2}$$

$$= 168.315 \times 10^3 \text{ J/kg}$$

Step - 2 : Calculate the impeller power and isentropic efficiency based on static conditions

Here, impeller power means power required.

$$P_{req} = \dot{m} \times WD_{act} = 3 \times 168.315 \times 10^3$$

$$\therefore \quad \mathbf{P_{req} = 504.945 \times 10^3 \ W} \qquad \textbf{... Ans.}$$

Isentropic efficiency for static conditons,

$$\eta_{isen} = \frac{WD_{isen}}{WD_{act}} = \frac{160.363 \times 10^3}{168.315 \times 10^3} = \mathbf{0.9527 = 95.27\ \%} \qquad \textbf{... Ans.}$$

Step - 3 : Calculate stagnation temperatures, pressures at points 1, 2, 2′

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

For point 1, $T_{01} - T_1 = \dfrac{V_1^2}{2C_p} \qquad \therefore T_{01} - 280 = \dfrac{200^2}{2 \times 1.005 \times 10^3}$

$$\therefore \quad T_{01} = 299.9004 \text{ K}$$

And, $\frac{T_{01}}{T_1} = \left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \therefore \frac{299.9004}{280} = \left(\frac{p_{01}}{0.8}\right)^{\frac{1.4-1}{1.4}}$

$\therefore \quad p_{01} = 1.0173$ bar

For point 2, $T_{02} - T_2 = \frac{V_2^2}{2C_p} \quad \therefore T_{02} - 343 = \frac{500^2}{2\times1.005\times10^3}$

$\therefore \quad T_{02} = 467.3781$ K

And, $\frac{T_{02}}{T_2} = \left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \therefore \frac{467.3781}{343} = \left(\frac{p_{02}}{1.5}\right)^{\frac{1.4-1}{1.4}}$

$\therefore \quad p_{02} = 4.4299$ bar

For point 2′, $p'_{02} = p_{02} = 4.4299$ bar

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{T'_{02}}{299.9004} = \left(\frac{4.4299}{1.0173}\right)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad T'_{02} = 456.5966$ K

Step - 4 : Calculate the isentropic efficiency based on stagnation conditions

Isentropic efficiency for stagnation conditions,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} = \frac{456.5966 - 299.9004}{467.3781 - 299.9004}$$

$\therefore \quad \eta_{isen} = 0.9356 = 93.56\ \%$ **... Ans.**

Example 8.7 : *A centrifugal compressor used as a supercharger for aeroengines handles 150 kg/min of air. The suction pressure and temperature are 1 bar and 290 K. The suction velocity is 80 m/sec. After compression, in the impeller, the conditions are 1.5 bar, 345 K and 220 m/sec.*

Find : a) Isentropic efficiency. b) Power required to drive the compressor.

c) Overall efficiency.

It is assumed that, K.E. of air in the impeller is entirely converted into pressure in the diffuser.

☞ **[GTU: Summer-16, Winter-17]**

Solution : Given data :

$\dot{m} = 150$ kg/min $= \frac{150}{60} = 2.5$ kg/sec, $p_1 = 1$ bar, $T_1 = 290$ K, $V_1 = 80$ m/sec,

$p_2 = 1.5$ bar, $T_2 = 345$ K, $V_2 = 220$ m/sec

To find : i) η_{isen} ii) P_{req} iii) η_o

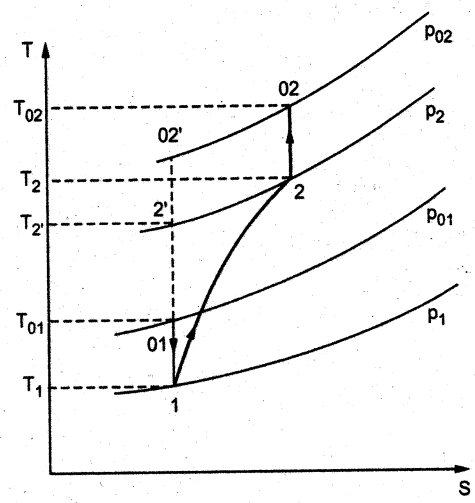


Fig. 8.23

Step - 1 : Calculate the isentropic work and actual work

Isentropic work, $WD_{isen} = C_p\left(T'_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}$

We know that,

$$\frac{T'_2}{T_1} = \left(\frac{p'_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{T'_2}{290} = \left(\frac{1.5}{1}\right)^{\frac{1.4-1}{1.4}} \quad \therefore T'_2 = 325.619\ \text{K}$$

$$\therefore \quad WD_{isen} = 1.005\times10^3 \times (325.619 - 290) + \frac{220^2 - 80^2}{2}$$

$$= 56.797\times10^3\ \text{J/kg}$$

Actual work, $WD_{act} = C_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$

$= 1.005 \times 10^3 \times (345 - 290) + \frac{220^2 - 80^2}{2}$

$= 76.275 \times 10^3$ J / kg

Step - 2 : Calculate the isentropic efficiency

Isentropic efficiency,

$$\eta_{isen} = \frac{WD_{isen}}{WD_{act}} = \frac{56.797 \times 10^3}{76.275 \times 10^3}$$

$$\eta_{isen} = \mathbf{0.74463 = 74.4634\ \%} \quad \textbf{... Ans.}$$

Step - 3 : Calculate the power required to drive the motor

Assuming mechanical efficiency $\eta_{mech} = 100\%$.

Power for the motor, $P_{req} = P_{cons} = \dot{m} \times WD_{act} = 2.5 \times 76.275 \times 10^3$

$\therefore$ $\mathbf{P_{req} = 190.6875 \times 10^3\ W}$ **... Ans.**

Step - 4 : Calculate stagnation temperatures, pressures at point 1, 2, 2′

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

For point 1, $T_{01} - T_1 = \frac{V_1^2}{2C_p}$ $\therefore$ $T_{01} - 290 = \frac{80^2}{2 \times 1.005 \times 10^3}$

$\therefore$ $T_{01} = 293.184$ K

And, $\frac{T_{01}}{T_1} = \left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore$ $\frac{293.184}{290} = \left(\frac{p_{01}}{1}\right)^{\frac{1.4-1}{1.4}}$

$\therefore$ $p_{01} = 1.0389$ bar

For point 2, $T_{02} - T_2 = \frac{V_2^2}{2C_p}$ $\therefore T_{02} - 345 = \frac{220^2}{2 \times 1.005 \times 10^3}$

$\therefore$ $T_{02} = 369.0796$ K

And, $\frac{T_{02}}{T_2} = \left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore$ $\frac{369.0796}{345} = \left(\frac{p_{02}}{1.5}\right)^{\frac{1.4-1}{1.4}}$

$\therefore$ $p_{02} = 1.8995$ bar

For point 2′, $p'_{02} = p_{02} = 1.8995$ bar

$\therefore$ $\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore \frac{T'_{02}}{293.184} = \left(\frac{1.8995}{1.0389}\right)^{\frac{1.4-1}{1.4}}$

$\therefore$ $T'_{02} = 348.3502$ K

Step - 5 : Calculate the overall efficiency

We know that, $\eta_o = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} = \frac{348.3502 - 293.184}{369.0796 - 293.184}$

$\therefore$ $\mathbf{\eta_o = 0.7268 = 72.6869\ \%}$ **... Ans.**

Example 8.8 : *The following data is used for a centrifugal compressor :*

Total pressure ratio	=	*3.6*
Diameter of inlet eye of compressor impeller	=	*35 cm*
Axial velocity at inlet	=	*140 m/sec*
Mass flow rate	=	*10 kg/sec*
Velocity in delivery pipe	=	*120 m/sec*
Tip speed of impeller	=	*460 m/sec*
Speed of compressor	=	*18,000 rpm*
Total head isentropic efficiency	=	*85%*
Ambient condition	=	*1.013 bar and* 15° C

Find : i) Static pressure and temperature at inlet and outlet ii) Static pressure ratio iii) Work done per kg iv) Theoretical power.

Solution : Given data :

$r_p = \frac{p_{02}}{p_{01}} = \frac{3.6}{1}$, $D_1 = 35$ cm $= 0.35$ m, $V_1 = 140$ m/sec, $\dot{m} = 10$ kg/sec,

$V_2 = 120$ m/sec, $u_2 = 460$ m/sec, $N = 18000$ rpm, $\eta_{isen} = 85\% = 0.85$,

$p_{01} = 1.013$ bar, $T_{01} = 15°\,C = 15 + 273 = 288$ K

To find : i) p_1, p_2, T_1, T_2 ii) $\frac{p_2}{p_1}$ iii) WD_{act} iv) P

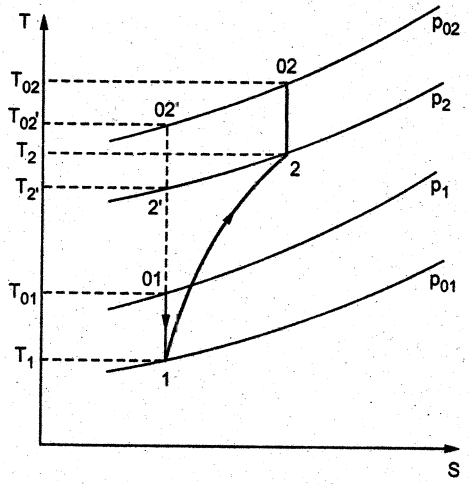


Fig. 8.24

Step - 1 : Calculate the static temperature and pressure at inlet and outlet, static pressure ratio

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

$\therefore$ At point 01, $T_{01} - T_1 = \dfrac{V_1^2}{2Cp} \quad \therefore 288 - T_1 = \dfrac{140^2}{2\times1.005\times10^3}$

$\therefore \quad T_1 = 278.2487$ K **...Ans.**

We know that,

$$\therefore \quad \frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T'_{02}}{288} = \left(\frac{3.6}{1}\right)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad T'_{02} = 415.2752$ K

Isentropic efficiency,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \quad \therefore 0.85 = \frac{415.2752 - 288}{T_{02} - 288}$$

$\therefore \quad T_{02} = 437.7355$ K

For stagnation state,

At point 02, $T_{02} - T_2 = \dfrac{V_2^2}{2C_p} \quad \therefore 437.7355 - T_2 = \dfrac{120^2}{2\times1.005\times10^3}$

$\therefore \quad T_2 = 430.5713$ K **...Ans.**

$p_{02} = r_p \times p_{01} = 3.6\times1.013 = 3.6468$ bar

We know that,

$$\frac{T_{02}}{T_2} = \left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{437.7355}{430.5713} = \left(\frac{3.6468}{p_2}\right)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad p_2 = 3.4421$ bar **...Ans.**

Also, $\dfrac{T_{01}}{T_1} = \left(\dfrac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \dfrac{288}{278.2487} = \left(\dfrac{1.013}{p_1}\right)^{\frac{1.4-1}{1.4}}$

$\therefore \quad p_1 = 0.8979$ bar **...Ans.**

Static pressure ratio,

$$\frac{p_2}{p_1} = \frac{3.4421}{0.8979} = 3.8335 \quad \textbf{... Ans.}$$

Step - 2 : Calculate the work of compressor per kg of air

Actual work done,

$$WD_{act} = C_p(T_{02} - T_{01}) = 1.005\times10^3 \times (437.7355 - 288)$$

$\therefore \quad WD_{act} = 150.4841\times10^3$ J / kg **... Ans.**

Step - 3 : Calculate the theoretical power required

Theoretical power,

$$P_{th} = \dot{m}\, C_p\, (\Delta T)_{isen} = \dot{m}\, C_p\, (T'_{02} - T_{01})$$
$$= 10\times1.005\times10^3 \times (415.2752 - 288)$$

$\therefore \quad P_{th} = 1279.1157\times10^3$ W **... Ans.**

Example 8.9 : *A rotary air compressor working between 1 bar and 2.5 bar has internal and external diameters of impeller as 300 mm and 600 mm respectively. The vane angle at inlet and outlet are* 30° and 45° *respectively. If air enters impeller at 15 m/s, find i) Speed of impeller in r.p.m. ii) Work done by compressor per kg of air.*

Solution : Given data :

$p_1 = 1$ bar, $p_2 = 2.5$ bar, $D_1 = 300$ mm $= 0.3$ m, $D_2 = 600$ mm $= 0.6$ m, $\theta = 30°$, $\phi = 45°$, $V_{f1} = V_{f2} = 15$ m/sec.

To find : i) N ii) WD

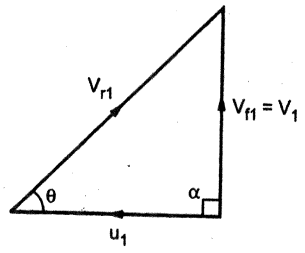


Fig. 8.25

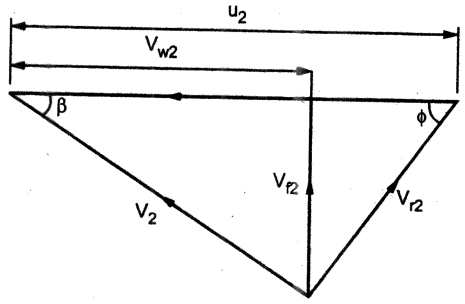


Fig. 8.26

Step - 1 : Calculate the speed of impeller

From Fig. 8.25 , by inlet velocity triangle,

$$\tan\theta = \frac{V_{f1}}{u_1} \quad \therefore \tan 30 = \frac{15}{u_1} \quad \therefore u_1 = 25.9807 \text{ m/sec}$$

Blade velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} \quad \therefore 25.9807 = \frac{\pi \times 0.3 \times N}{60}$$

$$\therefore \quad \mathbf{N = 1653.9827 \text{ rpm}} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the work done by compressor per kg of air

Outlet blade velocity,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1653.9827}{60} = 51.9613 \text{ m/sec}$$

From Fig. 8.29, by outlet velocity triangle,

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 51.9613 - \frac{15}{\tan 45} = 36.9613 \text{ m/sec}$$

- Work done by compressor per kg of air,

$$WD = V_{w2} u_2 = 36.9613 \times 51.9613$$

$$\therefore \quad \mathbf{WD = 1920.5571 \text{ J/kg}} \qquad \textbf{... Ans.}$$

Example 8.10 : *Following data is given for a centrifugal compressor.*
i) R. P. M. = 15000. ii) Air flow rate = 30 kg/sec.
iii) Air enters the compressor axially. iv) Conditions at exit : Radius = 0.3 m and relative velocity of air at the tip = 100 m/s at an angle of 80°.
Find the torque and power required to drive the compressor.

Solution : Given data :

$N = 15000$ rpm, $\dot{m} = 30$ kg/sec, $V_{w1} = 0$ ($\because$ Axial entry of air), $R_2 = 0.3$ m,

$\therefore D_2 = 2R_2 = 2 \times 0.3 = 0.6$ m, $V_{r2} = 100$ m/sec, $\phi = 80°$.

To find : i) T ii) P_{req}

Step - 1 : Calculate the torque required to drive the compressor

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 15000}{60} = 471.2388 \text{ m/sec}$$

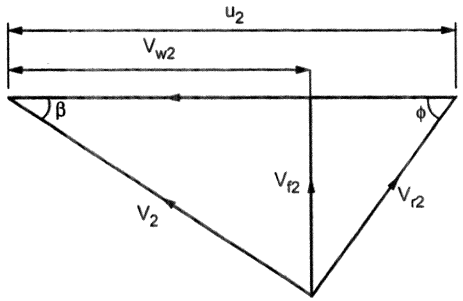


Fig. 8.27

From Fig. 8.27, by outlet velocity triangle,

$$V_{w2} = u_2 - V_{r2} \cos\phi = 471.2388 - 100 \times \cos 80 = 453.8739 \text{ m/sec}$$

$\therefore$ Torque required for compressor,

$$T = \dot{m} V_{w2} R_2 = 30 \times 453.8739 \times 0.3$$

$$\therefore \quad \mathbf{T = 4084.8651 \text{ N-m}} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate the power required to drive the compressor

Assuming mechanical efficiency $\eta_{mech} = 100$ %,

Power for the compressor,

$$P_{cons} = P_{req} = \dot{m} V_{w2} u_2 = 30 \times 453.8739 \times 471.2388$$

$$\therefore \quad \mathbf{P_{req} = 6416.4897 \times 10^3 \ W} \quad \textbf{... Ans.}$$

Example 8.11 : *A centrifugal compressor has inlet diameter of 180 mm. Impeller rotates at 21,000 rpm. Air at inlet has velocity of 115 m/sec. Inlet stagnation temperature = 300 K. Find blade angle at inlet and mach number of flow at tip of an eye.*

Solution : Given data :

$D_1 = 0.18$ m, N = 21000 rpm, $V_1 = 115$ m/sec, $T_{01} = 300$ K

To find : i) θ ii) M

Step - 1 : Calculate the blade angle of inlet

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

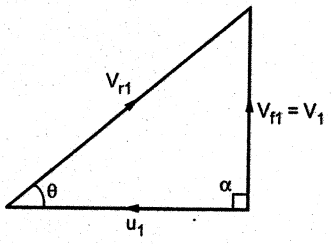


Fig. 8.28

∴ At inlet i.e. point 01,

$$T_{01} - T_1 = \frac{V_1^2}{2C_p}$$

$$\therefore \quad 300 - T_1 = \frac{115^2}{2 \times 1.005 \times 10^3}$$

$$\therefore \quad T_1 = 293.4203 \text{ K}$$

Blade velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.18 \times 21000}{60} = 197.9203 \text{ m/sec}$$

From Fig. 8.28, by inlet velocity triangle,

Inlet blade angle,

$$\tan\theta = \frac{V_1}{u_1} \quad \therefore \ \theta = \tan^{-1}\left(\frac{V_1}{u_1}\right)$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{115}{197.9203}\right) = \mathbf{30.1583°} \quad \textbf{... Ans.}$$

Step - 2 : Calculate the Mach number

Resultant inlet velocity,

$$V_{r1} = \sqrt{V_1^2 + u_1^2} = \sqrt{115^2 + 197.9203^2} = 228.9048 \text{ m/sec}$$

Sonic velocity,

$$V_s = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 293.4203} = 343.3602 \text{ m/sec}$$

Mach number,

$$\therefore \quad \mathbf{M = \frac{V_{r1}}{V_s} = \frac{228.9048}{343.3602} = 0.6666} \quad \textbf{... Ans.}$$

Example 8.12 : *A double sided centrifugal compressor has impeller eye root and tip diameters of 17.5 cm and 31.25 cm and is to deliver 18 kg of air/sec. at 16000 rpm. The ambient conditions are* 15° C *and 1 bar and the compressor is to be a part of stationary power plant. Find suitable values for the impeller vane angles at root and tip of the eye if the air is given* 20° *of prewhirl at all radii, The axial component of the inlet velocity is constant over the eye and is about 155 m/s. Find also the maximum Mach number at the eye.*

Solution : Given data :

$D_1 = 17.5$ cm $= 0.175$ m, $D_2 = 31.25$ cm $= 0.3125$ m, $\dot{m} = 18$ kg/sec, N = 16000 rpm,

$p_1 = 1$ bar, $T_1 = 15° C = 15 + 273 = 288$ K, $\psi = 20°$, $V_{f1} = V_{f2} = 155$ m/sec.

To find : i) θ ii) ϕ iii) M

Step - 1 : Calculate the vane angle at inlet

Blade velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.175 \times 16000}{60} = 146.6076 \text{ m/sec}$$

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3125 \times 16000}{60} = 261.7993 \text{ m/sec}$$

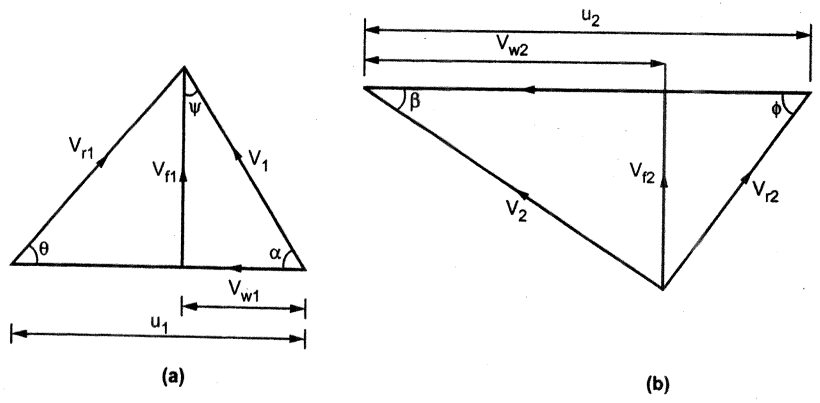


Fig. 8.29

From Fig. 8.29 (a), by inlet velocity triangle,

Whirl velocity at inlet, $V_{w1} = V_{f1} \tan\psi = 155 \times \tan(20) = 56.4153$ m/sec

$$\tan\theta = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{155}{146.6076 - 56.4153} \quad \therefore \theta = \mathbf{59.8054°}$$ **...Ans.**

Step - 2 : Calculate the vane angle at outlet

By neglecting friction, relative velocities at inlet and outlet,

$$V_{r1} = V_{r2} = \sqrt{V_{f1}^2 + (u_1 - V_{w1})^2} = \sqrt{155^2 + (146.6076 - 56.4153)^2}$$

$$= 179.3311 \text{ m/sec.}$$

From Fig. 8.32(b), by outlet velocity triangle,

$$\sin\phi = \frac{V_{f2}}{V_{r2}} = \frac{155}{179.3311} \quad \therefore \phi = \mathbf{59.8054°}$$ **...Ans.**

Step - 3 : Calculate the Mach number

Sonic Velocity,

$$V_s = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 288} = 340.174 \text{ m/sec.}$$

$\therefore$ Mach Number,

$$M = \frac{V_{r2}}{V_s} = \frac{179.3311}{340.174}$$

$\therefore$ $\mathbf{M = 0.5271}$ **... Ans.**

Example 8.13 : *A centrifugal compressor delivers 10 m^3 /sec of air when running at 10,000 rpm. The air is drawn in at 1 bar and 300 K and delivered at 4 bar. The isentropic efficiency is 80 %. Blades are radial at outlet and constant flow velocity is 64 m/s. The outer diameter of impeller is twice the inner and slip factor may be taken as 0.9. Calculate :*

i) Temperature of air at outlet

ii) Power required to drive the compressor

iii) Impeller diameter at inlet and outlet

iv) Impeller blade angle at inlet

v) Diffuser blade angle at inlet.

Solution : Given data :

$Q = 10 \text{ m}^3/\text{s}$, N = 10,000 rpm, p = 1 bar, $T_1 = 300$ K, $p_2 = 4$ bar,

$\eta_{isen} = 80\ \% = 0.8$, $V_{f1} = V_{f2} = 64$ m/sec, $\phi = 90°$, ($\because$ Blades are radial at outlet),

$D_2 = 2\ D_1$, $\phi_s = 0.9$

To find : i) T_1 ii) P_{req} iii) D_1, D_2 iv) θ v) β

Step - 1 : Calculate the final temperature of air

We know that, $\frac{T_2'}{T_1} = \left(\frac{p_2'}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T_2'}{300} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$

$$T_2' = 445.7982 \text{ K}$$

As isentropic efficiency,

$$\eta_{isen} = \frac{T_2' - T_1}{T_2 - T_1} \quad \therefore 0.80 = \frac{445.7982 - 300}{T_2 - 300}$$

$\therefore$ $\mathbf{T_2 = 482.2477 \text{ K}}$ **... Ans.**

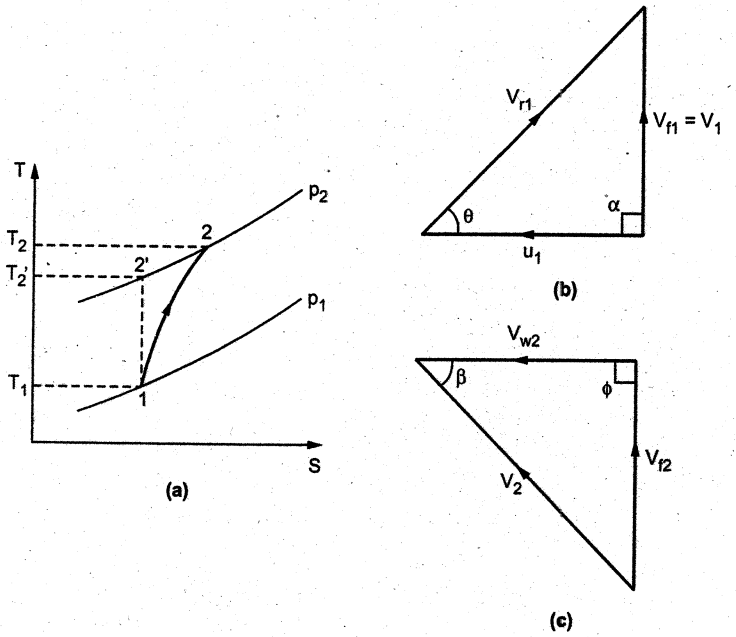


Fig. 8.30

Step - 2 : Calculate the power required to drive the compressor

By Ideal Gas Equation, $pV = mRT \therefore p = \rho RT$ or $p_1 = \rho RT_1$

$\therefore \quad 1\times10^5 = \rho\times287\times300 \therefore \rho = 1.1614\ kg/m^3$

Mass flow rate, $\dot{m} = \rho Q = 1.1614\times10 = 11.614$ kg/sec.

Assuming mechanical efficiency $\eta_{mech} = 100\ \%$

$\therefore$ Power for the compressor, $P_{req} = P_{cons} = \dot{m}\, W_{Dact}$

$= \dot{m} \times C_p (T_2 - T_1)$

$= 11.614\times1.005\times10^3 \times (482.2477 - 300)$

$\therefore \quad P_{req} = \mathbf{2127.2079\times10^3\ W}$ **... Ans.**

Step - 3 : Calculate the impeller diameter at inlet and outlet

Work done $WD_{act} = \dfrac{P_{req}}{\dot{m}} = \dfrac{2127.2079\times10^3}{11.614} = 183.1589\times10^3\ J/kg$

Slip factor, $\phi_s = \dfrac{V_{w2}}{u_2} \quad \therefore 0.9 = \dfrac{V_{w2}}{u_2} \quad \therefore V_{w2} = 0.9\, u_2$

Also, $WD_{act} = V_{w2}\, u_2 \therefore 183.1589\times10^3 = 0.9\, u_2 \times u_2$

$u_2^2 = 203.5098\times10^3 \quad \therefore u_2 = 451.1207$ m/sec

$\therefore$ Whirl velocity at outlet

$V_{w2} = 0.9\, u_2 = 0.9\times451.1207 = 406.0086$ m/sec

As blade velocity at outlet,

$u_2 = \dfrac{\pi D_2 N}{60}$

$\therefore \quad 451.1207 = \dfrac{\pi\times D_2\times10000}{60} \quad \therefore \mathbf{D_2 = 0.8615\ m}$ **... Ans.**

As $D_2 = 2D_1 \therefore \quad D_1 = \dfrac{1}{2}\times D_2 = \dfrac{1}{2}\times0.8615$

$\therefore \quad \mathbf{D_1 = 0.43075\ m}$ **...Ans.**

Step - 4 : Calculate the impeller blade angle, diffuser blade angle at inlet

Blade velocity at inlet, $u_1 = \dfrac{\pi D_1 N}{60}$

$\therefore \quad u_1 = \dfrac{\pi\times0.43075\times10000}{60} = 225.5401$ m/sec

From Fig. 8.33(b)., by inlet velocity triangle,

$\tan\theta = \dfrac{V_{f1}}{u_1} \quad \therefore \theta = \tan^{-1}\left(\dfrac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\dfrac{64}{225.5401}\right)$

$\therefore \quad \theta = \mathbf{15.8419^\circ}$ **... Ans.**

From Fig. 8.33(c), by outlet velocity triangle,

$\tan\beta = \dfrac{V_{f2}}{V_{W2}} \quad \therefore \beta = \tan^{-1}\left(\dfrac{V_{f2}}{V_{W2}}\right) = \tan^{-1}\left(\dfrac{64}{406.0086}\right)$

$\therefore \quad \beta = \mathbf{8.9579^\circ}$ **... Ans.**

Example 8.14 : *A centrifugal blower compresses 4.8 m^3 / s of air from 1 bar and 20°C to 1.5 bar. The index of compression n is 1.5. The flow velocity at inlet and outlet of the machine is the same and equal to 65 m/s. The inlet and outlet impeller diameters are 0.34 m and 0.64 m respectively. The blower rotates at 7000 r.p.m. Calculate :*

i) The blade angles at inlet and outlet of impeller.

ii) The absolute angle at tip of impeller.

iii) The breadth of blade at inlet and outlet.

It is assumed that no diffuser is employed and the whole pressure increase takes place in the impeller and the blades have negligible thickness.

Solution : Given data :

$Q = 4.8\ m^3$ / sec, $p_1 = 1$ bar, $T_1 = 20°C = 20 + 273 = 293$ K, $n = 1.5$, $p_2 = 1.5$ bar,

$V_{f1} = V_{f2} = 65$ m / sec, $D_1 = 0.34$ m, $D_2 = 0.64$ m, $\therefore R_1 = \frac{0.34}{2} = 0.17$ m,

$R_2 = \frac{0.64}{2} = 0.32$ m, $N = 7000$ rpm.

To find : i) θ, ϕ ii) β iii) B_1, B_2

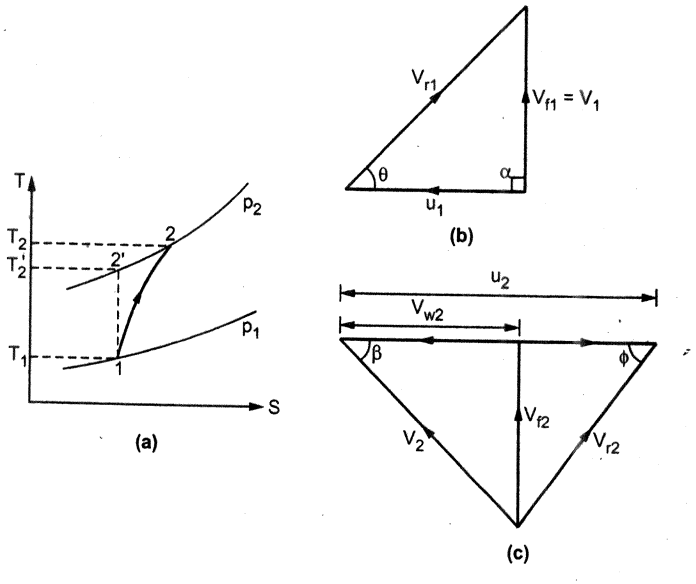


Fig. 8.31

Step - 1 : Calculate the blade angles at inlet and outlet of the impeller

For polytropic process,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \therefore \frac{T_2}{293} = \left(\frac{1.5}{1}\right)^{\frac{1.5-1}{1.5}} \qquad \therefore T_2 = 335.4012 \text{ K}$$

Blade velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.34 \times 7000}{60} = 124.6165 \text{ m/sec}$$

Blade velocity at inlet,

$$u_2 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.64 \times 7000}{60} = 234.5722 \text{ m/sec}$$

Work done,

$$WD_{act} = C_p (T_2 - T_1) = 1.005 \times 10^3 \times (335.4012 - 293)$$

$$= 42.6132 \times 10^3 \text{ J/kg}$$

Also, $WD_{act} = V_{w2}\, u_2 \qquad \therefore 42.6132 \times 10^3 = V_{W2} \times 234.5722$

$\therefore \quad V_{w2} = 181.6634$ m/sec

From Fig. 8.31(b), by inlet velocity triangle,

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{65}{124.6165}$$

$\therefore \quad \theta = \mathbf{27.5465°}$ **... Ans.**

From Fig. 8.31(c), by outlet velocity triangle,

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{65}{234.5722 - 181.6634}$$

$\therefore \quad \phi = \mathbf{50.855°}$ **... Ans.**

Step - 2 : Calculate the absolute angle at impeller tip

From Fig. 8.31(c), by outlet velocity triangle,

$$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{65}{181.6634}$$

$\therefore \quad \beta = \mathbf{19.6874°}$ **... Ans.**

Step - 3 : Calculate the breadth of blade at inlet and outlet

Discharge at inlet,

$$Q_1 = 2\pi R_1 B_1 V_{f1} \quad \therefore \quad 4.8 = 2\pi \times 0.17 \times B_1 \times 65$$

$$\therefore \quad \mathbf{B_1 = 0.06913\ m} \quad \text{... Ans.}$$

For polytropic process,

$$p_1 v_1^{\,n} = p_2 v_2^{\,n} \therefore 1 \times 4.8^{1.5} = 1.5 \times v_2^{\,1.5} \quad \text{... } (\because v_1 = Q_1)$$

$$\therefore \quad v_2 = 3.663\ m^3/sec = Q_2 \quad \text{... [Volume flow rate } (v_2 = Q_2)]$$

$\therefore$ Discharge at outlet,

$$Q_2 = 2\pi R_2 B_2 V_{f2} \quad \therefore 3.663 = 2\pi \times 0.32 \times B_2 \times 65$$

$$\therefore \quad \mathbf{B_2 = 0.028\ m} \quad \text{... Ans.}$$

Example 8.15 : *A centrifugal compressor running at 10,000 rpm delivers 660 m^3/min of free air. The air is compressed from 1 bar, 20° C to a pressure ratio of 4 with isentropic efficiency 82 %. The blades are radial at outlet of impeller and flow velocity is 62 m/sec (constant). The outer radius of impeller is twice of inner and slip factor is 0.9. The blade area coefficient is 0.9 at inlet. Find :*

a) Final temperature of air
b) Power
c) Impeller diameter at inlet and outlet
d) Impeller blade angle at inlet
e) Diffuser blade angle at inlet
f) Width of impeller at inlet.

Solution : Given data :

$N = 10{,}000$ rpm, $Q = 660\ m^3/min = \frac{660}{60} = 11\ m^3/sec$, $p_1 = 1$ bar,

$T_1 = 20°C = 20 + 273 = 293$ K, $r_p = 4$,

$\eta_{isen} = 0.82 = 82\ \%$, $R_2 = 2\,R_1$, $\phi_s = 0.9$,

$K_a = 0.9$, $V_{f1} = V_{f2} = 62$ m/sec, $\phi = 90°$ ($\because$ Blades are radial at outlet)

To find : i) T_2 ii) P iii) D_1, D_2 iv) θ v) β vi) B_1

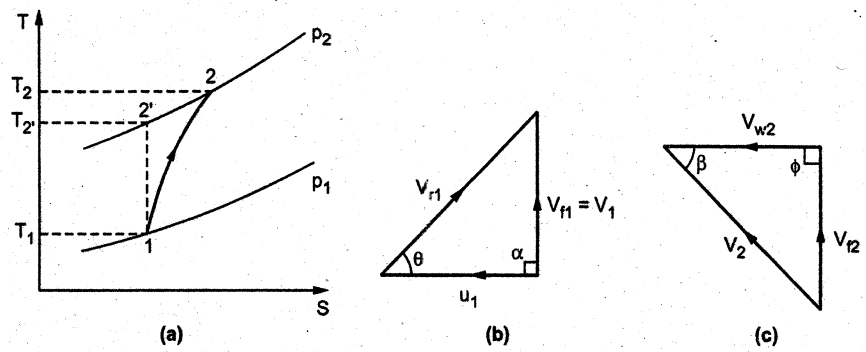


Fig. 8.32

Step - 1 : Calculate the final temperature of air

We know that,

$$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2'}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{T_2'}{293} = (4)^{\frac{1.4-1}{1.4}} \quad \therefore T_2' = 435.3963\ K$$

As isentropic efficiency,

$$\eta_{isen} = \frac{T_2' - T_1}{T_2 - T_1} \quad \therefore 0.82 = \frac{435.3963 - 293}{T_2 - 293}$$

$$\therefore \quad \mathbf{T_2 = 466.654\ K} \quad \text{... Ans.}$$

Step - 2 : Calculate the power

By ideal Gas Equation,

$$pV = mRT \quad \therefore p = \rho RT$$

or $$p_1 = \rho R T_1$$

$$\therefore \quad 1 \times 10^5 = \rho \times 287 \times 293 \quad \therefore \rho = 1.1891\ kg/m^3$$

Mass flow rate, $\dot{m} = \rho Q = 1.1891 \times 11 = 13.0801$ kg/sec

Assuming mechanical efficiency $\eta_{mech} = 100\ \%$

$$\eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

$$\therefore \quad P_{cons} = P_{req} = \dot{m}\, W D_{act}$$

$$= \dot{m} \times C_p (T_2 - T_1)$$

$$= 13.0801 \times 1.005 \times 10^3 \times (466..654 - 293)$$

$$\therefore \quad P_{req} = \mathbf{2282.7687 \times 10^3\ W} \qquad \textbf{... Ans.}$$

Step - 3 : Calculate the impeller diameter at inlet and outlet

As we know that work done,

$$WD_{act} = \frac{P_{req}}{\dot{m}} = \frac{2282.7687 \times 10^3}{13.0801} = 174.5222 \times 10^3\ J/kg$$

Slip factor, $\phi_s = \dfrac{V_{w2}}{u_2} \quad \therefore 0.9 = \dfrac{V_{w2}}{u_2} \quad \therefore V_{w2} = 0.9\, u_2$

Also, $WD_{act} = V_{w2}\, u_2 \quad \therefore 174.5222 \times 10^3 = 0.9\, u_2 \times u_2$

$$u_2^2 = 193.9135 \times 10^3 \quad \therefore u_2 = 440.3561 \text{ m/sec}$$

Whirl velocity at outlet, $V_{w2} = 0.9 u_2 = 0.9 \times 440.3561$

$$= 396.3204 \text{ m/sec}$$

As blade velocity at outlet, $u_2 = \dfrac{\pi D_2 N}{60}$

$$\therefore \quad 440.3204 = \frac{\pi \times D_2 \times 10000}{60} \quad \therefore \mathbf{D_2 = 0.841\ m} \qquad \textbf{... Ans.}$$

As $D_2 = 2D_1 \therefore D_1 = \dfrac{1}{2} \times D_2 = \dfrac{1}{2} \times 0.841$

$$\therefore \quad \mathbf{D_1 = 0.4205\ m} \qquad \textbf{...Ans.}$$

Step - 4 : Calculate the impeller blade angle, diffuser blade angle and width of impeller at inlet

Blade velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.4205 \times 10000}{60}$$

$$= 220.1732 \text{ m/sec}$$

From Fig. 8.35(b), inlet velocity triangle,

$$\tan\theta = \frac{V_{f1}}{u_1 - V_{w1}} \quad \therefore \theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right)$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{62}{220.1732}\right) = \mathbf{15.727^\circ} \qquad \textbf{... Ans.}$$

From Fig. 8.35(c), outlet velocity triangle,

$$\tan\beta = \frac{V_{f2}}{V_{w2}} \quad \therefore \beta = \tan^{-1}\left(\frac{V_{f2}}{V_{w2}}\right)$$

$$= \tan^{-1}\left(\frac{62}{396.3145}\right)$$

$$\therefore \quad \beta = \mathbf{8.8913^\circ} \qquad \textbf{... Ans.}$$

As discharge, $Q = K_a \times \pi \times D_1 \times B_1 \times V_{f1}$

$$\therefore \quad 11 = 0.9 \times \pi \times 0.4205 \times B_1 \times 62$$

$$\therefore \quad \mathbf{B_1 = 0.1492\ m} \qquad \textbf{... Ans.}$$

Example 8.16 : *A centrifugal compressor runs at 16000 r.p.m. and the power required to run the compressor amounts to 2300 kW. The outer diameter of the impeller is 50 cm. The uniform width of the casing of vortex chamber between impeller and diffuser is 4 cm. Static conditions at a radius of 27 cm were measured and it were equal to 2.4 bar and 390 K. Surrounding pressure is 1 bar.*

Assuming a slip factor of 0.94, find the mass flow rate of gas, the resultant velocity and the temperature at given radius of 27 cm. Assuming, C_p = 1.05 kJ/kgK and γ = 1.4.

Solution : Given data :

N = 16,000 rpm, P_{req} = 2300 kW, D_2 = 50 cm = 0.5 m, B = 4 cm = 0.04 m, p_1 = 1 bar, ϕ_s = 0.94, For R_c = 27 cm = 0.27 m, p_C = 2.4 bar, T_C = 390 K, $C_p = 1.05 \times 10^3$ J/kg K, γ = 1.4

To find : i) $\dot{m}$ ii) $(V_R)_C$ iii) T_C

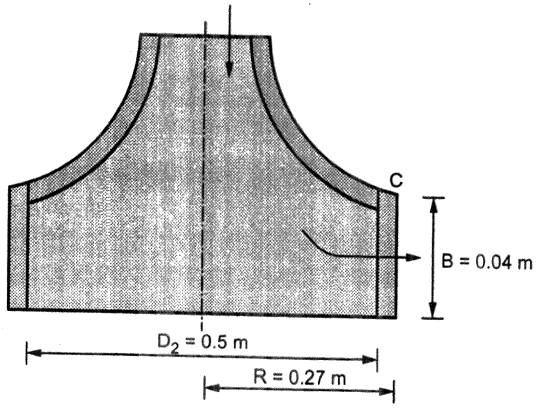


Fig. 8.33

Step - 1 : Calculate the mass flow rate of gas

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 16000}{60} = 418.879 \text{ m/sec}$$

Slip factor,

$$\phi_s = \frac{V_{w2}}{u_2} \quad \therefore \quad 0.94 = \frac{V_{w2}}{u_2} \quad \therefore V_{w2} = 0.94\, u_2$$

Work done per kg of gas

$$WD_{act} = V_{w2}\, u_2 = 0.94 u_2 \times u_2$$
$$= 0.94 \times 418.879^2 = 164.932 \times 10^3 \text{ J/kg}$$

Assuming mechanical efficiency $\eta_{mech} = 100\ \%$

$$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

Power for the compressor

$$P_{cons} = P_{req} = \dot{m}\, WD_{act} \quad \therefore \quad 2300 \times 10^3 = \dot{m} \times 164.932 \times 10^3$$

$$\therefore \quad \dot{m} = \mathbf{13.9451 \text{ kg/sec}} \qquad \textbf{... Ans.}$$

Step - 2 : Calculate resultant velocity and total temperature at radius of 0.27 m

Area of flow at section C,

$$A_C = 2\pi R_C \times B = 2\pi \times 0.27 \times 0.04 = 0.06785 \text{ m}^2$$

Gas constant,

$$R = \frac{(\gamma - 1)}{\gamma} C_p = \left(\frac{1.4 - 1}{1.4}\right) \times 1.05 \times 10^3 = 300 \text{ N-m/kg-K}$$

By Ideal Gas Equation,

For unit mass, m = 1 kg

$$p_C v_C = R T_C$$

$$\therefore \quad 2.4 \times 10^5 \times v_C = 300 \times 390$$

$$\therefore \quad v_C = 0.4875 \text{ m}^3/\text{kg}$$

Total discharge of gas,

$$Q = \dot{m} \cdot v_C = 13.9451 \times 0.4875 = 6.7982 \text{ m}^3/\text{sec}$$

Also, $Q = A_C V_{fC} \quad \therefore \quad 6.7982 = 0.06785 \times V_{fC}$

$$\therefore \quad V_{fC} = 100.1945 \text{ m/sec}$$

Whirl velocity at outlet,

$$V_{w2} = \phi_s u_2 = 0.94 \times 418.879 = 393.7462 \text{ m/sec.}$$

For free vortex,

$$\text{Whirl velocity} \propto \frac{1}{\text{Radius}}$$

$$\frac{V_{w2}}{V_{wC}} = \frac{R_C}{R_2}$$

$$\therefore \quad \frac{393.7462}{V_{wC}} = \frac{0.27}{0.25}$$

$$\therefore \quad V_{wC} = 364.5792 \text{ m/sec}$$

$\therefore$ Resultant velocity at C

$$(V_T)_C = \sqrt{(V_{fC})^2 + (V_{fC})^2} = \sqrt{100.1945^2 + 364.5798^2}$$

$$\therefore \quad (V_T)_C = 378.097 \text{ m/sec}$$

Total temperature i.e. stagnation temperature at C

For stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

At C, $T_{0C} - T_C = \dfrac{V_C^2}{2C_p} \quad \therefore \quad T_{0C} - 390 = \dfrac{378.097^2}{2 \times 1.05 \times 10^3}$ ... $[\because V_C = (V_R)_C]$

$$\therefore \quad T_{0C} = \mathbf{458.0749 \text{ K}} \qquad \textbf{...Ans.}$$

Example 8.17 : *A single inlet centrifugal compressor handles 540 kg/min of air with ambient air conditions as 1 bar and 20° C. The compressor runs at 18000 r.p.m. with isentropic efficiency 90 %. The air is compressed in the compressor from 1 bar static pressure to 4 bar total pressure. The air enters the impeller eye with a velocity of 145 m/sec. Assume the ratio of whirl speed to tip speed as 0.9. Calculate :*

i) Rise in total temperature in compressor if change in kinetic energy is negligible.

ii) Tip diameter of impeller

iii) Required power

iv) Eye diameter if diameter of the hub is 12 cm.

Solution : Given data :

$\dot{m} = 540$ kg/min $= \frac{540}{60} = 9$ kg/sec, $p_1 = 1$ bar, $T_1 = 20°C = 20+273 = 293$ K,

$N = 18{,}000$ rpm, $p_{02} = 4$ bar, $\eta_{isen} = 90\% = 0.9$, $V_1 = 145$ m/sec, $\frac{V_{w2}}{u_2} = 0.9 = \phi_s$,

$D_h = 12$ cm $= 0.12$ m

To find : i) $(\Delta T)_{act}$ ii) D_2 iii) P_{req} iv) D_1

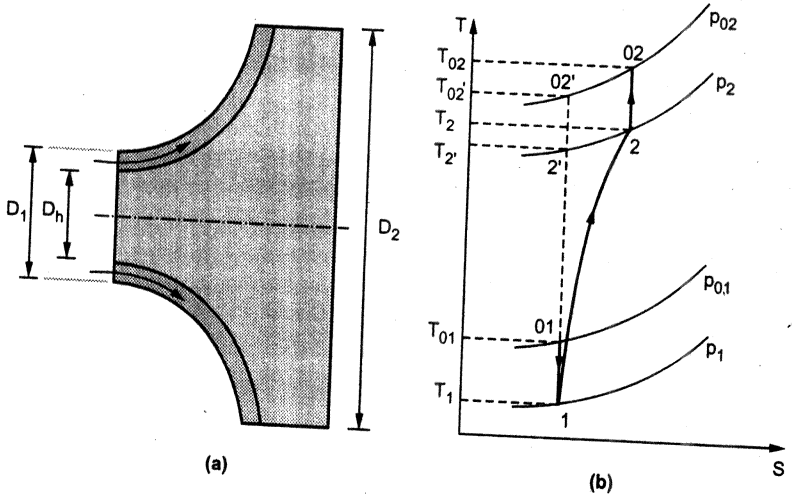


Fig. 8.34

Step - 1 : Calculate the rise in total temperature during compression if change in K.E. is negligible

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

For point 1, $T_{01} - T_1 = \frac{V^2}{2C_p}$ $\therefore$ $T_{01} - 293 = \frac{145^2}{2 \times 1.005 \times 10^3}$

$$T_{01} = 303.4601 \text{ K}$$

We know that,

$$\frac{T_{01}}{T_1} = \left(\frac{p_{01}}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \frac{303.4601}{293} = \left(\frac{p_{01}}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad p_{01} = 1.1306 \text{ bar}$$

Also,

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots [\because p'_{02} = p_{02}]$$

$$\therefore \quad \frac{T'_{02}}{303.4601} = \left(\frac{4}{1.1306}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T'_{02} = 435.3991 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{(\Delta T)_{isen}}{(\Delta T)_{act}} = \frac{T'_{02} - T_{01}}{(\Delta T)_{act}}$$

$$\therefore \quad 0.9 = \frac{435.3991 - 303.4601}{(\Delta T)_{act}}$$

$$\therefore \quad (\Delta T)_{act} = \mathbf{146.5988 \text{ K}} \quad \ldots \textbf{Ans.}$$

Step - 2 : Calculate the tip diameter of impeller

Actual work,

$$WD_{act} = C_p \times (\Delta T)_{act} = 1.005 \times 10^3 \times 146.5988$$

$$WD_{act} = 147.3317 \times 10^3 \text{ J/kg}$$

Slip factor $\phi_s = \frac{V_{w2}}{u_2} \quad \therefore 0.9 = \frac{V_{w2}}{u_2} \quad \therefore V_{w2} = 0.9\, u_2$

Also, $WD_{act} = V_{w2}\, u_2 \quad \therefore 147.3317 \times 10^3 = 0.9\, u_2 \times u_2$

$$u_2^2 = 163.7018 \times 10^3 \quad \therefore \quad u_2 = 404.6007 \text{ m/sec}$$

Blade blade at outlet,

$$u_2 = \frac{\pi D_2 N}{60} \quad \therefore 404.6007 = \frac{\pi \times D_2 \times 18000}{60}$$

$$\therefore \quad \mathbf{D_2 = 0.4292 \text{ m}} \quad \textbf{... Ans.}$$

Step - 3 : Calculate the power required

Assuming mechanical efficiency $\eta_{mech} = 100\ \%$

$$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

Power required, $P_{req} = P_{cons} = \dot{m} \times WD_{act} = 9 \times 147.3317 \times 10^3$

$$\therefore \quad \mathbf{P_{req} = 1325.9853 \times 10^3 \text{ W}} \quad \textbf{... Ans.}$$

Step - 4 : Calculate the eye diameter

By Ideal Gas Equation,

$$pv = mRT \quad \therefore \quad p = \rho R T \quad \ldots \left(\rho = \frac{m}{v}\right)$$

$$\therefore \quad p_1 = \rho R T_1 \quad \therefore \quad 1 \times 10^5 = \rho \times 287 \times 293$$

$$\therefore \quad \rho = 1.1891 \text{ kg/m}^3$$

Mass flow rate, $\dot{m} = \rho A V_1 = \rho \times \frac{\pi}{4} \times \left(D_1^2 - D_h^2\right) \times V_1$

$$\therefore \quad 9 = 1.1891 \times \frac{\pi}{4} \times \left(D_1^2 - 0.12^2\right) \times 145$$

$$\therefore \quad \mathbf{D_1 = 0.2843 \text{ m}} \quad \textbf{... Ans.}$$

Example 8.18 : *The impeller of a centrifugal compressor has the inlet and outlet diameter of 0.3 and 0.6 m respectively. The intake is from the atmosphere at 100 kPa and 300 K, without any whirl component. The outlet blade angle is 75°. The speed is 10000 rpm and the velocity of flow is constant at 120 m/s. If the blade width at intake is 6 cm, calculate :*

i) Specific work. ii) Exit pressure. iii) Mass flow rate.

iv) Power required to drive compressor if the overall efficiency can be assumed at 0.7.

Solution : $D_1 = 0.3$ m, $D_2 = 0.6$ m, $p_1 = 100$ kPa, $T_1 = 300$ K, $V_{w1} = 0$, $\phi = 75°$, $N = 10{,}000$ rpm, $V_{f1} = V_{f2} = 120$ m/s,

$B_1 = 0.06$ m, $\eta_0 = 0.7$

To find : i) Specific work ii) Exit pressure iii) $\dot{m}$ iv) power

Step - 1 : Calculate specific work

Refer Fig. 8.35.

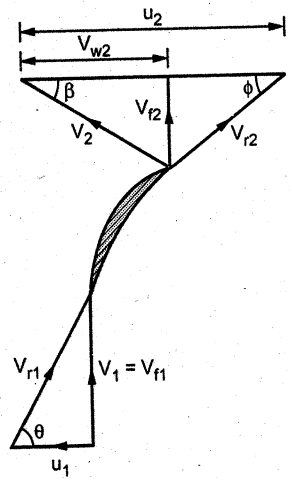


Fig. 8.35

We know that,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 10{,}000}{60} = 157.79 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 10{,}000}{60} = 314.16 \text{ m/s}$$

From velocity triangle, (Refer Fig. 8.35)

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 314.16 - \frac{120}{\tan 75}$$

$$V_{w2} = 282 \text{ m/s}$$

Specific work is given by,

Specific work = $V_{w2} u_2 = 282 \times 314.16 = 88.593 \times 10^3$ J / kg ... **Ans.**

Step - 2 : Calculate exit pressure

We know that,

$$W = C_p (\Delta T)_{th} = C_p (T_2' - T_1)$$

$$\therefore \quad 88.593 \times 10^3 = 1.005 \times 10^3 (T_2' - 300)$$

$$\therefore \quad T_2' = 388.153 \text{ K}$$

But, $$\frac{p_2'}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{388.153}{300}\right)^{\frac{1.4}{1.4-1}}$$

$$\therefore \quad \frac{p_2'}{p_1} = 2.463 \quad \therefore \quad p_2 = 2.463\ p_1$$

$$\therefore \quad p_2 = 2.463 \times 100 = \mathbf{2.463 \text{ kPa}}$$ ... **Ans.**

Step - 3 : Calculate mass flow rate

Volume flow rate is given by,

$$Q = \pi D_1 B_1 V_{f1} = \pi \times 0.3 \times 0.06 \times 120 = 6.786 \text{ m}^3/\text{s}$$

We know that,

$$\dot{m} = \rho Q = 1.16 \times 6.786 \quad \text{... (Assume } \rho = 1.16 \text{ kg/m}^3\text{)}$$

$$\therefore \quad \dot{m} = \mathbf{7.871 \text{ kg/s}}$$ ...**Ans.**

Step - 4 : Calculate power required to drive the compressor

Theoretical power is given by,

$$P_{th} = \text{Specific work} \times \dot{m} = 88.593 \times 10^3 \times 7.871 = 697.315 \text{ kW}$$

Overall efficiency is given by

$$\eta_0 = \frac{\text{Theoretical Power}}{\text{Actual Power}} \quad \therefore \quad 0.7 = \frac{697.315}{\text{Actual Power}}$$

$\therefore$ **Actual Power = 996.164 kW** ... **Ans.**

Example 8.19 : *A centrifugal compressor impeller admits 20 kg/s air at static state of 1 bar, 300 K and runs at 15000 rpm. Isentropic efficiency is 90 % for the compression upto 5 bar total pressure. The air enters the impeller eye without prewhirl with the velocity of 120 m/s. Considering the ratio of whirl velocity to tip speed as 0.9 and the internal diameter of the impeller eye as 20 cm, determine :*
i) Rise in the total temperature in the compressor ii) Impeller tip speed
iii) Impeller tip diameter. iv) Power required to drive compressor.
v) Outer diameter of the impeller eye.

Solution : Given data :

$\dot{m}$ = 20 kg/sec, p_1 = 1 bar, T_1 = 300 K, N = 15000 rpm, η_{isen} = 90 % = 0.9, p_{02} = 5 bar, V_{w1} = 0,

V_1 = 120 m/sec, $\frac{V_{w2}}{u_2}$ = 0.9, D_i = 20 cm = 0.2 m.

To find : Total temperature rise, u_2, D_2, p, D_0.

Step - 1 : Calculate total temperature rise.

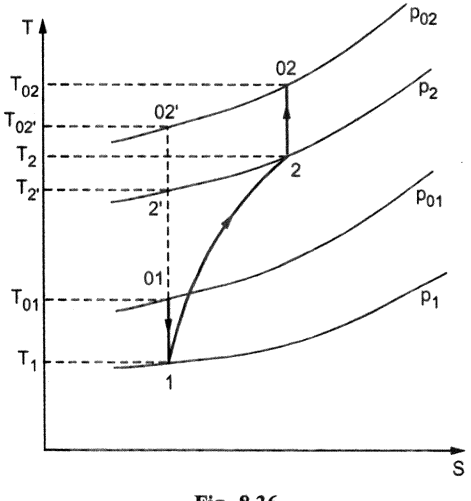


Fig. 8.36

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

At point 01,

$$T_{01} - T_1 = \frac{V_1^2}{2C_p} \qquad \therefore \quad T_{01} - 300 = \frac{120^2}{2\times 1.005\times 10^3}$$

$$\therefore \qquad T_{01} = 307.16 \text{ K}$$

Now, $$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad \therefore \quad \frac{p_{01}}{1} = \left(\frac{307.16}{300}\right)^{\frac{1.4}{1.4-1}}$$

$$\therefore \qquad p_{01} = 1.086 \text{ bar}$$

We know that,

$$\therefore \qquad \frac{T'_{02}}{T_{01}} = \left(\frac{p_{02'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \qquad \frac{T_{02'}}{307.16} = \left(\frac{5}{1.086}\right)^{\frac{1.4-1}{1.4}} \qquad \therefore \quad T_{02'} = 475.151 \text{ K}$$

Now, Isentropic efficiency is,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \qquad \therefore \quad 0.9 = \frac{475.151 - 307.16}{T_{02} - T_{01}}$$

$$\therefore \qquad \textbf{Total temperature rise} = T_{02} - T_{01} = 186.656 \text{ K}$$

Step - 2 : Calculate power required to run compressor

$$\text{Power (P)} = \dot{m} C_p (T_{02} - T_{01})$$

$$= 20\times 1.005\times 10^3 \times 186.656$$

$$\therefore \qquad \mathbf{P = 3.7517\times 10^6 \text{ W}} \qquad \textbf{... Ans.}$$

Step - 3 : Calculate impeller tip speed (u)

$$W = \dot{m} C_p (T_{02} - T_{01}) = 4148.64\times 10^3 \text{ W}$$

Also $$W = \dot{m}\, u_2 \cdot V_{w2}$$

$$\therefore \qquad 3.7517\times 10^6 = 20\times u_2 \times 0.9 u_2 \qquad \left(\because \frac{V_{w2}}{u_2} = 0.9\right)$$

$$\therefore \qquad \mathbf{u_2 = 456.544 \text{ m/sec.}} \qquad \textbf{... Ans.}$$

Step - 4 : Calculate impeller tip diameter (D_2)

We know that, $$u_2 = \frac{\pi D_2 N}{60} \qquad \therefore \quad 456.544 = \frac{\pi\times D_2 \times 15000}{60}$$

$$\therefore \qquad \mathbf{D_2 = 0.5812 \text{ m}} \qquad \textbf{... Ans.}$$

Step - 5 : Calculate outer diameter of the impeller eye

Annular Area , $$A_1 = \frac{\pi}{4}\left(D_0^2 - D_i^2\right)$$

We know that $$\dot{m} = \rho A_1 V_1$$

$$\therefore \qquad 20 = 1.2\times \frac{\pi}{4}\left(D_0^2 - 0.2^2\right)\times 120 \qquad (\text{Assume } \rho = 1.23 \text{ kg/m}^3)$$

$$\therefore \qquad \mathbf{D_0 = 0.4656 \text{ m}} \qquad \textbf{... Ans.}$$

Example 8.20 : *A centrifugal compressor develops a pressure ratio of 5.0 and an air consumption of 30 kg/s. The inlet temperature and pressure are 15 °C and 1 bar respectively. If isentropic efficiency is 0.85, Calculate*
i) The work done ii) Exit total temperature iii) The power required

Solution : Given data :

$\frac{p_{02}}{p_{01}} = 5$, $\dot{m} = 30$ kg/s, $T_{01} = 15$ °C $= 288$ K, $p_{01} = 1$ bar, $\eta_{isen} = 0.85$

To find : i) WD ii) T_{02} iii) P_{req}

Refer Fig. 6.20.

Step - 1 : Calculate the exit temperature and work done of the compressor

We know that, $$\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \qquad \frac{T'_{02}}{288} = (5)^{\frac{1.4-1}{1.4}} \qquad \therefore \quad T'_{02} = 456.14 \text{ K}$$

Isentropic efficiency is,

$$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \quad \therefore \quad 0.85 = \frac{456.14 - 288}{T_{02} - 288}$$

$$\therefore \quad T_{02} = \mathbf{485.8117\ K} \qquad \textbf{...Ans.}$$

Work done of the compressor is,

$$WD_{act} = C_P(T_{02} - T_{01})$$

$$= 1.005 \times 10^3 \times (485.8117 - 288)$$

$$\therefore \quad \mathbf{WD_{act} = 198.8 \times 10^3\ J/kg} \qquad \textbf{...Ans.}$$

Step 2 : Calculate the required power of compressor

Required power of the compressor is,

$$P_{req} = \dot{m} \times WD_{act}$$

$$= 30 \times 198.8 \times 10^3$$

$$\therefore \quad \mathbf{P_{req}} = 5.964 \times 10^6 \text{ watts} = \mathbf{5.964\ MW} \qquad \textbf{...Ans.}$$

Example 8.21 : *A centrifugal compressor runs at 15000 rpm with overall stagnation pressure ratio of 4. An ambient air conditions are 25 °C and 1 bar, vanes are radial, slip factor is 0.96 and power input factor (work factor) is 1.04. Flow in the inlet section up to the impeller entry is isentropic and that in the impeller and diffuser is adiabatic, there is no Prewhirl at axial entry to the impeller. Mechanical efficiency is 96 % and the electric motor driving the compressor is 98 % efficient. The stagnation pressure loss from impeller exit to diffuser exit is 0.1 bar isentropic efficiency of the impeller alone is 90 %, taking $\gamma = 1.4$, $C_p = 1.005$ kJ/kgK for air, Determine i) Electrical energy consumed by electric motor per kg of air,*
ii) Overall efficiency of the compressor, iii) Impeller tip diameter. Draw a neat T - S diagram.

Sol. : Given data :

$N = 15000$ rpm, $\frac{p_{02}}{p_{01}} = 4$,

$T_{01} = 25\ °C = 298\ K$, $p_{01} = 1$ bar,

$\phi_s = 0.96$, $\phi_w = 1.04$, $\eta_m = 0.96$, $\eta_{motor} = 0.98$,

Pressure loss from impeller to diffuser $(p_{02} - p_2) = 0.1$ bar,

$\eta_i = 0.9$, $\gamma = 1.4$, $C_p = 1.005$ kJ/kgK

To find : i) Electrical energy of motor ii) η_o iii) D_2

Step - 1 : Calculate the total head temperature of the exit of compressor and energy consumed by motor

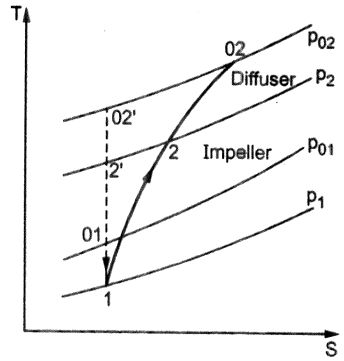


Fig. 8.37 T-S diagram

We know that, $\frac{T'_{02}}{T_{01}} = \left(\frac{p'_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ ... $(\because p'_{02} = p_{02})$

$$\therefore \quad \frac{T'_{02}}{298} = (4)^{\frac{1.4-1}{1.4}} \quad \therefore \quad T'_{02} = 442.8262\ K$$

Isentropic efficiency is, $\eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \quad \therefore \quad 0.9 = \frac{442.8262 - 298}{T_{02} - 298}$

$$\therefore \quad T_{02} = 458.918\ K$$

Electrical energy consumed by motor per kg of air is,

$$W_{motor} = W_{act} \times \frac{1}{\eta_m} \times \frac{1}{\eta_{motor}} = C_p\,(T_{02} - T_{01}) \times \frac{1}{\eta_m} \times \frac{1}{\eta_{motor}}$$

$$\therefore \quad W_{motor} = 1.005 \times 10^3 \times (458.918 - 298) \times \frac{1}{0.96} \times \frac{1}{0.98}$$

$$\therefore \quad \mathbf{W_{motor}} = \mathbf{171.9 \times 10^3\ J/kg} \qquad \textbf{...Ans.}$$

Overall efficiency is given by,

$$\eta_o = \eta_i \times \eta_m \times \eta_{motor} = 0.9 \times 0.96 \times 0.98 = \mathbf{0.8467\ or\ 84.67\ \%} \quad \textbf{...Ans.}$$

Step - 2 : Calculate the tip diameter of impeller

Slip factor $\phi_s = \frac{V_{w2}}{u_2}$ $\therefore$ $0.96 = \frac{V_{w2}}{u_2}$ $\therefore V_{w2} = 0.96\, u_2$

Also,

$$WD_{act} = C_p (T_{02} - T_{01}) = 1.005 \times 10^3 \times (458.918 - 298)$$

$$= 161.7225 \times 10^3 \text{ J/kg}$$

But, $WD_{act} = \phi_w \times V_{w2} \times u_2 = 1.04 \times 0.96\, u_2 \times u_2$

$\therefore$ $161.7225 \times 10^3 = 0.9984\, u_2^2$ $\therefore$ $u_2 = 402.4694$ m/sec

and $u_2 = \frac{\pi D_2 N}{60}$ $\therefore$ $402.4694 = \frac{\pi \times D_2 \times 15000}{60}$

$\therefore$ $\mathbf{D_2 = 0.5124\ m}$ **... Ans.**

8.11 Losses in Centrifugal Compressor and its Characteristics

- The actual pressure head obtained from a centrifugal compressor is less than the theoretical head (Euler's head) due to the following types of losses :

i) Channel losses	ii) Exit losses in diffuser
iii) Inlet losses	iv) Incidence losses

i) Channel losses : Channel or friction losses are due to skin friction and turbulence in the flow passage. The losses are proportional to square of velocity (V^2).

ii) Exit losses in diffuser : These losses represents the loss of head equivalent to non-conversion of kinetic energy of air between the entry and exit of the diffuser.

iii) Inlet losses : These losses are due to the fact that the angle of incidence of air on the impeller blades varies with the discharge rates and it does not coincide with the angle of entrance.

iv) Incidence losses : These losses are in terms of drag coefficient (C_D) and it is proportional to ($C_D V^2$).

- If the above losses and the effect of slip and non-uniform distribution of radial velocity around the periphery of impeller are considered, it indicates that the combined losses $(H_f) \propto Q^2$.

Fig. 8.38 shows the head (H) - capacity (Q) characteristic for the backward curved vanes.

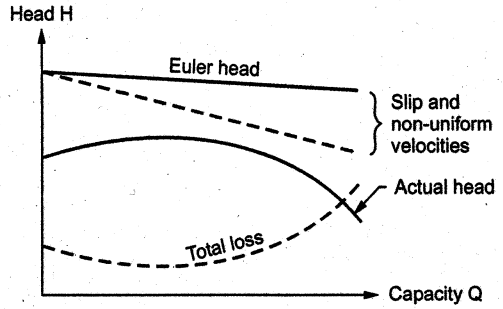


Fig. 8.38 : Characteristics of centrifugal compressor

8.12 Surging and Choking of Compressor

- Fig. 8.39 shows the actual head (H) v/s mass flow rate ($\dot{m}$ and Q) in a dynamic compressor.
- In Fig. 8.39 part ABC has positive slope and part CDEF has negative slope.

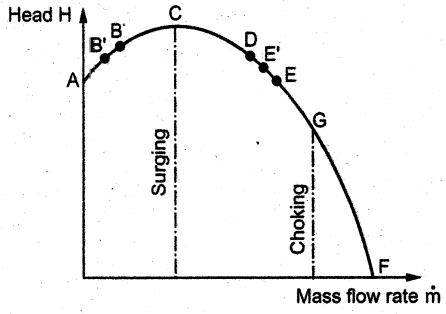


Fig. 8.39 : Surging and choking in compressor

- The mass flow rate is zero when the discharge valve is closed, but the air entrapped in blade area is compressed and the pressure of air increases. This condition is shown by point A.
- The point F represents the maximum mass flow rate i.e. fully open discharge valve but the developed head is zero.
- The point C represents the condition of maximum head developed at some mass flow rate. Also, at point C the compression efficiency is maximum.

Surging

- Now consider any point E on the characteristic curve. The head developed by the compressor at the given mass flow rate includes friction and other losses of the compressor.
- When the load on the compressor increases due to frictional losses, the mass flow rate decreases and hence the operation of the curve will shift to E′.
- At this condition the operation of the compressor is stable as it can develop the required head including the increased frictional losses.
- After sometime its mass flow rate again increases and the compressor returns back to its operating point E. It means the operation of compressor is comparatively stable in portion FC.
- Now consider the operation of any point B on the curve. Due to frictional losses in compressor the mass flow rate reduced and the operating point shifts to B'.
- At point B' the compressor will develop less head than the required head but the pressure in delivery pipe is more. This results is reversal of the flow.
- After sometime, due to flow of fluid from discharge line, the back pressure will fall and compressor again start to deliver the fluid or air.
- Thus, the compressor operation again shift to point B where the similar unstable condition will occur.
- It means, the air flow is pulsating throughout the compressor when it has to operate at reduced mass flow rates. This pulsating air flow phenomena is called as **surging** or **pumping.**
- Surging occurs in centrifugal as well as axial compressors. It causes overheating and stress reversal in the vanes and damages the compressor.
- The region between the points A and C is known as surging region and the operation of compressor in this region is unstable.
- The surging does not take place in the region CG because the reduction in mass flow rate is accompanied with increase in delivery head and reversal of flow is not possible. Thus, operation is stable.

Chocking

- It is already discussed that at point F the mass flow rate is maximum but the developed pressure head is zero.
- Practically, the maximum mass flow is limited by the point G, because if the mass flow exceeds the design mass flow, the air angles are different from vane angles.
- This maximum mass flow rate which is possible (at point G) is called as **choking mass flow rate.**

- Choking means the maximum mass flow rate at a given rotational speed. At this point G the compressor can no longer assure a fixed discharge head and the energy of compression is dissipated.
- Therefore, the slope of the characteristic steepens and finally it becomes vertical.
- Because of the above reasons the rotary compressors are always operated between the limits of surging and choking.

8.13 Non-Dimensional Parameters for Plotting Compressor Characteristics

- The performance of compressor depends upon the following variables :
  - Inlet and outlet pressures p_{01} and p_{02}
  - Inlet and outlet temperatures T_{01} and T_{02}
  - Mass flow rate $\dot{m}$
  - Specific volume at entry and exit v_1 and v_2
  - Adiabatic index γ
  - Impeller diameter D and speed N
- To express the performance of compressor all the above variables are considered and the results are plotted.
- The obtained curves are very complicated hence the method of dimensional analysis is used. In this method, a convenient number of parameters are expressed in dimensionless groups.
- If p_{02} be the outlet temperature of air then,

$$p_{02} = f(p_{01}, T_{01}, D, \dot{m}, N) \quad \text{... (i)}$$

- As the flowing fluid is air, the variable γ is not included in the above analysis. Also, the specific volume is dependent on pressure and temperature hence not considered. In addition to this, the diameter D is the characteristic dimension of the compressor.
- Now solving the above equation (i) by using Buckingham π - theorem we will get the following different dimensionless groups (π-terms) :

$$\pi_1 = \frac{p_{02}}{p_{01}}$$

$$\pi_2 = \frac{\dot{m}\sqrt{T_{01}}}{p_{01} D^2}$$

$$\pi_3 = \frac{DN}{\sqrt{T_{01}}}$$

$$\therefore \quad \frac{p_{02}}{p_{01}} = \left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}} \right) \qquad \ldots (ii)$$

- It means, the performance of a compressor of fixed size with respect to variation in delivery pressure and temperature with mass flow rate, speed and inlet conditons can be expressesed in the form of dimensionless groups.

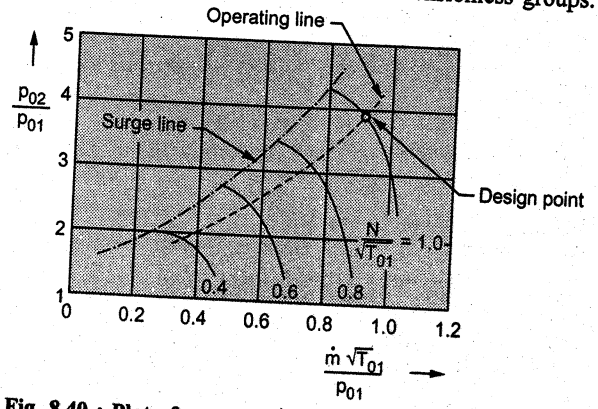


Fig. 8.40 : Plot of pressure ratio v/s mass flow parameter

8.13.1 Performance of Centrifugal Compressor

- In order to predict the performance of the compressor to that of a geometrically similar model for the above discussed variables, the performance curves are plotted with dimensionless parameters.
- A plot of number of curves between the pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$ and mass flow parameter by varying the speed parameter is shown in Fig. 8.41.
- The unstable range of curve is limited by surge line which corresponds to low mass flow rates and by choking at high mass flow rates.
- The performance may also be plotted for a compressor in terms of pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$ and discharge flow rate. Refer Fig. 8.41.
- The following conclusions are obtained from Fig. 8.41 :
  - Surge line limits the operation of compressor at low mass flow rates.
  - At low speeds, the curves are almost flat.
  - Maximum mass flow rates are limited by choking point.
  - As pressure ratio increases, there is decrement in discharge at a given speed.
  - At a given pressure ratio, increase in speed give rise to the discharge but reduces the efficiency.

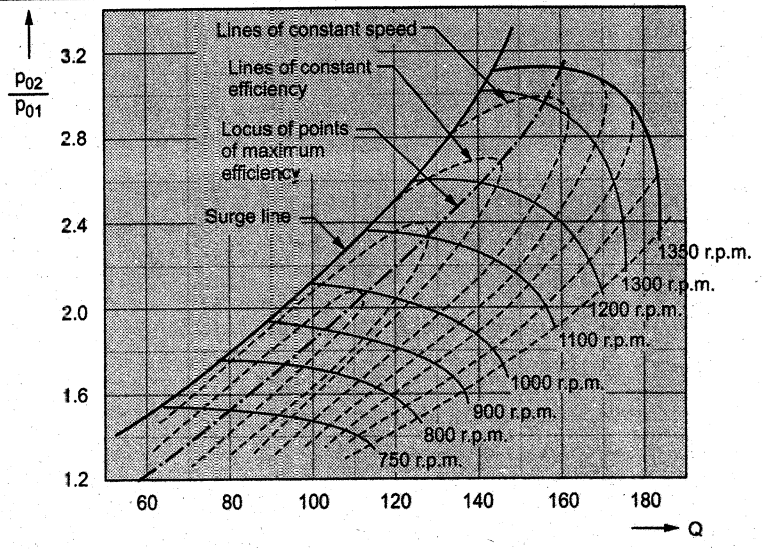


Fig. 8.41 : Performance characteristic curves of centrifugal compressor

Review Questions

1. *State the advantages and applications of centrifugal compressors.*
2. *Give the classification of rotary compressors.*
3. *With neat sketch explain the components of centrifugal compressor.*
4. *Explain the processes involved in a centrifugal compressor on T-S diagram.*
5. *Derive the expression for isentropic efficiency of a centrifugal compressor, based on total values.*
6. *Explain the analysis of centrifugal compressors.*
7. *Write a short note on vane shapes and their characteristics.*
8. *Explain : i) Slip factor ii) Power input factor iii) Pressure coefficient iv) Pre whirl*
9. *Explain the different types of losses in centrifugal compressors.*
10. *Discuss the non-dimensional parameters used for plotting compressor characteristics and state their importance.*
11. *Write short note on : i) Surging ii) Choking.*
12. *Explain the performance characteristic curves of rotary compressors.*

Centrifugal Compressors ends ...

Notes

CHAPTER - 9

AXIAL FLOW COMPRESSORS

Syllabus : Construction of an axial flow compressor, Aerofoil blading, Liftand drag, Performance characteristics.

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9.1 Introduction

- In case of jet aircraft applications, the compressors should have low weight to power ratio with high volume flow rate. Also, it should have high compression ratio with less frontal area.
- The centrifugal compressor could not meet these requirements and this leads to development of axial flow compressors.
- Axial flow compressors are capable of deliverying high discharge rates required for aircraft applications.
- In this type of compressor, the air flows parallel to the axis of compressor hence these compressors are suitable for multistaging as compared to centrifugal compressors.
- Generally, the running speed of these compressors is about 10,000 to 30,000 rpm.
- Axial flow compressors are also used in gas turbine power plants as it can develop high pressure ratio with high volume flow rates.

9.2 Construction and Working of Axial Flow Compressor

Construction ☞ **[GTU : Summer-16, Winter-17]**

- The axial flow compressor is often described as a *reversed reaction turbine*. The energy transfer in the compressor is the reverse of that in the turbine.
- *In this compressor the air flows parallel to the axis of compressor.* It consists of adjacent rows of moving blades (rotor) and fixed blades (stator).
- The moving blades are mounted on the rotating drum and the fixed blades are attached to the casing.
- A row of moving blades followed by a row of fixed blades is called as **one stage**.
- The main function of the fixed blades is to receive high velocity air from the preceding rotor blades and to direct the flow to the succeeding rotor blades.
- Fig. 9.1 shows the axial flow compressor with drum type rotor. The compressor blades have aerofoil section whereas the turbine blades have profile section.
- At the inlet of compressor an extra row of fixed vanes is provided which is called as **Inlet Guide Vane (IGV)**.
- The acceleration process carried in the converging blade passage of reaction turbine is more efficient and stable as compared to diffusing (decelerating) process carried in the diverging passage of blades of axial compressor.
- The annular area is reduced from inlet to outlet of the compressor which results in *constant flow velocity throughout the compressor length.*
- Due to diffusion there is rise in temperature in the diverging passage of the moving blades. Also, the absolute velocity is increased due to work input.

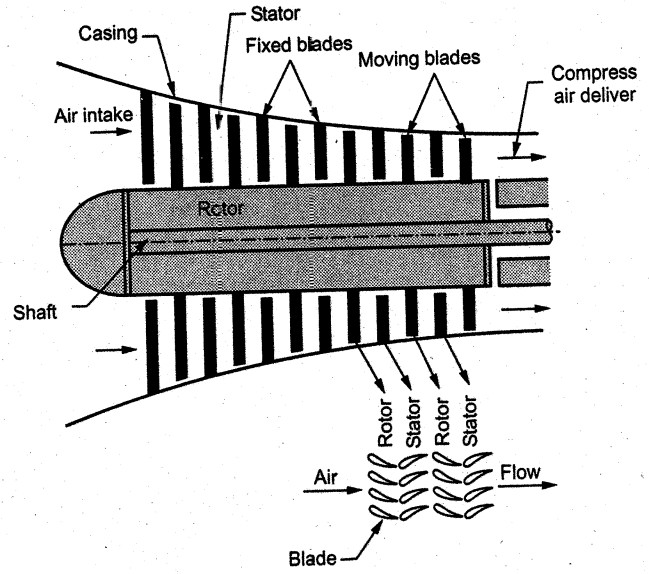


Fig. 9.1 : Drum type axial flow compressor

Working

- The basic working principle of axial flow compressor is similar to the centrifugal compressor.
- The input energy of the rotor shaft is transferred to the air by moving blades and thus accelerating the air.
- The blades are arranged in such a way that the spacing between the blades form a diffuser. Hence, the velocity of air relative to the blades is decreased $(V_{r2} < V_{r1})$ and there is a rise in pressure.
- After moving blades, the air is further diffused in stator blades which also form diffuser passage.
- While passing through the stator blades, the air is turned through an angle so that it can easily pass to a second row of moving blades.
- The pressure rise per stage in axial compressors is 1.1 to 1.3 hence a number of stages (generally 5 to 14 stages) are used for pressure ratio upto 12.
- The multistage axial compressor can supply air upto 30,000 m^3 / min.

Note : *An axial flow compressor may have a drum type rotor or disc type rotor. The drum type rotor is suitable for static industrial applications whereas disc type rotor is used where low weight is important. Fig. 9.1 shows the drum type rotor.*

9.3 Velocity Diagram for a Stage of Axial Flow Compressor

- Fig. 9.2 shows the velocity diagram for a single stage of axial flow compressor.
- *It is important to note that, in axial flow compressor all angles are measured from the axial direction (with vertical) and not from tangential direction as in case of centrifugal compressor.*
- As the air enters and leaves the blades at almost equal radii, the blade velocity (u) is considered as same at entry and exit $(u_1 = u_2 = u)$.

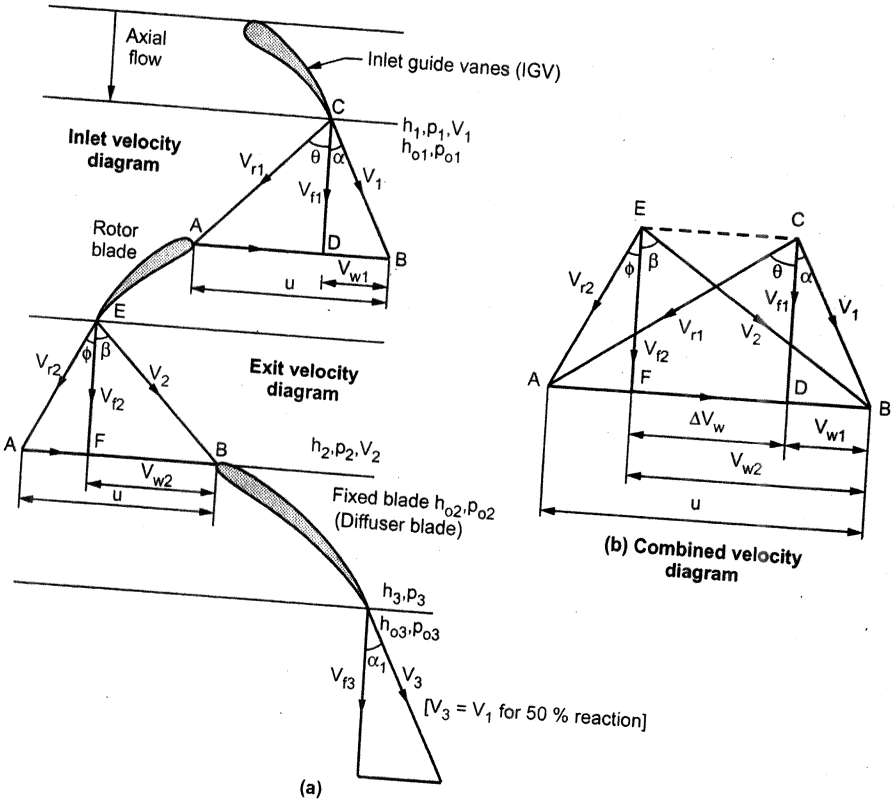


Fig. 9.2 : Velocity diagram for axial compressor stage

- Let, u = Mean blade velocity
  V_{r1} and V_{r2} = Relative velocity of air at inlet and outlet
  V_1 and V_2 = Absolute velocity of air at inlet and outlet
  V_{f1} and V_{f2} = Flow velocity of air at inlet and outlet = V_f
  V_{w1} and V_{w2} = Whirl velocity of air at inlet and outlet
  α = Exit angle of fixed blade
  β = Inlet angle of fixed blade
  θ = Inlet angle of moving or rotor blade
  ϕ = Exit angle of moving or rotor blade
  V_3 = Absolute velocity leaving the compressor stage at angle α_1
- For any stage, the rotar receives air with an absolute velocity V_1 and angle α from the previous stage for only first stage the air is received in required direction by using Inlet Guide Vanes (IGV).
- In Fig. 9.2 the outlet velocity diagram is drawn for 50 % reaction axial flow compressor.
- Due to diffusion in diverging passage of rotor blades, there is rise in pressure and hence $V_{r2} < V_{r1}$.
- Also, the work is done on the air by rotor blades, the air leaves the rotor with increased absolute velocity V_2 $(V_2 > V_1)$.
- During the flow of air in an axial compressor, the axial velocity of air remains constant.
- As the blade velocity is same at inlet and outlet of the rotor blade, the inlet and outlet velocity diagrams are superimposed. Refer Fig. 9.2 (b).
- The work absorbed by the stage per kg of air is,

$$WD = (V_{w2} - V_{w1})u \quad \ldots (9.1)$$

- Assuming the process to be adiabatic, the work of compression can be written as (similar to centrifugal compressor),

$$WD = C_p (T_{02} - T_{01}) \quad \ldots (9.2)$$

Equating equation (9.1) and (9.2),

$$(V_{w2} - V_{w1})u = C_p (T_{02} - T_{01})$$

$$\therefore \quad T_{02} - T_{01} = \frac{(V_{w2} - V_{w1})u}{C_p} \quad \ldots (9.3)$$

9.4 Enthalpy - Entropy (h-S) Diagram for Single Stage Compression

☞ **[GTU : Winter-15, Summer-16]**

- The stage of a single stage compressor on h-S diagram is shown in Fig. 9.3.
- The air enters the rotor blades with velocity V_1 and leaves the compressor at higher velocity V_2. This velocity is reduced in stator or diffuser blades.
- From stagnation properties, we know that,

$$T_0 = T + \frac{V^2}{2C_p} \qquad \therefore T_0 C_p = T C_p + \frac{V^2}{2}$$

$$\therefore \qquad h_0 = h + \frac{V^2}{2} \qquad \ldots (\because h = C_p T)$$

At inlet of compressor,

$$h_{01} = h_1 + \frac{V_1^2}{2} \qquad \text{and} \qquad p_{01} = p_1 + \frac{1}{2}\rho_1 V_1^2$$

At outlet of rotor,

$$h_{02} = h_2 + \frac{V_2^2}{2} \qquad \text{and} \qquad p_{02} = p_2 + \frac{1}{2}\rho_2 V_2^2$$

Similarly at outlet of stator,

$$h_{03} = h_3 + \frac{V_3^2}{2} \qquad \text{and} \qquad p_{03} = p_3 + \frac{1}{2}\rho_3 V_3^2$$

- But at the outlet of stator the condition is similar to the first stage i.e. $V_3 \approx V_1$ (for R = 0.5).
- Thus, there is increase in enthalpy of the stage which causes the temperature rise $(T_{03} - T_{01})$.
- The fixed blades in the stator mainly transform the kinetic energy to increase the static pressure.
- Hence, $T_{03} \approx T_{02}$ and $p_{03} < p_{02}$ due to stagnation pressure loss. But the total stagnation enthalpy remains the same i.e. $h_{03} = h_{02}$.
- The total temperature rise per stage is,

$$\Delta T_0 = T_{03} - T_{01} = T_{02} - T_{01}$$

- If $T_{03'}$ is the isentropic temperature after compression, the isentropic efficiency or stage efficiency is,

$$\eta_i = \frac{\text{Isentropic work input}}{\text{Actual work input}} = \frac{h_{03'} - h_{01}}{h_{03} - h_{01}} \qquad \ldots (i)$$

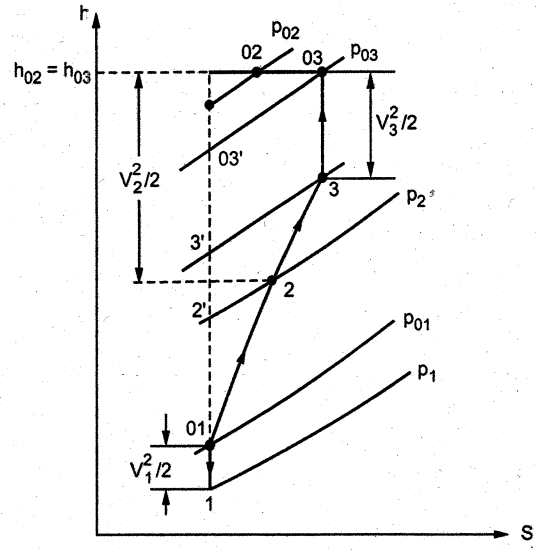


Fig. 9.3 : (h-S) diagram for single stage axial flow compressor

$$\therefore \qquad \eta_i = \frac{T_{03'} - T_{01}}{T_{03} - T_{01}} = \frac{T_{03'} - T_{01}}{T_{02} - T_{01}} \qquad \ldots (\because T_{03} = T_{02}) \quad \ldots (9.4)$$

or $$T_{03'} - T_{01} = \eta_i (T_{02} - T_{01})$$

$$\therefore \qquad T_{01}\left(\frac{T_{03'}}{T_{01}} - 1\right) = \eta_i (T_{02} - T_{01})$$

$$\therefore \qquad T_{01}\left[\left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \eta_i (T_{02} - T_{01})$$

$$\therefore \qquad \frac{p_{03}}{p_{01}} = \left[\frac{\eta_i (T_{02} - T_{01})}{T_{01}} + 1\right]^{\frac{\gamma}{\gamma-1}} \qquad \ldots (ii)$$

But, $$C_p (T_{02} - T_{01}) = u(V_{w2} - V_{w1})$$

$$\therefore \qquad C_p (T_{02} - T_{01}) = u [V_f(\tan\theta - \tan\phi)]$$

$$\therefore \qquad T_{02} - T_{01} = \frac{u [V_f (\tan\theta - \tan\phi)]}{C_p}$$

Substituting this value in equation (ii),

$$\frac{p_{03}}{p_{01}} = \left[\frac{\eta_i \, u \times V_f(\tan\theta - \tan\phi)}{C_p} + 1\right]^{\frac{\gamma}{\gamma-1}} \qquad \ldots (9.5)$$

9.5 Degree of Reaction

- Degree of reaction for axial flow compressor is defined as the ratio of pressure rise in rotor blades to the pressure rise in the compressor stage. It is denoted by R_D.
- Mathematically it is given as,

$$R_D = \frac{\text{Pressure rise in rotor blades}}{\text{Pressure rise in compressor stage}} \qquad \ldots (9.6)$$

But, pressure rise in the rotor blades at the expense of kinetic energy is,

$$= \frac{V_{r1}^2 - V_{r2}^2}{2} \qquad \ldots (i)$$

Also, the pressure rise in the compressor stage is equal to the work input per stage i.e.

$$= (V_{w2} - V_{w1})u \qquad \ldots (ii)$$

Substituting these values in the above equation (9.6),

$$\therefore \quad R_D = \frac{(V_{r1}^2 - V_{r2}^2)/2}{(V_{w2} - V_{w1})u} = \frac{(V_{r1}^2 - V_{r2}^2)}{2u\,(V_{w2} - V_{w1})} \qquad \ldots (9.7)$$

- The combined velocity diagram for a stage of axial flow compressor is shown in Fig. 9.4.

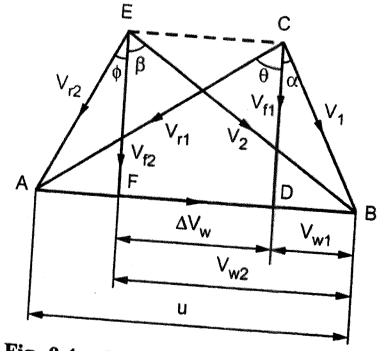


Fig. 9.4 : Combined velocity diagram

From Δ ACD,

$$\tan\theta = \frac{u - V_{w1}}{V_{f1}} \qquad \therefore V_{w1} = u - V_{f1}\tan\theta \qquad \ldots (iii)$$

From Δ AEF,

$$\tan\phi = \frac{u - V_{w2}}{V_{f2}} \qquad \therefore V_{w2} = u - V_{f2}\tan\phi \qquad \ldots (iv)$$

Subtracting equation (iv) and (iii),

$$\therefore \quad V_{w2} - V_{w1} = (u - V_{f2}\tan\phi) - (u - V_{f1}\tan\theta)$$

$$\therefore \quad V_{w2} - V_{w1} = -V_{f2}\tan\phi + V_{f1}\tan\theta$$

$$\therefore \quad V_{w2} - V_{w1} = V_f(\tan\theta - \tan\phi) \qquad \ldots (V_{f2} = V_{f1} = V_f) \quad \ldots (v)$$

- Again consider Δ ACD,

$$\tan\theta = \frac{AD}{CD} = \frac{AD}{V_{f1}} \qquad \therefore AD = V_{f1}\tan\theta$$

Also, $$AC^2 = AD^2 + CD^2$$

$$\therefore \quad V_{r1}^2 = (V_{f1}\tan\theta)^2 + V_{f1}^2 = V_{f1}^2(1 + \tan^2\theta) \qquad \ldots (vi)$$

- Again consider Δ AEF,

$$\tan\phi = \frac{AF}{EF} = \frac{AF}{V_{f2}} \qquad \therefore AF = V_{f2}\tan\phi$$

Also, $$AE^2 = AF^2 + EF^2$$

$$\therefore \quad V_{r2}^2 = (V_{f2}\tan\phi)^2 + V_{f2}^2 = V_{f2}^2(1 + \tan^2\phi) \qquad \ldots (vii)$$

- Now substituting equations (v), (vi) and (vii) in equation (9.7),

$$\therefore \quad R_D = \frac{V_{f1}^2(1 + \tan^2\theta) - V_{f2}^2(1 + \tan^2\phi)}{2u \times V_f(\tan\theta - \tan\phi)}$$

$$\therefore \quad R_D = \frac{V_f^2 + V_f^2\tan^2\theta - V_f^2 - V_f^2\tan^2\phi}{2u \times V_f(\tan\theta - \tan\phi)}$$

$$\therefore \quad R_D = \frac{V_f(\tan^2\theta - \tan^2\phi)}{2u\,(\tan\theta - \tan\phi)}$$

$$\therefore \quad R_D = \frac{V_f(\tan\theta - \tan\phi)\,(\tan\theta + \tan\phi)}{2u\,(\tan\theta - \tan\phi)}$$

$$\therefore \quad R_D = \frac{V_f(\tan\theta + \tan\phi)}{2u}$$

- Generally, for axial flow compressors the degree of reaction is 50 % ($R_D = 0.5$).

$$\therefore \quad \frac{1}{2} = \frac{V_f(\tan\theta + \tan\phi)}{2u}$$

$$\therefore \quad \frac{u}{V_f} = \tan\theta + \tan\phi \qquad \ldots (9.8)$$

- From equation (iii),

$$\tan\theta = \frac{u - V_{w1}}{V_{f1}}$$

Also from ΔDCB,

$$\tan\alpha = \frac{V_{w1}}{V_{f1}}$$

Adding $\tan\theta$ and $\tan\alpha$,

$$\therefore \quad \tan\theta + \tan\alpha = \frac{u - V_{w1}}{V_{f1}} + \frac{V_{w1}}{V_{f1}} = \frac{u - V_{w1} + V_{w1}}{V_{f1}}$$

$$\therefore \quad \tan\theta + \tan\alpha = \frac{u}{V_f} \qquad \ldots (9.9)$$

Equating equations (9.8) and (9.9),

$$\therefore \quad \tan\theta + \tan\phi = \tan\theta + \tan\alpha$$

$$\therefore \quad \tan\phi = \tan\alpha \text{ or } \phi = \alpha \qquad \ldots (9.10)$$

- From equation (iv),

$$\tan\phi = \frac{u - V_{w2}}{V_{f2}}$$

Also from ΔFEB,

$$\tan\beta = \frac{V_{w2}}{V_{f2}}$$

Adding $\tan\phi$ and $\tan\beta$,

$$\therefore \quad \tan\phi + \tan\beta = \frac{u - V_{w2}}{V_{f2}} + \frac{V_{w2}}{V_{f2}} = \frac{u}{V_f} \qquad \ldots (9.11)$$

Equating the above equation (9.11) with equation (9.8),

$$\therefore \quad \tan\theta + \tan\phi = \tan\phi + \tan\beta$$

$$\therefore \quad \tan\theta = \tan\beta \text{ or } \theta = \beta \qquad \ldots (9.12)$$

- *From equations (9.10) and (9.12) it is clear that, for 50 % reaction axial flow compressors the blades are symmetric i.e.*

$$\alpha = \phi \text{ and } \theta = \beta$$

9.6 Work Input Factor, Flow Coefficient and Blade Loading Coefficient

(i) Work input factor

- The work input factor or work done factor (ϕ_w) for axial flow compressor is same as that contrifugal compressor i.e.

$$\phi_w = \frac{C_p(T_{02} - T_{01})}{u(V_{w2} - V_{w1})} \qquad \ldots (9.13)$$

(ii) Flow coefficient

- The flow coefficient (ϕ_f) of axial flow compressor is defined as the ratio of axial flow velocity (V_f) to the mean blade velocity. It is given as,

$$\phi_f = \frac{\text{Axial flow velocity}}{\text{Mean blade velocity}} = \frac{V_f}{u} \qquad \ldots (9.14)$$

(iii) Blade loading coefficient

- The blade loading coefficient (ψ) for axial flow compressor is defined as the ratio of work done of compressor to the square of mean blade velocity. It is given as,

$$\psi = \frac{\text{Work done of compressor}}{\text{Square of mean blade velocity}} = \frac{WD}{u^2}$$

$$\therefore \quad \psi = \frac{u(V_{w2} - V_{w1})}{u^2} = \frac{(V_{w2} - V_{w1})}{u} \qquad \ldots (9.15)$$

9.6.1 Work Coefficient and Pressure Coefficient

i) Work or head coefficient (ϕ_h)

- It is defined as the ratio of actual work done of the compessor to the kinetic energy corresponding to the mean blade velocity i.e.

$$\phi_h = \frac{\text{Actual work done}}{\text{Kinetic energy}} = \frac{u\,(V_{w2} - V_{w1})}{\frac{1}{2}u^2}$$

$$\therefore \quad \phi_h = \frac{2\,(V_{w2} - V_{w1})}{u} = 2\psi \qquad \ldots (9.16)$$

ii) Pressure coefficient (ϕ_p)

- It is defined as the ratio of isentropic work done of compressor to the kinetic energy corresponding to the mean blade velocity i.e.

$$\phi_p = \frac{\text{Isentropic work done}}{\text{Kinetic energy}} = \frac{C_p\,(\Delta T)_{isen}}{\frac{1}{2}u^2}$$

$$\therefore \quad \phi_p = \frac{2\,C_p\,(\Delta T)_{isen}}{u^2} \qquad \ldots (9.17)$$

9.7 Polytropic Efficiency of Axial Flow Compressor

- The work input to the compressor is given by,

$$W = C_p(T_{02} - T_{01})$$

Multiplying and dividing the above equation by $(T_{02'} - T_{01})$,

$$\therefore \quad W = C_p(T_{02} - T_{01}) \times \left(\frac{T_{02'} - T_{01}}{T_{02'} - T_{01}}\right)$$

$$\therefore \quad W = C_p \times \eta_{isen} \times (T_{02'} - T_{01}) \quad \text{... [From equation (9.4)]}$$

$$\therefore \quad W = C_p \times \eta_{isen} \times T_{01}\left[\frac{T_{02'}}{T_{01}} - 1\right]$$

$$\therefore \quad W = C_p \times \eta_{isen} \times T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad \text{... (9.18)}$$

- From the above equation (9.18) it is clear that for the same isentropic efficiency and pressure ratio the input work is proportional to the initial temperature.
- Thus, in a multistage compressor of equal isentropic efficiency, each succeeding stage will have to perform more work because it deals with the increased temperature fluid delivered to it from the preceding stage.
- Therefore, overall isentropic efficiency which represents the overall performance of a compressor cannot be used to compare two compressors of different pressure ratios.
- To compare the performance of compressors with different stages, the concept of polytropic efficiency is used.
- **Polytropic efficiency** is the isentropic efficiency of one stage of a multistage compressor. It is constant for all stages of a compressor with infinite number of stages.
- Consider a T-S diagram for four stages of a multistage axial flow compressor as shown in Fig. 9.5.
- For a pressure ratio of (p_{02}/p_{01}) the overall isentropic temperature rise is $(T_{02'} - T_{01})$ whereas the total isentropic temperature rise for all stages is $\Sigma(\Delta T)$ which is greater than the overall isentropic temperature rise.
- The gas is compressed from pressure p_{01} to p_{02} in four stages of equal pressure ratio. Let, p_{0a}, p_{0b} and p_{0c} be the intermediate pressures.
- Mathematically polytropic efficiency (η_p) is given as,

$$\eta_p = \frac{\text{Polytropic work}}{\text{Actual work}} \quad \text{... (9.19)}$$

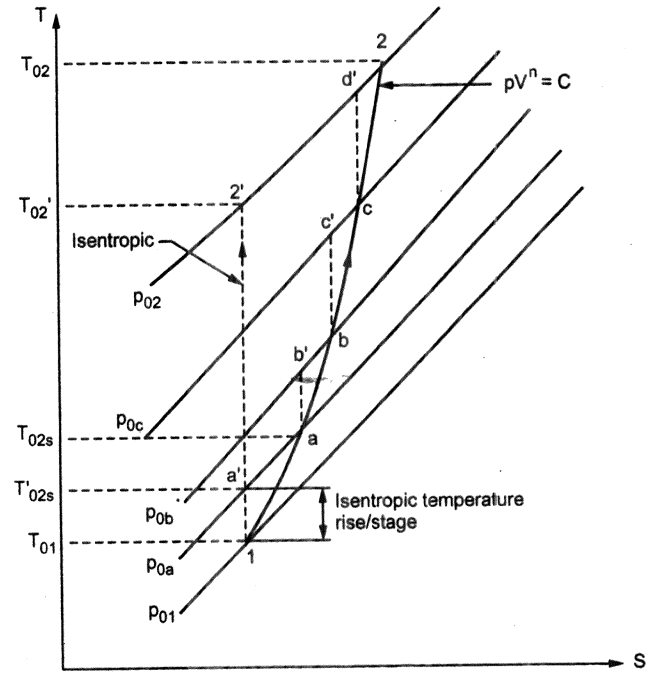


Fig. 9.5 : Polytropic efficiency

$$\text{Polytropic work} = \left(\frac{n}{n-1}\right) m(T_{02} - T_{01})R$$

and $$\text{Actual work} = mC_p(T_{02} - T_{01})$$

$$\therefore \quad \eta_p = \frac{\left(\frac{n}{n-1}\right) m(T_{02} - T_{01})R}{mC_p(T_{02} - T_{01})} = \left(\frac{n}{n-1}\right) \times \frac{R}{C_p} \quad \text{... (i)}$$

- From thermodynamics, we know that,

$$C_p - C_v = R \quad \text{and} \quad \frac{C_p}{C_v} = \gamma \quad \text{or} \quad C_p = \gamma C_v$$

$$\therefore \quad \gamma C_v - C_v = R \quad \therefore \quad C_v(\gamma - 1) = R$$

$$\therefore \quad C_v = \frac{R}{\gamma - 1}$$

As $$\frac{C_p}{C_v} = \gamma \quad \therefore \quad \frac{C_p}{(R/\gamma - 1)} = \gamma$$

$$\therefore \quad C_p = \gamma \times \frac{R}{(\gamma - 1)}$$

Substituting this value in equation (i),

$$\therefore \quad \eta_p = \left(\frac{n}{n-1}\right) \times \frac{R}{\gamma \times R} \times (\gamma - 1)$$

$$\therefore \quad \eta_p = \left(\frac{n}{n-1}\right) \times \left(\frac{\gamma - 1}{\gamma}\right) \quad \ldots (9.20)$$

- To find the value of polytropic index 'n' we can write,

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{n-1}{n}}$$

Taking $\log_e$ on both sides,

$$\log_e\left(\frac{T_{02}}{T_{01}}\right) = \left(\frac{n-1}{n}\right) \log_e\left(\frac{p_{02}}{p_{01}}\right)$$

$$\therefore \quad \left(\frac{n-1}{n}\right) = \frac{\log_e(T_{02}/T_{01})}{\log_e(p_{02}/p_{01})} \quad \ldots (9.21)$$

Substituting this value in polytropic efficiency,

$$\eta_p = \left(\frac{\gamma - 1}{\gamma}\right) \times \frac{\log_e(p_{02}/p_{01})}{\log_e(T_{02}/T_{01})} \quad \ldots (9.22)$$

9.8 Surging, Choking and Stalling in Axial Compressor

☞ **[GTU : Summer-16, Winter-17]**

- The phenomenon of surging and choking in axial flow compressor is simillar to as that of in centrifugal compressor. So *refer section 8.14 of chapter 8.*

Stalling

- Stalling of a stage of axial compressor is defined as the breakaway of the flow from the suction side of the aerofoil blade.
- It may be due to lesser flow rate than designed value or non-uniformity in the profile of blade. It means, stalling is ahead of surging phenomenon.
- A multistage compressor can operate stable in unsurged region with some stages stalled and other stages unstalled.
- Also, we can say that stalling is a local phenomenon and surging is a complete system phenomena.
- The rotating stall can produce resonant vibration and may cause the blade failure.

9.9 Losses in Axial Flow Compressor

- When the air flows through the compressor stage some losses takes place. Refer Fig. 9.6.
- The total pressure loss arises in the following three ways :

(i) Profile losses	(ii) Annulus losses	(iii) Secondary flow losses

i) Profile losses :

- Profile losses occurs on the surface of blade.
- It represents the pressure losses due to skin friction on the blade surfaces. It can be obtained from experimental cascade testing.
- It dependes on Reynold's number and the angle of incidence.

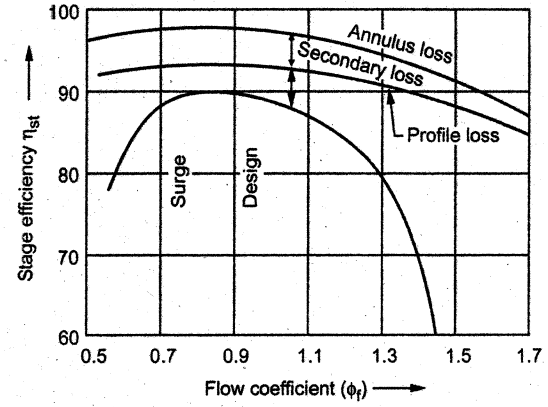


Fig. 9.6 : Losses in compressor stage

ii) Annulus losses :

- It occurs on the annulus walls of the compressor.
- Annulus losses are attributed to the growth of boundary layer along the axis of compressor.
- It is quite significant for multistage axial compressors.

iii) Secondary flow losses

- Secondary flow losses occur as flow pases through a curved blade passage. Also it is produced by combined effect of curvature and boundary layer.

9.10 Performance Characteristics of Axial Flow Compressor

- The characteristic curves of axial compressor are plotted on the same non-dimensional parmeters as that of in centrifugal compressor.

- The non-dimensional parameters are as follows :
  - Pressure ratio parameter (p_{02}/p_{01})
  - Speed parameter ($N/\sqrt{T_{01}}$)
  - Mass flow parameter ($m\sqrt{T_{01}}/p_{01}$)
- A characteristic curve between pressure ratio parameter against mass flow parameter at various speed parameters is shown in Fig. 9.7.

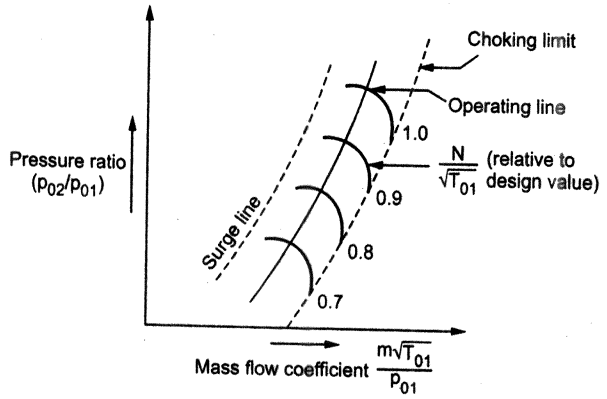


Fig. 9.7 : Plot of pressure ratio against mass flow coefficient at various speed parameters

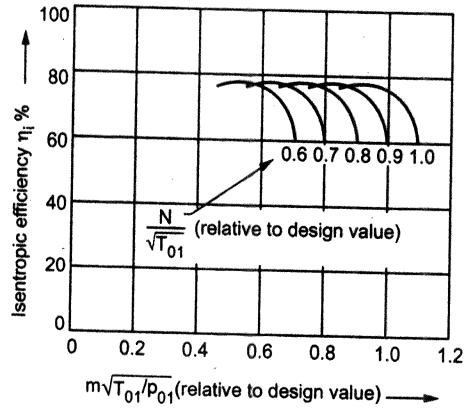


Fig. 9.8 : Plot of isentropic efficiency against mass flow coefficient at various speed parameters

- The unstable range of curve is limited by surge line which corresponds to low mass flow rates and by the choking at high mass flow rates.

- A characteristic curve between the isentropic efficiency and mass flow parameter at various speed parameters is shown in Fig. 9.8.
- The isentropic efficiency varies with mass flow rate at a given speed similar to pressure ratio. It is almost equal at all speeds.
- The operating line is obtained by joining the locus of points of maximum efficiency.
- The characteristic curves for the compressor with constant efficiency curves is shown in Fig. 9.9.

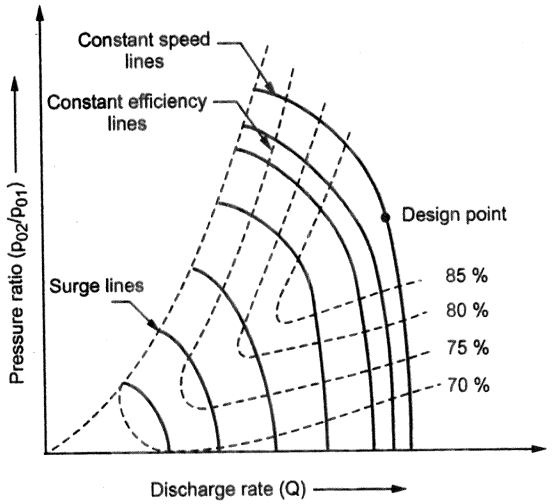


Fig. 9.9 : Characteristic curves of an axial flow compressor

- From the above curves the following conclusions are made :
  - The optimum performance of the compressor is obtained at large flow rates and high speeds.
  - Compressor efficiency reduces with reduction in pressure ratio at a given mass flow rate.
  - The rise in pressure does not affect the discharge rates as the speed lines are almost vertical (except at low speeds).

9.11 Comparison between Centrifugal and Axial Flow Compressor

☞ **[GTU : Summer-17, Winter-18]**

Sr. No.	Centrifugal compressor	Axial flow compressor
1.	The flow of fluid proceeds radially.	The flow of fluid proceeds axially parallel to the axis of compressor.

2.	High pressure ratio per stage (about 4)	Low pressure ratio per stage (about 1.5)
3.	These compressors are easy to manufacture.	These compressors are difficult to manufacture.
4.	Isentropic efficiency of centrifugal compressor is less than axial flow compressor (about 70 %).	Isentropic efficiency of axial flow compressor is about 85 to 88 %.
5.	Power required per kg of air flow is more.	Power required per kg of air flow is less.
6.	Flexible in operation due to adjustable pre-whirl and diffuser vanes.	Not flexible in operation as there is no such arrangement.
7.	It needs large frontal area for given mass flow rates.	It needs less frontal area for given mass flow rates.
8.	Deposite formation on the surface of an impeller does not affect the performance of compressor.	Deposite formation on the surface of an impeller affect the performance of compressor.
9.	It has low starting torque.	It has high starting torque.
10.	It is not suitable for multistaging due to large losses	It is suitable for multistaging .
11.	Delivery pressure is upto 400 bar.	Delivery pressure is upto 20 bar.
12.	The efficiency V/s speed curve is more flat.	The efficiency V/s speed curve is less flat.
13.	**Applications :** Super charging of IC engines, refrigerants and industrial gases, Blowing engine in steel mills, used in air conditoning, fertiliser industry, gas pumping in long distance pipe lines, etc.	**Applications :** In jet engines, gas turbine power plant, steel mills, etc.

9.12 List of Formulae

- Actual workdone : $WD_{act} = C_P (T_{02} - T_{01})$

  Also, $WD = (V_{w2} - V_{w1})u$
- Isentropic efficiency :

$$\eta_i = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{h_{02'} - h_{01}}{h_{02} - h_{01}}$$

$$= \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

- Pressure ratio

$$\frac{p_{03}}{p_{01}} = \left[\frac{\eta_i \times u \times V_f(\tan\theta - \tan\phi)}{C_p} + 1\right]^{\frac{\gamma}{\gamma-1}}$$

- Degree of reaction :

$$R_D = \frac{\text{Pressure rise in rotor blades}}{\text{Pressure rise in compressor stage}} = \frac{V_{r1}^2 - V_{r2}^2}{2u(V_{w2} - V_{w1})}$$

- For $R_D = \frac{1}{2}$ or 50 %

$$\alpha = \phi \quad \text{and} \quad \theta = \beta$$

- Work input factor :

$$\phi_w = \frac{C_p(T_{02} - T_{01})}{(V_{w2} - V_{w1})u}$$

- Flow coefficient :

$$\phi_f = \frac{V_f}{u}$$

- Blade loading coefficient :

$$\psi = \frac{WD}{u^2} = \frac{(V_{w2} - V_{w1})}{u}$$

- Work or head coefficient :

$$\phi_h = \frac{2(V_{w2} - V_{w1})}{u} = 2\psi$$

- Pressure coefficient :

$$\phi_p = \frac{2\,C_p\,(\Delta T)_{isen}}{u^2}$$

- Polytropic efficiency :

$$\eta_p = \frac{\text{Polytropic work}}{\text{Actual work}} = \left(\frac{n}{n-1}\right) \times \frac{R}{C_p}$$

$$= \left(\frac{n}{n-1}\right) \times \left(\frac{\gamma - 1}{\gamma}\right)$$

Also, $$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{n-1}{n}}$$

$$\eta_p = \left(\frac{\gamma - 1}{\gamma}\right) \times \frac{\log_e(p_{02}/p_{01})}{\log_e(T_{02}/T_{01})}$$

- Number of stages :

$$z = \frac{\text{Total work done}}{\text{Work done of each stage}}$$

9.13 Solved Examples

Example 9.1 : *For an axial compressor, the intake air pressure is 1 bar and 18° C. The pressure head and temperature in delivery pipe are 3.5 bar and 185 °C respectively. Static pressure in the delivery pipe is 3 bar. Determine :*
(i) The total head isentropic efficiency
(ii) Air velocity in delivery pipe.

Solution : Given data :

p_{01} = 1 bar; T_{01} = 18° C = 18 + 273 = 291 K, p_{02} = 3.5 bar,

T_{02} = 185° C = 185 + 273 = 458 K, p_2 = 3 bar, Assume C_p = 1.005 kJ/kg K

To find : i) η_{isen} ii) V_2

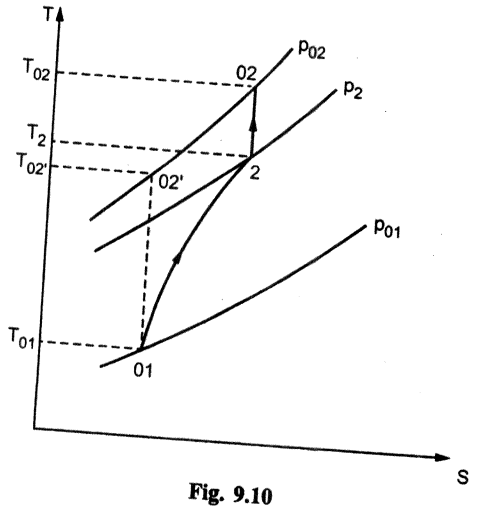


Fig. 9.10

Step 1 : Calculate the isentropic efficiency

We know that,

$$\frac{T_{02'}}{T_{01}} = \left(\frac{p_{02'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{02'}}{291} = \left(\frac{3.5}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_{02'} = 416.2373 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} = \frac{416.2373 - 291}{458 - 291}$$

$$\therefore \quad \eta_{isen} = 0.74992 = \mathbf{74.9923\ \%} \quad \text{... Ans.}$$

Step 2 : Calculate the velocity at exit

We know that,

$$\frac{T_{02}}{T_2} = \left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{458}{T_2} = \left(\frac{3.5}{3}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_2 = 438.266 \text{ K}$$

As we know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2\,C_p}$$

At point 02, $$T_{02} - T_2 = \frac{V_2^2}{2\,C_p} \quad \therefore 458 - 438.266 = \frac{V_2^2}{2 \times 1.005 \times 10^3}$$

$$\therefore \quad \mathbf{V_2 = 199.1615\ m/sec} \quad \text{... Ans.}$$

Example 9.2 : *For a multistage axial flow compressor the mass flow rate is 20 kg/sec. The inlet stagnation condition is 1 bar and 17 °C. The power required for compressor is 4350 kW. Find :*
i) Delivery pressure of compressor ii) Number of stages required
iii) Overall isentropic efficiency of compressor.
Assume temperature rise in the first stage as 15 °C and polytropic efficiency is 90 % and stagnation pressure ratio is constant. ☞ **[GTU : Summer-16, Winter-17]**

Solution : Given data :

$\dot{m}$ = 20 kg/sec., p_{01} = 1 bar, T_{01} = 17 °C = 17 + 273 = 290 K, P = 4350 kW,

$(\Delta T)_{01}$ = 15 °C, η_p = 90 % = 0.9, $(r_p)_{stage}$ = Constant, C_p = 1.005 kJ/kg K

To find : i) p_{0N} ii) z iii) η_{isen}

Step 1 : Calculate the delivery pressure of compressor

Power required by the compressor,

$$P = \dot{m}\, WD_{act} = \dot{m} C_p\, (T_{0N} - T_{01})$$

$$\therefore \quad 4350 \times 10^3 = 20 \times 1.005 \times 10^3 \times (T_{0N} - 290)$$

$$\therefore \quad T_{0N} = 506.4179 \text{ K}$$

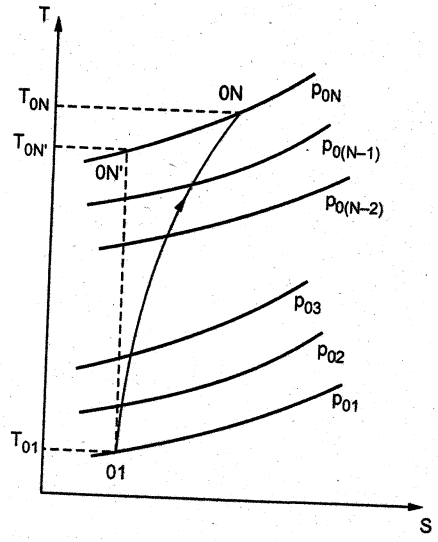


Fig. 9.11

For stagnation state,

at point 02, $T_{02} = T_{01} + (\Delta T)_{01} = 17 + 15 = 32°C$

$= 32 + 273 = 305$ K

For states 01 and 02, polytropic efficiency,

$$\eta_p = \frac{\frac{\gamma-1}{\gamma}\log_e\left(\frac{p_{02}}{p_{01}}\right)}{\log_e\left(\frac{T_{02}}{T_{01}}\right)} \quad \therefore \quad 0.9 = \frac{\frac{1.4-1}{1.4}\times\log_e\left(\frac{p_{02}}{p_{01}}\right)}{\log_e\left(\frac{305}{290}\right)}$$

$$\therefore \quad \frac{p_{02}}{p_{01}} = (r_p)_{stage} = 1.1721$$

Also, $$\eta_p = \frac{\frac{\gamma-1}{\gamma}\log_e\left(\frac{p_{0N}}{p_{01}}\right)}{\log_e\left(\frac{T_{0N}}{T_{01}}\right)}$$

$$\therefore \quad 0.9 = \frac{\frac{1.4-1}{1.4}\times\log_e\left(\frac{p_{0N}}{p_{01}}\right)}{\log_e\left(\frac{506.4179}{290}\right)} \quad \therefore \quad \frac{p_{0N}}{p_{01}} = r_p = 5.7896$$

$$\therefore \quad p_{0N} = 1 \times 5.7896 = \mathbf{5.7896\ bar} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the number of stages

As the pressure ratio is constant i.e. same for each stage,

$$\frac{p_{02}}{p_{01}} = \frac{p_{03}}{p_{02}} = \frac{p_{04}}{p_{03}} = \text{.....} = \frac{p_{0(N-1)}}{p_{0(N-2)}} = \frac{p_{0N}}{p_{0(N-1)}} = 1.1721$$

$$\therefore \quad \left(\frac{p_{02}}{p_{01}}\right)^{z-1} = \left(\frac{p_{0N}}{p_{01}}\right) \quad \therefore \quad 1.1721^{(z-1)} = 5.7896$$

$$\therefore \quad \mathbf{z = 12} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the overall isentropic efficiency

We know that,

$$\frac{T_{012'}}{T_{01}} = \left(\frac{p_{012'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{012}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{012'}}{290} = (5.7896)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_{012'} = 478.9581 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{012'} - T_{01}}{T_{012} - T_{01}} = \frac{478.9581 - 290}{506.4179 - 290} \quad \text{... } [\because T_{0N} = T_{012} = 506.4179 \text{ K}]$$

$$\therefore \quad \eta_{isen} = \mathbf{0.87311 = 87.311\ \%} \qquad \textbf{... Ans.}$$

Example 9.3 : *An axial flow air compressor having 50 % reaction design, has blades with inlet and outlet angles of 45° and 10° respectively. The compressor produces a pressure ratio of 5 : 1 with an overall isentropic efficiency of 85 % when the air inlet temperature is 40 °C. The blade speed and axial velocity are constant throughout the compressor. Assuming a value of 200 m/s for the blade speed, find the number of stages required when the work factor is (i) Unity (ii) 0.8 for all stages.*

Solution : Given data :

$R_D = 50\ \% = 0.5$, $\theta = 45°C$, $\phi = 10°C$, $r_p = 5$, $\eta_{isen} = 85\ \% = 0.85$,

$T_1 = 40\ °C = 40 + 273 = 313\ K$, $V_{f1} = V_{f2} = V_f$, $u = 200$ m/sec.

To find : z when i) $\phi_w = 1$ ii) $\phi_w = 0.8$

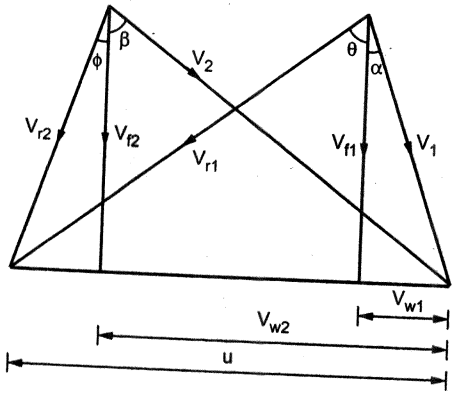


Fig. 9.12

Step 1 : Calculate the stagnation temperatures

For 50 % reaction design i.e. $R_D = 0.5$,

$$\theta = \beta = 45\ °C,\ \alpha = \phi = 10°$$

$$\frac{u}{V_f} = \tan\theta + \tan\phi \quad \therefore \quad \frac{200}{V_f} = \tan(45) + \tan(10)$$

$$\therefore \quad V_f = 170.0207 \text{ m/sec}$$

From Fig. 9.12 by inlet velocity triangle,

$$\cos\alpha = \frac{V_{f1}}{V_1} \quad \therefore \quad \cos(10) = \frac{170.0207}{V_1}$$

$$\therefore \quad V_1 = 172.6435 \text{ m/sec}$$

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

For point 01, $T_{01} - T_1 = \dfrac{V_1^2}{2C_p}$

$$\therefore \quad T_{01} - 313 = \frac{172.6435^2}{2\times1.005\times10^3} \quad \text{... [Taking } C_p = 1.005\times10^3\text{]}$$

$$\therefore \quad T_{01} = 327.8287 \text{ K}$$

We know that,

$$\frac{T_{0N'}}{T_{01}} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{0N'}}{327.8287} = (5)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_{0N'} = 519.2215 \text{ K}$$

Step 2 : Calculate number of stages for given work factors

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{0N'} - T_{01}}{T_{0N} - T_{01}} = \frac{(\Delta T)_{isen}}{(\Delta T)_{act}}$$

$$0.85 = \frac{519.2215 - 327.8287}{(\Delta T)_{act}} \quad \therefore \ (\Delta T)_{act} = 225.168 \text{ K}$$

Work consumed by the compressor,

$$\therefore \quad WD_{act} = C_p(\Delta T)_{act} = 1.005\times10^3 \times 225.168 \quad \text{... [Taking } C_p = 1.005\times10^3\text{]}$$

$$= 226.2938\times10^3 \text{ J/kg}$$

From Fig. 9.12, by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = u - V_f \tan\theta = 200 - 170.0207 \times \tan(45) = 29.9793 \text{ m/sec}$$

From Fig. 9.12, by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = u - V_f \tan\phi = 200 - 170.0207 \times \tan(10) = 170.0207 \text{ m/sec}$$

Total work done for $\phi_w = 1$,

$$\therefore \quad WD_{act} = \phi_w\ u(V_{w2} - V_{w1})\ z$$

$$\therefore \quad 226.2938\times10^3 = 1\times200\times(170.0207 - 29.9793)\times z$$

$$\therefore \quad z = 8.0795 \approx \mathbf{9} \qquad \textbf{... Ans.}$$

Total work done for $\phi_w = 0.8$,

$$WD_{act} = \phi_w\ u(V_{w2} - V_{w1})\ z$$

$$\therefore \quad 226.2938\times10^3 = 0.8\times200\times(170.0207 - 29.9793)\times z$$

$$\therefore \quad z = 10.0994 \approx \mathbf{11} \qquad \textbf{... Ans.}$$

Example 9.4 : *An axial flow compressor is required to deliver 50 kg/sec of air at a stagnation pressure of 5 bar. At inlet to first stage the stagnation pressure is 1 bar and stagnation temperature is 300 K. The hub and tip diameters at this location are 0.436 m and 0.728 m. At mean radius, which is constant through all stages of the compressor, the reaction is 0.5 and absolute air angles at stator exit is 28.8° for all stages. The speed of rotor is 8000 rpm.*

Determine the number of similar stages needed, assuming polytropic efficiency is 0.89 and that the axial velocity at the mean radius is constant through the stages equal to 1.05 times the average axial velocity.

Solution : Given data :

$\dot{m}$ = 50 kg/sec, p_{0N} = 5 bar, p_{01} = 1 bar, T_{01} = 300 K, D_h = 0.436 m, D_1 = 0.728 m,

R_D = 50 % = 0.5, $\phi = \alpha = 28.8°$, N = 8000 rpm, η_p = 89 % = 0.89,

$V_{f1} = V_{f2} = V_f = 1.05\,(V_f)_{avg}$

To find : z

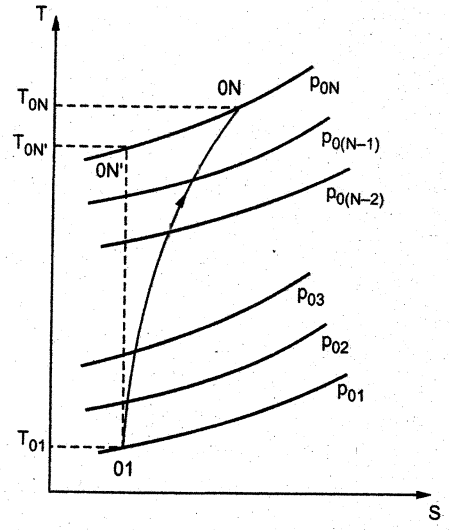


Fig. 9.13 (a)

Step 1 : Calculate the total work done

By ideal Gas Equation,

$$p\,V = m\,R\,T \quad \therefore \quad p = \rho RT$$

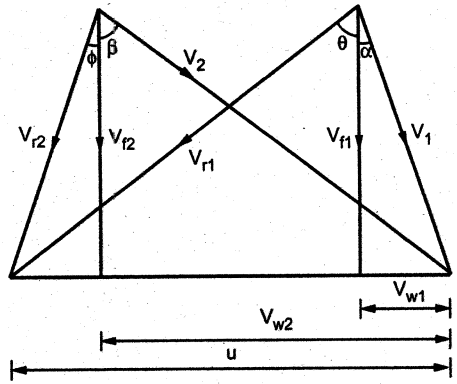


Fig. 9.13 (b)

For stagnation state 01,

$$p_{01} = \rho R\,T_{01} \quad \therefore \quad 1\times10^5 = \rho\times287\times300$$

$$\therefore \quad \rho = 1.1614\ kg/m^3$$

Mass flow rate,

$$\dot{m} = \rho\,A(V_f)_{avg} = \rho\frac{\pi}{4}(D_1^2 - D_h^2)\,(V_f)_{avg}$$

$$\therefore \quad 50 = 1.1614\times\frac{\pi}{4}\times(0.728^2 - 0.436^2)\times(V_f)_{avg}$$

$$\therefore \quad (V_f)_{avg} = 161.2732\ \text{m/sec}$$

Axial velocity at mean radius,

$$V_{f1} = V_{f2} = V_f = 1.05\ (V_f)_{avg} = 1.05\times161.2732 = 169.3368\ \text{m/sec}$$

Polytropic efficiency,

$$\eta_p = \frac{\left(\frac{\gamma-1}{\gamma}\right)\log_e\left(\frac{p_{0N}}{p_{01}}\right)}{\log_e\left(\frac{T_{0N}}{T_{01}}\right)}$$

$$\therefore \quad 0.89 = \frac{\frac{1.4-1}{1.4}\times\log_e\left(\frac{5}{1}\right)}{\log_e\left(\frac{T_{0N}}{300}\right)}$$

$$\therefore \quad T_{0N} = 502.9324\ K$$

Total work done,

$$WD_{act} = C_p(T_{0N} - T_{01})$$
$$= 1.005 \times 10^3 \times (502.9324 - 300)$$
$$= 203.947 \times 10^3 \text{ J/kg}$$

Step 2 : Calculate the number of stages

From Fig. 9.13 (b), by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = V_{f1} \tan\alpha = 169.3368 \times \tan(28.8) = 93.0936 \text{ m/sec}$$

Mean diameter,

$$D_m = \frac{D_1 + D_h}{2} = \frac{0.728 + 0.436}{2} = 0.582 \text{ m}$$

Blade velocity,

$$u = \frac{\pi D_m N}{60} = \frac{\pi \times 0.582 \times 8000}{60} = 243.7875 \text{ m/sec}$$

From Fig. 9.13 (b), by outlet velocity triangle,

$$V_{w2} = u - V_{f2} \tan\phi = 243.7875 - (169.3368 \times \tan(28.8))$$
$$= 150.6938 \text{ m/sec}$$

Also, $WD_{act} = u(V_{w2} - V_{w1})z$

$\therefore \quad 203.947 \times 10^3 = 243.7875 \times (150.6938 - 93.0936) \times z$

$\therefore \quad z = 14.5238 \approx \mathbf{15}$ **... Ans.**

Example 9.5 : *If an axial flow compressor with overall isentropic efficiency 90 % draws air at 20 °C and compresses it to a pressure ratio of 4. Assuming 50 % reaction and blade velocity as 180 m/sec, find flow velocity and number of stages (z). Take* $\alpha = 12°$ *and* $\theta = 42°$. *Assume work input factor = 0.8.*

Solution : Given data :

$\eta_{isen} = 90\ \% = 0.9$, $T_{01} = 20\ °C = 20 + 273 = 293$ K, $r_p = 4$, $u = 180$ m/sec,

$\phi_w = 0.8$, $V_{f1} = V_{f2} = V_f$, For $R_D = 50\ \% = 0.5$, $\alpha = \phi = 12°$, $\theta = \beta = 42°$

To find : i) V_f ii) z

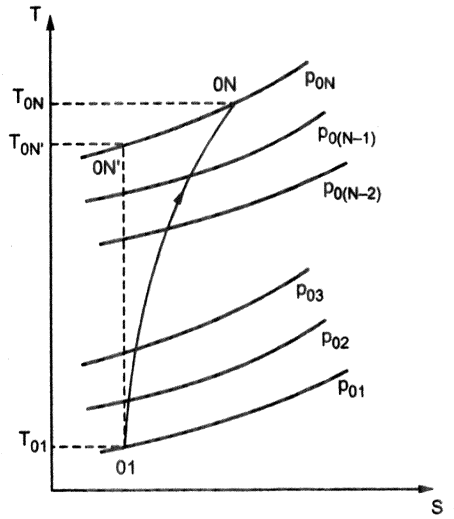


Fig. 9.14 (a)

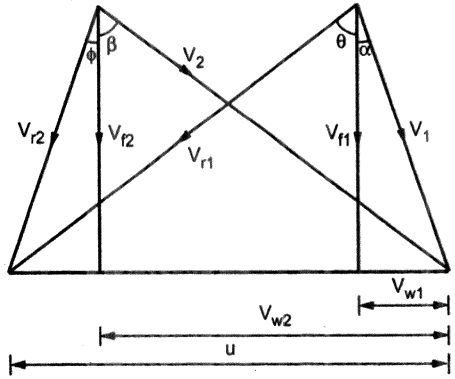


Fig. 9.14 (b)

Step 1 : Calculate the flow velocity

For 50 % reaction design i.e. $R_D = 0.5$,

$$\frac{u}{V_f} = \tan\theta + \tan\phi = \tan(42) + \tan(12) = 1.1129$$

$\therefore \quad V_f = \frac{u}{1.1129} = \frac{180}{1.1129}$

$\therefore \quad \mathbf{V_f = 161.7395 \text{ m/sec}}$ **... Ans.**

Step 2 : Calculate the number of stages

We know that,

$$\frac{T_{0N'}}{T_{01}} = \left(\frac{p_{0N}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T_{0N'}}{293} = (4)^{\frac{1.4-1}{1.4}} \quad \therefore T_{0N'} = 435.3963 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{0N'} - T_{01}}{T_{0N} - T_{01}} = \frac{(\Delta T)_{isen}}{(\Delta T)_{act}}$$

$$\therefore \quad 0.9 = \frac{435.3963 - 293}{(\Delta T)_{act}} \quad \therefore (\Delta T)_{act} = 158.2181 \text{ K}$$

Work done per kg of air,

$$WD_{act} = C_p (\Delta T)_{act} = 1.005 \times 10^3 \times 158.2181$$

$$= 159.0091 \times 10^3 \text{ J/kg}$$

From Fig. 9.14 (b), by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = u - V_f \tan\theta = 180 - 161.7395 \times \tan(42) = 34.3691 \text{ m/sec}$$

From Fig. 9.14 (b), by outlet velocity triangle,

Whril velocity at outlet,

$$V_{w2} = u - V_f \tan\phi = 180 - 161.7395 \times \tan(12) = 145.6212 \text{ m/sec}$$

Also,

$$WD_{act} = \phi_w \, u(V_{w2} - V_{w1}) z$$

$$\therefore \quad 159.0091 \times 10^3 = 0.8 \times 180 \times (145.6212 - 34.3691) \times z$$

$$\therefore \quad z = 9.9254 \approx \mathbf{10} \quad \text{... Ans.}$$

Example 9.6 : *An axial flow compressor having 8 stages and 50 % reaction design compresses air in the pressure ratio of 4. The air enters the compressor at 20 °C and flows through it with constant speed of 90 m/sec. The rotating blades of compressor rotate with mean speed of 180 m/sec. Isentropic efficiency of compressor is 82 %. Find work done and blade angles.*

Solution : Given data :

$z = 8$, $R_D = 50\ \% = 0.5$, $\frac{p_{08}}{p_{01}} = 4$, $T_{01} = 20 + 273 = 293$ K, $V_{f1} = V_{f2} = V_f = 90$ m/sec,

$u = 180$ m/sec, $\eta_{isen} = 82\ \% = 0.82$

To find : i) WD_{act} ii) θ, ϕ

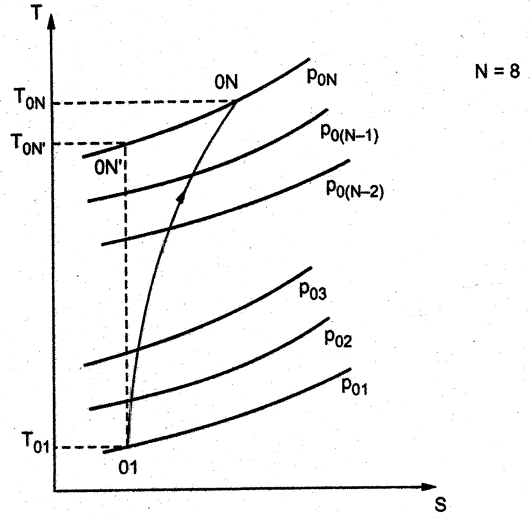


Fig. 9.15 (a)

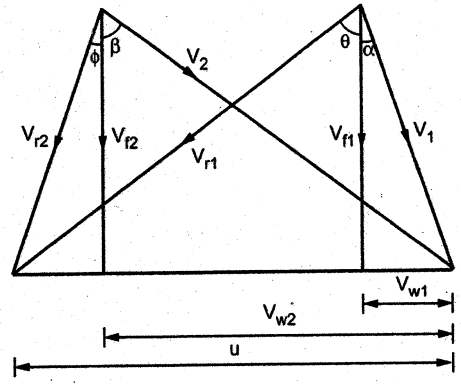


Fig. 9.15 (b)

Step 1 : Calculate the work by the compressor

We know that,

$$\frac{T_{08'}}{T_{01}} = \left(\frac{p_{08}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \frac{T_{08'}}{293} = (4)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad T_{08'} = 435.3963\ K$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{08'} - T_{01}}{T_{08} - T_{01}} \quad \therefore 0.82 = \frac{435.3963 - 293}{T_{08} - 293}$$

$\therefore \quad T_{08} = 466.654\ K$

Work done by the compressor,

$$WD_{act} = C_p(T_{08} - T_{01}) = 1.005 \times 10^3 \times (466.654 - 293)$$

$\therefore \quad \mathbf{WD_{act} = 174.5222 \times 10^3\ J/kg}$ **... Ans.**

Step 2 : Calculate the blade angles at inlet and outlet

From Fig. 9.15 (b), by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = u - V_f \tan\theta = 180 - 90 \tan\theta$$

From Fig. 9.15 (b), by outlet veloity triangle

Whirl velocity at outlet,

$$V_{w2} = u - V_f \tan\phi = 180 - 90 \tan\phi$$

$\therefore$ Total work done,

$$WD_{act} = u\,(V_{w2} - V_{w1})\,z$$

$$\therefore \quad 174.5222 \times 10^3 = 180 \times [(180 - 90\tan\phi) - (180 - 90\tan\theta)] \times 8$$

$$\therefore \quad \tan\theta - \tan\phi = 1.3466 \qquad \ldots (9.23)$$

For 50 % reaction design i.e. $R_D = 0.5$

$$\frac{u}{V_f} = \tan\theta + \tan\phi \quad \therefore \quad \frac{180}{90} = \tan\theta + \tan\phi$$

$$\tan\theta + \tan\phi = 2 \qquad \ldots (9.24)$$

Solving Equations (9.23) and (9.24),

$$\tan\theta = 1.6733, \quad \tan\phi = 0.3267$$

$\therefore \quad \beta = \theta = \mathbf{59.1365°}$ **... Ans.**

$\therefore \quad \alpha = \phi = \mathbf{18.0922°}$ **... Ans.**

Example 9.7 : *In 8 stage axial compressor the overall stagnation pressure ratio is 5 with isentropic efficiency of 92 %. The inlet stagnation temperature and pressure are 290 K and 1 bar. The mean blade speed is 160 m/sec. and 50 % reaction design is used. The axial velocity is constant which is 90 m/sec. Find : i) Blade angles ii) Power required.*

Solution : Given data :

$z = 8, r_p = \frac{p_{08}}{p_{01}} = 5, \eta_{isen} = 92\ \% = 0.92, T_{01} = 290\ K, p_{01} = 1\ bar, u = 160\ m/sec.,$

$R_D = 50\ \% = 0.5, V_{f1} = V_{f2} = V_f = 90\ m/sec.$

To find : i) θ, ϕ ii) P

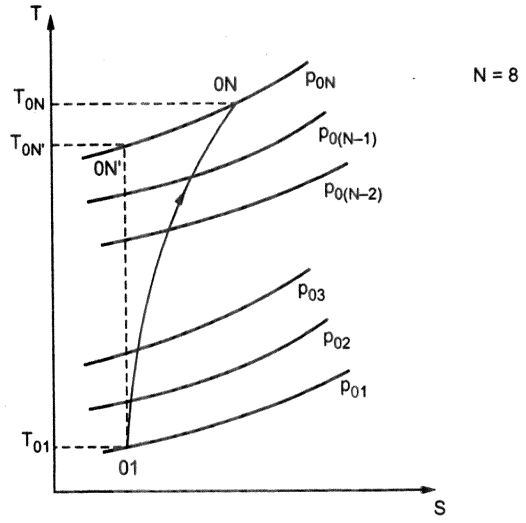


Fig. 9.16 (a)

Step 1 : Calculate the blade angles

We know that,

$$\frac{T_{08'}}{T_{01}} = \left(\frac{p_{08'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{08}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{08'}}{290} = (5)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad T_{08'} = 459.3076\ K$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{08'} - T_{01}}{T_{08} - T_{01}} \quad \therefore 0.92 = \frac{459.3076 - 290}{T_{08} - 290}$$

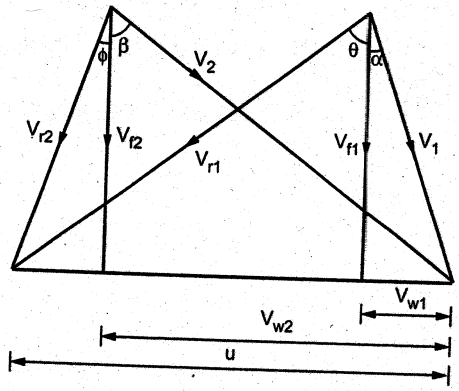


Fig. 9.16 (b)

$\therefore \quad T_{08} = 474.03 \text{ K}$

Work consumed by the compressor,

$$WD_{act} = C_p(T_{08} - T_{01}) = 1.005 \times 10^3 \times (474.03 - 290)$$

$$\therefore \quad WD_{act} = 184.9501 \times 10^3 \text{ J/kg}$$

Also,

$$WD_{act} = u(V_{w2} - V_{w1})z$$

$$\therefore \quad 184.9501 \times 10^3 = 160 \times (V_{w2} - V_{w1}) \times 8$$

$$\therefore \quad V_{w2} - V_{w1} = 144.4922 \qquad \ldots (9.25)$$

From Fig. 9.16 (b), by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = u - V_f \tan\theta = 160 - 90 \tan\theta$$

From Fig. 9.16 (b), by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = u - V_f \tan\phi = 160 - 90 \tan\phi$$

$$V_{w2} - V_{w1} = (160 - 90\tan\phi) - (160 - 90\tan\theta) = 90\tan\theta - 90\tan\phi$$

$$\therefore \quad V_{w2} - V_{w1} = 90(\tan\theta - \tan\phi) \qquad \ldots (9.26)$$

Equating equations (9.25) and (9.26),

$$90(\tan\theta - \tan\phi) = 144.4922$$

$$\therefore \quad \tan\theta - \tan\phi = 1.6054 \qquad \ldots (9.27)$$

For 50 % reaction design i.e. $R_D = 0.5$.

$$\tan\theta + \tan\phi = \frac{u}{V_f} = \frac{160}{90}$$

$$\therefore \quad \tan\theta + \tan\phi = 1.7778 \qquad \ldots (9.28)$$

Solving equations (9.27) and (9.28),

$$\tan\theta = 1.6916 \text{ and } \tan\phi = 0.0862$$

$$\therefore \quad \theta = \mathbf{59.4102°} \qquad \textbf{... Ans.}$$

$$\therefore \quad \phi = \mathbf{4.9267°} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the power required

Assuming mechanical efficiency $\eta_{mech} = 100$ %,

$$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

Power required,

$$\therefore \quad P_{cons} = P_{req} = \dot{m}\, WD_{act} = 1 \times 184.9501 \times 10^3$$

$$\therefore \quad \mathbf{P = 184.9501 \times 10^3 \text{ W}} \qquad \textbf{... Ans.}$$

Example 9.8 : *An eight stage axial flow compressor takes in air at a temperature of 293 K at the rate of 3 kg/s. The pressure ratio obtained is 6 and isentropic efficiency is 0.89. The compressor is designed for 50 % reaction. The blade speed for each stage may be assumed constant as 180 m/s and the velocity of flow as 100 m/s. Determine the power required to drive the compressor and the direction of air at entry and exit from the rotor and stator. Assume that the total work is equally shared between the stages. Assume, $\gamma = 1.4$ and $C_p = 1.005$ kJ/kg K.*

Solution : Given data :

$z = 8$, $T_1 = 293$ K, $\dot{m} = 3$ kg/sec, $r_p = \frac{p_{08}}{p_{01}} = 6$, $\eta_{isen} = 0.89 = 89$ %,

$R_D = 50$ % $= 0.5$, $u = 180$ m/sec, $V_{f1} = V_{f2} = V_f = 100$ m/sec, $\gamma = 1.4$,

$C_p = 1.005$ kJ/kg K.

To find : i) P_{cons} ii) $\theta, \alpha, \phi, \beta$

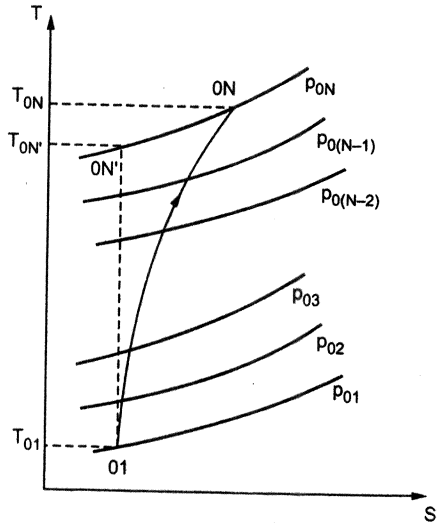


Fig. 9.17 (a)

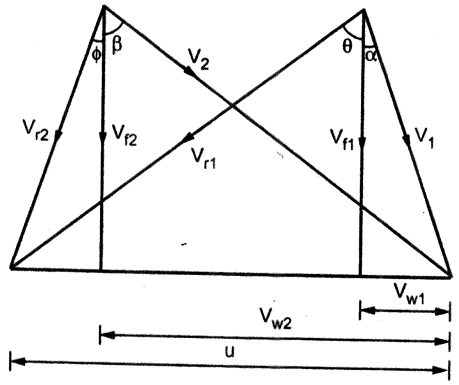


Fig. 9.17 (b)

Step 1 : Calculate the power required to drive the compressor

We know that,

$$\frac{T_{08'}}{T_{01}} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{08'}}{293} = (6)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_{08'} = 488.8735 \text{ K}$$

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{08'} - T_{01}}{T_{08} - T_{01}} \quad \therefore 0.89 = \frac{488.8735 - 293}{T_{08} - 293}$$

$$\therefore \quad T_{08} = 513.0825 \text{ K}$$

Work done per kg of air,

$$WD_{act} = C_p(T_{08} - T_{01})$$

$$= 1.005 \times 10^3 \times (513.0825 - 293)$$

$$\therefore \quad WD_{act} = 221.1829 \times 10^3 \text{ J/kg}$$

Power required,

Assuming mechanical efficiency η_{mech} = 100 %

$$\therefore \quad \eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

$$\therefore \quad P_{cons} = P_{req} = \dot{m}\, WD_{act} = 3 \times 221.1829 \times 10^3$$

$$\therefore \quad \mathbf{P_{cons} = 663.5487 \times 10^3 \text{ W}} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the blade angles

From Fig. 9.17 (b), by inlet velocity triangle,

$$V_{w1} = u - V_f \tan\theta = 180 - 100 \tan\theta$$

From Fig. 9.17 (b), by outlet velocity triangle,

$$V_{w2} = u - V_f \tan\phi = 180 - 100 \tan\phi$$

Also $WD_{act} = u(V_{w2} - V_{w1})z$

$$\therefore \quad 221.1829 \times 10^3 = 180 \times [(180 - 100 \tan\phi) - (180 - 100 \tan\theta)] \times 8$$

$$\therefore \quad \tan\theta - \tan\phi = 1.5359 \qquad \ldots (1)$$

For 50 % reaction design i.e. R_D = 0.5,

$$\frac{u}{V_f} = \tan\theta + \tan\phi \qquad \therefore \quad \frac{180}{100} = \tan\theta + \tan\phi$$

$$\therefore \quad \tan\theta + \tan\phi = 1.8 \qquad \ldots (2)$$

Solving equations (1) and (2),

$\tan\theta = 1.6679, \quad \tan\phi = 0.132$

$\therefore \quad \theta = \beta = 59.0549°$ ... Ans.

$\therefore \quad \phi = \alpha = 7.5195°$ ... Ans.

Example 9.9 : *In axial flow compressor the overall stagnation pressure ratio is 4 and isentropic efficiency is 85 %. The inlet stagnation pressure is 1 bar and temperature is 300 K. The mean blade speed is 180 m/sec. The degree of reaction is 50 % and the inlet angles are 12° and 32°. The work factor is 0.9; find :*
i) Stagnation polytropic efficiecy. ii) Number of stages.
iii) Inlet temperature and pressure iv) Blade height in first stage if hub to tip ratio is 0.42 and mass flow rate is 19.5 kg/sec.

Solution : Given data :

$\frac{p_{0N}}{p_{01}} = r_p = 4$, $\dot{m} = 19.5$ kg/sec, $\eta_{isen} = 85\ \% = 0.85$, $p_{01} = 1$ bar, $T_{01} = 300$ K,

$u = 180$ m/sec, $\phi_w = 0.9$, For $R_D = 50\ \% = 0.5$, $\alpha = \phi = 12°$, $\beta = \theta = 32°$, $\frac{D_h}{D_1} = 0.42$.

To find : i) η_p ii) z iii) p_1, T_1 iv) H

Step 1 : Calculate the stagnation polytropic efficiency

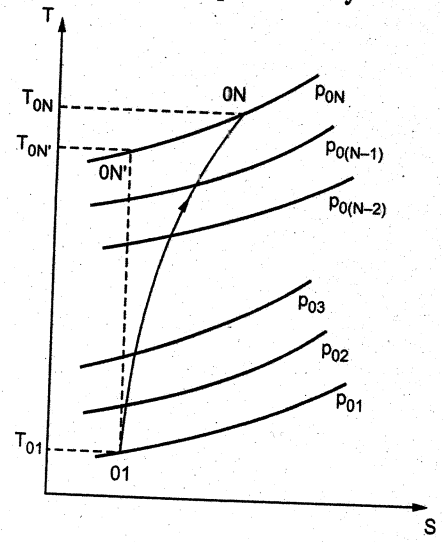


Fig. 9.18 (a)

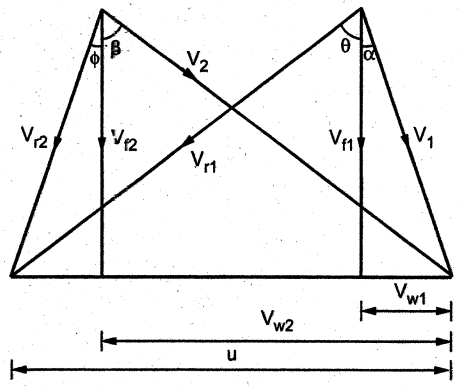


Fig. 9.18 (b)

We know that,

$$\frac{T_{0N'}}{T_{01}} = \left(\frac{p_{0N}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \qquad \therefore \quad \frac{T_{0N'}}{300} = (4)^{\frac{1.4-1}{1.4}}$$

$\therefore \quad T_{0N'} = 445.7982$ K

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{0N'} - T_{01}}{T_{0N} - T_{01}}$$

$$\therefore \quad 0.85 = \frac{445.7982 - 300}{T_{0N} - 300}$$

$\therefore \quad T_{0N} = 471.5272$ K

Polytropic stagnation efficiency,

$$\eta_p = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\log_e\left(\frac{p_{0N}}{p_{01}}\right)}{\log_e\left(\frac{T_{0N}}{T_{01}}\right)}\right)$$

$$\therefore \quad = \frac{1.4-1}{1.4} \times \frac{\log_e(4)}{\log_e\left(\frac{471.5272}{300}\right)}$$

$\therefore \quad \eta_p = 0.8759 = 87.5915\ \%$ ... Ans.

Step 2 : Calculate the number of stages

Work consumed by the compressor,

$$WD_{act} = C_p(T_{0N} - T_{01}) = 1.005\times10^3 \times (471.5272 - 300)$$

$$\therefore \quad WD_{act} = 172.3848\times10^3 \text{ J/kg}$$

For 50 % reaction design i.e., $R_D = 0.5$

$$\frac{u}{V_f} = \tan\theta + \tan\phi \quad \therefore \frac{180}{V_f} = \tan(32) + \tan(12)$$

$$\therefore \quad V_f = 214.9443 \text{ m/sec}$$

From Fig. 9.18 (b), by inlet velocity triangle,

Whirl velocity at inlet,

$$V_{w1} = u - V_f \tan\theta = 180 - 214.9443 \times \tan(32) = 45.6878 \text{ m/sec}$$

From Fig. 9.18 (b), by outlet velocity triangle,

Whirl velocity at outlet,

$$V_{w2} = u - V_f \tan\phi = 180 - 214.9443 \times \tan(12) = 134.3121 \text{ m/sec}$$

Also, $WD_{act} = \phi_w \, u(V_{w2} - V_{w1}) z$

$$\therefore \quad 172.3848\times10^3 = 0.9 \times 180 \times (134.3121 - 45.6878) \times z$$

$$\therefore \quad \mathbf{z = 12} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the inlet temperature and pressure

From Fig. 9.18 (b), by inlet velocity triangle,

$$\cos\alpha = \frac{V_{f1}}{V_1} \quad \therefore \cos(12) = \frac{214.9443}{V_1}$$

$$\therefore \quad V_1 = 219.7462 \text{ m/sec.}$$

We know that, for stagnation state,

$$T_0 - T = \frac{V^2}{2C_p}$$

At point 01, $T_{01} - T_1 = \frac{V_1^2}{2C_p}$

$$\therefore \quad 300 - T_1 = \frac{219.7462^2}{2\times1.005\times10^3}$$

$$\therefore \quad \mathbf{T_1 = 275.9759 \text{ K}} \qquad \textbf{... Ans.}$$

We know that,

$$\frac{T_{01}}{T_1} = \left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{300}{275.9759} = \left(\frac{1}{p_1}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad \mathbf{p_1 = 0.7466 \text{ bar}} \qquad \textbf{... Ans.}$$

Step 4 : Calculate the height of blade in first stage

For first stage, by ideal gas equation,

$$pV = mRT \quad \therefore \quad p = \rho R T$$

$$\therefore \quad p_{01} = \rho_1 R T_{01} \quad \therefore \; 1\times10^5 = \rho_1 \times 287 \times 300$$

$$\therefore \quad \rho_1 = 1.161 \text{ kg/m}^3$$

Mass flow rate,

$$\dot{m} = \rho_1 A_1 V_{f1} = \rho_1 \frac{\pi}{4}(D_1^2 - D_h^2) V_{f1}$$

$$\therefore \quad 19.5 = 1.161 \times \frac{\pi}{4} \times [D_1^2 - (0.42\, D_1)^2] \times 214.9443$$

$$\therefore \quad D_1 = 0.347 \text{ m}$$

$$D_h = 0.42\, D_1 = 0.42 \times 0.347 = 0.145 \text{ m}$$

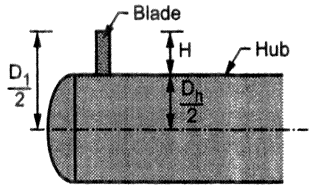


Fig. 9.18 (c)

Height of blade,

$$H = \frac{D_1 - D_h}{2} = \frac{0.347 - 0.145}{2}$$

$$\therefore \quad \mathbf{H = 0.101 \text{ m}} \qquad \textbf{... Ans.}$$

Example 9.10 : *Derive the following relation for efficiencies and degree of reactions of axial compressor,*

$$\eta_{st} = R\eta_R + (1 - R)\,\eta_D$$

Calculate the value of the stage efficiency of 50 % reaction compressor stage with the following efficiencies of the blade rows,

$$\eta_R = 0.849 \text{ and } \eta_D = 0.849$$

Solution :

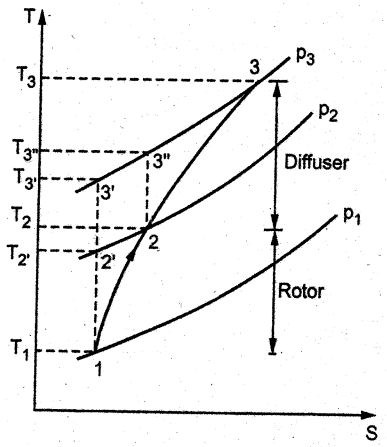


Fig. 9.21

- From Fig. 9.21

Stage efficiency $\eta_{st} = \dfrac{h_{3'} - h_1}{h_3 - h_1} = \dfrac{C_p(T_{3'} - T_1)}{C_p(T_3 - T_1)}$

$$\eta_{st} = \frac{T_{3'} - T_1}{T_3 - T_1}$$

Adding and subtracting $T_{2'}$ in numarator,

$$\eta_{st} = \frac{T_{3'} - T_1 + T_{2'} - T_{2'}}{T_3 - T_1} = \frac{(T_{3'} - T_{2'}) + (T_{2'} - T_1)}{T_3 - T_1}$$

$$\eta_{st} = \frac{T_{2'} - T_1}{T_3 - T_1} + \frac{T_{3'} - T_{2'}}{T_3 - T_1}$$

Multiplying and dividing the first term by $(T_2 - T_1)$

$$\eta_{st} = \frac{T_{2'} - T_1}{T_2 - T_1} \times \frac{T_2 - T_1}{T_3 - T_1} + \frac{T_{3'} - T_{2'}}{T_3 - T_1} \quad \ldots (i)$$

- We know that, degree of reaction (R) is,

$$R = \frac{\text{Pressure rise in rotor}}{\text{Pressure rise in a stage}}$$

$$\therefore \quad R = \frac{(\Delta h)_{rotor}}{(\Delta h)_{rotor} + (\Delta h)_{Diffuser}}$$

$$\therefore \quad R = \frac{(h_2 - h_1)}{(h_2 - h_1) + (h_3 - h_2)} = \frac{h_2 - h_1}{h_3 - h_1}$$

or $$R = \frac{T_2 - T_1}{T_3 - T_1} \quad \ldots (ii)$$

- Also rotor and diffuser efficiencies are,

$$\eta_{Rot} = \frac{h_{2'} - h_1}{h_2 - h_1} = \frac{T_{2'} - T_1}{T_2 - T_1} \quad \ldots (iii)$$

and $$\eta_{Diff} = \frac{h_{3''} - h_2}{h_3 - h_2} \approx \frac{h_{3'} - h_{2'}}{h_3 - h_2} = \frac{T_{3'} - T_{2'}}{T_3 - T_2} \quad \ldots (iv)$$

Substituting equations (ii), (iii) and (iv) in equation (i),

$$\eta_{st} = \eta_R \times R + \frac{T_{3'} - T_{2'}}{T_3 - T_1} \quad \ldots (v)$$

Now, $$\frac{T_{3'} - T_{2'}}{T_3 - T_1} = \frac{T_{3'} - T_{2'}}{T_3 - T_1} \times \frac{T_3 - T_2}{T_3 - T_2} = \frac{T_{3'} - T_{2'}}{T_3 - T_2} \times \frac{T_3 - T_2}{T_3 - T_1}$$

$$\therefore \quad \frac{T_{3'} - T_{2'}}{T_3 - T_1} = \eta_{Diff} \times \left(\frac{T_3 - T_2}{T_3 - T_1}\right)$$

Adding and subtracting 1 in the bracketed terms,

$$\therefore \quad \frac{T_{3'} - T_{2'}}{T_3 - T_1} = \eta_{Diff} \times \left(\frac{T_3 - T_2}{T_3 - T_1} + 1 - 1\right)$$

$$\therefore \quad \frac{T_{3'} - T_{2'}}{T_3 - T_1} = \eta_{Diff} \times \left(1 + \frac{T_3 - T_2 - T_3 + T_1}{T_3 - T_1}\right)$$

$$\therefore \quad \frac{T_{3'} - T_{2'}}{T_3 - T_1} = \eta_{Diff} \times \left[1 + \left(\frac{T_1 - T_2}{T_3 - T_1}\right)\right] = \eta_{Diff} \times \left[1 - \frac{(T_2 - T_1)}{(T_3 - T_1)}\right]$$

$$\therefore \quad \frac{T_{3'} - T_{2'}}{T_3 - T_1} = \eta_{Diff} \times (1 - R) \quad \ldots \text{[From equation (ii)]} \quad \ldots (vi)$$

- Put the value of equation (vi) in equation (v),

$\therefore \quad \eta_{st} = R \times \eta_R + \eta_{Diff} \times (1 - R)$

or $\quad \eta_{st} = R \times \eta_R + (1 - R)\ \eta_D$ **... Hence proved.**

Substitute the given values in the above equation,

$\therefore \quad \eta_{st} = 0.5 \times 0.849 + (1 - 0.5) \times 0.849$

$\therefore \quad \eta_{st} = \mathbf{0.849}$ **... Ans.**

Example 9.11 : *Draw velocity trinagles at entry and exit for following axial compressor stages : i) R = 1/2 ii) R = 1 iii) R > 0.5 iv) R < 0.5*

Solution : We know that, degree of reaction (R) is,

$$R = \frac{(V_{r1}^2 - V_{r2}^2)/2}{u(V_{w2} - V_{w1})} \quad \text{... [From equation (9.7)]}$$

or

$$R = \frac{(V_{r1}^2 - V_{r2}^2)/2}{\frac{V_2^2 - V_1^2}{2} + \frac{V_{r1}^2 - V_{r2}^2}{2}}$$

(i) For $R = \frac{1}{2} = 0.5$, the blades are symmetric i.e. $\alpha = \phi$ and $\theta = \beta$. For this case the combined velocity triangle is shown in Fig. 9.22 (a).

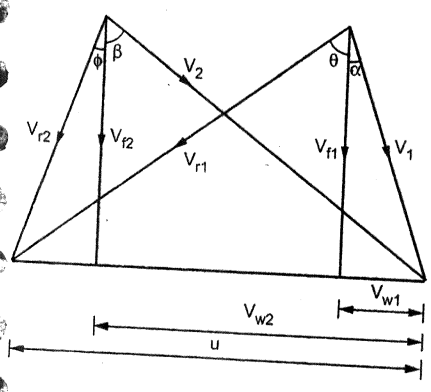


Fig. 9.22 (a) : R = 0.5

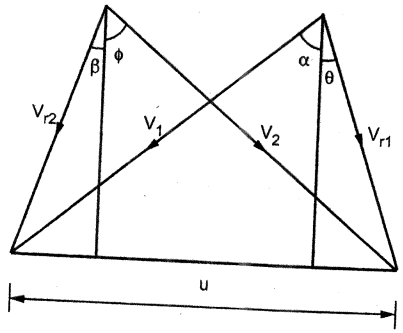


Fig. 9.22 (b) : R = 1

(ii) For R = 1 : If R = 1 then the above equation becomes,

$$R = 1 = \frac{(V_{r1}^2 - V_{r2}^2)/2}{\frac{V_2^2 - V_1^2}{2} + \frac{V_{r1}^2 - V_{r2}^2}{2}}$$

$\therefore \quad V_{r1}^2 - V_{r2}^2 = V_2^2 - V_1^2 + V_{r1}^2 - V_{r2}^2$

$\therefore \quad V_2^2 = V_1^2 \quad$ or $\quad V_2 = V_1$ Refer Fig. 9.22 (b).

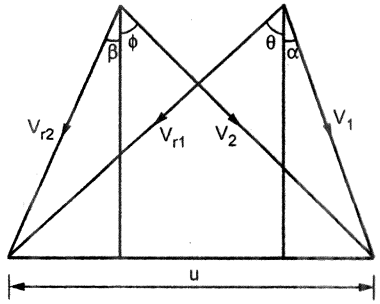


Fig. 9.22 (c) : R > 0.5 (β > α)

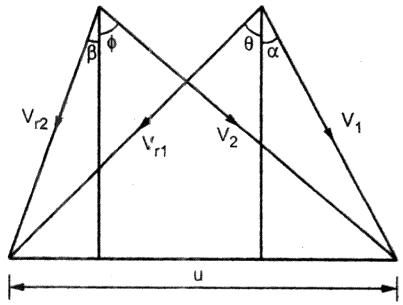


Fig. 9.22 (d) : R < 0.5 (β < α)

(iii) For R > 0.5 : If R > 0.5 it mean enthalpy rise and pressure rise in stator blades is less than that of in rotor blades. For this case, $\beta > \alpha$. Refer Fig. 9.22 (c).

Also if R < 0.5 then the enthalpy and pressure in stator blades is greater than that of in rotor blades. For this case $\beta < \alpha$. Refer Fig. 9.22 (d).

Example 9.12 : *For a multistage axial flow air compressor, initial state of air is 1 bar, 30° C and final state is 6 bar, 300° C. Calculate the overall isentropic and polytropic efficiencies. When the actual air temperature rise per stage as 16 °C, Calculate the number of stages required, assuming polytropic efficiency as the stage efficiency.*

Solution : Given data :

$p_{01} = 1$ bar, $T_{01} = 30\ °C = 30 + 273 = 303$ K, $p_{0N} = 6$ bar,

$T_{0N} = 300\ °C = 300 + 273 = 573$ K, $((\Delta T)_{act})_{stage} = 16\ °C$, $\eta_p = \eta_{stage}$

To find : i) η_{isen} ii) η_p iii) z

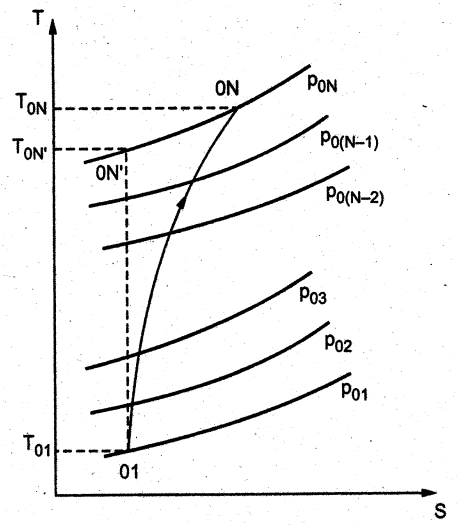


Fig. 9.23

Step 1 : Calculate the overall isentropic efficiency

We know that,

$$\frac{T_{0N'}}{T_{01}} = \left(\frac{p_{0N'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{0N}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad \frac{T_{0N'}}{303} = \left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore \quad T_{0N'} = 505.5586 \text{ K}$$

Overall isentropic efficiency,

$$\eta_{isen} = \frac{T_{0N'} - T_{01}}{T_{0N} - T_{01}} = \frac{505.5586 - 303}{573 - 303}$$

$$\therefore \quad \eta_{isen} = \mathbf{0.7502 = 75.02\ \%} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the overall polytropic efficiency

We know that,

$$\frac{T_{0N}}{T_{01}} = \left(\frac{p_{0N}}{p_{01}}\right)^{\frac{n-1}{n}} \quad \therefore \frac{573}{303} = \left(\frac{6}{1}\right)^{\frac{n-1}{n}} \quad \therefore n = 1.5518$$

Overall polytropic efficiency,

$$\eta_p = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right) = \left(\frac{1.5518}{1.5518 - 1}\right) \times \left(\frac{1.4 - 1}{1.4}\right)$$

$$\therefore \quad \eta_p = \mathbf{0.8035 = 80.35\ \%} \qquad \textbf{... Ans.}$$

Step 3 : Calculate the number of stages

As $\eta_p = \eta_{stage}$

Polytropic efficiency,

$$\eta_p = \frac{(\Delta T)_p}{((\Delta T)_{act})_{stage}} \quad \therefore \quad 0.8035 = \frac{(\Delta T)_p}{16}$$

$$\therefore \quad (\Delta T)_p = 12.856\,^\circ\text{C}$$

Considering only the first stage,

$$T_{02} = T_{01} + (\Delta T)_p = 30 + 12.856$$
$$= 42.856^\circ\text{C} = 315.856 \text{ K}$$

We know that,

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{315.856}{303}\right)^{\frac{1.4}{1.4-1}} = 1.1565$$

For the total process,

$$\frac{p_{0N}}{p_{01}} = \left(\frac{p_{02}}{p_0}\right)^z \quad \therefore \frac{6}{1} = (1.1565)^z$$

$$\therefore \quad z = 12.3231 \approx \mathbf{13} \qquad \textbf{... Ans.}$$

Example 9.13 : *An axial compressor draws air at a suction pressure of 1 bar and temperature 300 K with velocity of 80 m/sec. Isentropic efficiency of compressor is 70 % with stagnation pressure ratio as 4. If mechanical efficiency is 90 %, find the stagnation pressure at exit and power of driving motor for a flow rate of 27 kg/min.*

Solution : Given data :

$p_1 = 1$ bar, $T_1 = 300$ K, $V_1 = 80$ m/sec, $\eta_{isen} = 70\ \% = 0.7$, $\frac{p_{02}}{p_{01}} = 4$,

$\dot{m} = 27$ kg/min $= \frac{27}{60} = 0.45$ kg/sec, $\eta_{mech} = 90\ \% = 0.9$

To find : i) p_{02} ii) P_{req}

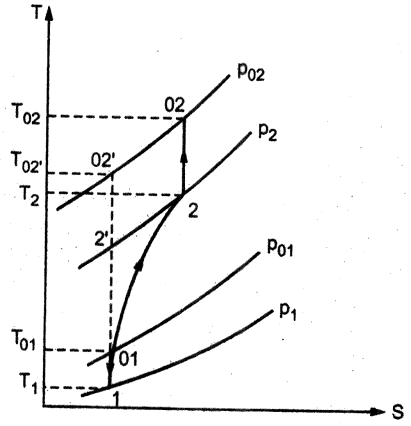


Fig. 9.24

Step 1 : Calculate the stagnation pressure at exit

We know that, for stagnation point,

$$T_0 - T = \frac{V^2}{2\,C_p}$$

For point 01, $T_{01} - T_1 = \dfrac{V_1^2}{2\,C_p}$ $\therefore$ $T_{01} - 300 = \dfrac{80^2}{2\times 1.005\times 10^3}$

$\therefore$ $T_{01} = 303.184$ K

Also,

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \quad \therefore \quad \frac{p_{01}}{1} = \left(\frac{303.184}{300}\right)^{\frac{1.4}{1.4-1}}$$

$\therefore$ $p_{01} = 1.0376$ bar

As $\dfrac{p_{02}}{p_{01}} = 4$ $\therefore$ $\dfrac{p_{02}}{1.0376} = 4$

$\therefore$ $\mathbf{p_{02} = 4.1504\ bar}$ **... Ans.**

Step 2 : Calculate the power of driving motor

As, $\dfrac{T_{02'}}{T_{01}} = \left(\dfrac{p_{02'}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore$ $\dfrac{T_{02'}}{303.184} = (4)^{\frac{1.4-1}{1.4}}$

$\therefore$ $T_{02'} = 450.5296$ K

Isentropic efficiency,

$$\eta_{isen} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} \quad \therefore \quad 0.7 = \frac{450.5296 - 303.184}{T_{02} - 303.184}$$

$\therefore$ $T_{02} = 513.6778$ K

Power consumed,

$$P_{cons} = \dot{m} \times WD_{act} = \dot{m}\, C_p\, (T_{02} - T_{01})$$
$$= 0.45 \times 1.005 \times 10^3 \times (513.6778 - 303.184)$$

$\therefore$ $P_{cons} = 951958 \times 10^3$ W

Here power of the driving motor means the power required (P_{req}).

Mechanical efficiency is given as,

$\therefore$ $\eta_{mech} = \dfrac{P_{cons}}{P_{req}}$ $\therefore 0.9 = \dfrac{951958 \times 10^3}{P_{req}}$

$\therefore$ $\mathbf{P_{req} = 105.7731 \times 10^3\ W}$ **... Ans.**

9.14 Drag and Lift

- Consider a stationary body in a stream of real fluid. Let, the fluid is flowing at a constant velocity 'U' as shown in Fig. 9.25.

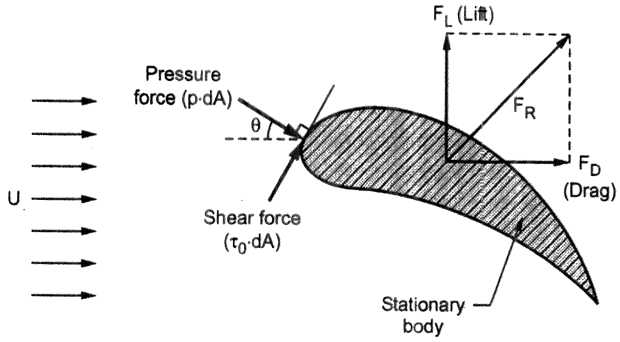


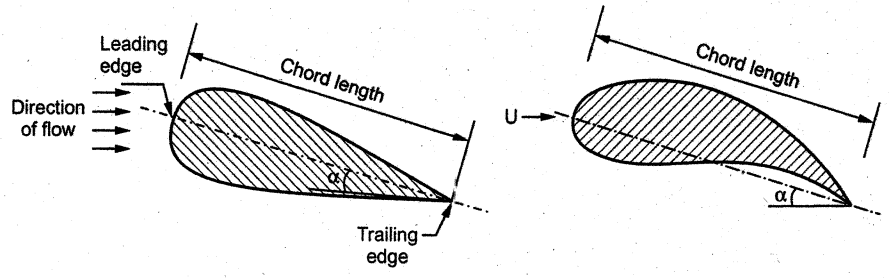
Fig. 9.25 : Force on a stationary body

- The fluid will exert a total force (F_R) on the body which may be inclined at an angle 'θ' to the direction of velocity.
- This total force can be resolved in two components, one in the direction of flow and another is perpendicular to the direction of flow.

- The component of total force in the parallel direction of flow is called as **drag** which is denoted by F_D. It is defined as the force exerted by the fluid on the body in the direction of flow. This force resist the motion of the body or fluid.
- For example, *Wind resistance to a moving car*. To overcome the drag extra power is required hence it is reduced to its minimum value.
- The component of total force in the perpendicular direction of flow is called as **lift** which is denoted by F_L. It is defined as the force exerted by the fluid normal to the direction of motion.
- Lift force will exist only when the axis of body is inclined to the direction of fluid flow. If the axis of body is parallel to the direction of flow and the body is symmetrical, the lift force is zero and only drag force will exist.

9.15 Aerofoil Blading

- An aerofoil is a streamlined body which may be either symmetrical or unsymmetrical. It is characterized by the following parameters (Refer Fig. 9.26) :



(a) Symmetrical airfoil (b) Unsymmetrical airfoil

Fig. 9.26 : Characteristic of an aerofoil

1. **Chord length :** It is the length of the line joining the leading edge and the trailing edge. It is also called as **chord line.** For a symmetrical aerofoil the chord line coincides with the axis of symmetry. It is denoted by **C**.
2. **Angle of attack :** It is the angle between the chord line and the direction of flow of fluid. It is denoted by α.
3. **Span :** It is the length of aerofoil normal to the cross-section. It is denoted by **L.**
4. **Aspect ratio :** It is the ratio of span (L) and chord length (C) of an aerofoil. It is denoted by **A.R.**
5. **Camber :** It is the curvature of the aerofoil from the leading edge to the trailing edge. Upper camber refers to the curvature of upper surface whereas lower camber refers to the curvature of lower surface.

6. **Stall :** The point corresponding to maximum coefficient of lift is known as **stall point** and corresponding angle of attack is called as **stalling angle.** The reduction in the coefficient of lift with increase in angle of attack beyond the stalling angle is due to the separation of flow at some point on the upper side of aerofoil. The condition in which the flow separates from the upper surface of aerofoil is called as stall.

Review Questions

1. *With neat sketch, explain the components of axial flow compressor, along with its working.*
2. *Explain the analysis of axial flow compressor.*
3. *Explain the Enthalpy-Entropy (h-S) diagram for single stage compression in axial flow compressors.*
4. *Explain : i) Degree of reaction (R_D) ii) Work input factor (ϕ_w)*
5. *Show that the axial flow compressor has symmetrical blades for 50 % reaction.*
6. *Derive an expression for polytropic efficiency of an axial flow compressor.*
7. *Define : i) Flow Coefficient ii) Rotor blade loading coefficient iii) Pressure coefficient.*
8. *Write short note on i) Surging ii) Choking iii) Stalling*
9. *Explain the different types of losses in axial flow compressors.*
10. *Explain the performance characteristic curves of axial flow compressors.*
11. *Give the comparison between centrifugal and axial flow compressors.*

Axial Flow Compressors ends ...

Notes

SOLVED MODEL QUESTION PAPER

[As per New Syllabus]

Applied Thermodynamics

Semester - VI (Mechanical)

Time : $2\frac{1}{2}$ Hours] [Total Marks : 70

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **a)** *What is ideal gas and write equation for Ideal gas.* **(Refer sections 1.1 and 1.1.1)** **[3]**

b) *Explain practical vapour absorption cycle.* **(Refer section 3.31)** **[4]**

c) *Define the following : a) Sensible heating b) Sensible cooling c) Humidification d) De-humidification* **(Refer section 2.7)** **[7]**

Q.2 **a)** *Write the classification refrigerant.* **(Refer section 3.3)** **[3]**

b) *Explain cooling and de-humidification.* **(Refer section 2.7.6)** **[4]**

c) *Explain practical Li-Br absorption refrigeration system.* **(Refer section 3.34.1)** **[7]**

OR

c) *With help of neat sketch explain rope brake dynamometer.* **(Refer section 5.4.1)** **[7]**

Q.3 **a)** *Define Willan's Line method.* **(Refer section 6.8)** **[3]**

b) *Explain stagnation properties for turbine.* **[4]**

c) *Moist air enters the cooling coil at 40 °C DBT and 20 °C WBT. Finally, the air is sensibly cooled to 26 °C DBT. Plot the process on psychrometric chart and determine (i) Final WBT of air. (ii) The total heat transferred in kW, if air is flowing at the rate of 100 m^3/min. If the cooling coil surface temperature is 22 °C, find the bypass factor of coil.* **(Refer example 2.11)** **[7]**

OR

Q.3 **a)** *Define Bernoullies equation.* **(Refer section 6.4.2)** **[3]**

b) *Write the applications of compressed air.* **(Refer section 7.1.1)** **[4]**

c) *In refrigeration plant working on Bell - coleman cycle, air is compressed to 1 bar from 7 bar. Its initial temperature is 10 °C. After compression air is cooled upto 20 °C in cooler before expanding back to pressure of 1 bar. Determine theoretical COP of the plant and net refrigerating effect, Take* $C_P = 1.005\ kJ/kg\ K$ *and* $C_V = 0.718\ kJ/kg\ K$. **(Refer example 3.6)** **[7]**

Q.4 **a)** *Define capacity control for compressor.* **(Refer section 7.9)** **[3]**

b) *Drive volumeteric efficiency for compressor.* **(Refer section 7.6.2)** **[4]**

c) *A data pertaining to refrigeration system is as follows :*
Two separate evaporators E_1 *(STR) and* E_2 *(10TR) at temperatures 5 °C and 15 °C respectively. Common condenser is at a temperature 38 °C. The liquid from the condenser is saturated. After evaporator* E_2*, pressure reducing valve bring the pressure to the suction pressure of the compressor. Single compressor and individual expansion valves are used in the system. Use refrigerant data provided. Make the neat flow diagram and a cycle diagram on p-h plane, Find; i) Power required to run the system ii) COP of the system*
(Refer example 3.32) **[7]**

Refrigerant Data								
T	p	v_f	v_g	h_f	h_g	h_{fg}	s_f	s_g
°C	bar	m^3/kg		kJ/kg			kJ/kgK	
5	3.63	0.000725	0.0475	40.69	189.65	148.96	0.1586	0.6943
15	4.91	0.000743	0.0355	50.1	193.79	143.69	0.1915	0.6902
38	9.14	0.000794	0.0191	72.66	202.51	129.95	0.2654	0.683

R134-a Data								
T	p	v_g	h_f	h_g	s_f	S_g	(C_p)	(C_p)
°C	bar	m^3/kg	kJ/kg	kJ/kg	kJ/kgK	kJ/kgK	kJ/kgK	kJ/kgK
−10	2.0052	0.09963	186.78	-	0.9509	1.7337	-	0.842
−6	2.3418	0.8591	192.03	395.15	0.9707	1.731	-	0.858
2	3.145	0.0647	202.68	399.84	-	1.7263	1.341	0.892
12	4.4289	0.04636	-	405.51	1.0579	1.7215	1.374	0.939
24	6.4566	0.03189	233.05	411.93	-	1.7169	-	1.006
30	7.7008	0.02667	241.65	414.94	1.1432	1.7149	-	1.044
50	13.177	-	271.59	423.63	1.2373	1.7078	1.569	1.218

OR

Q.4 **a)** *Draw p-V and T-S diagram for single stage compression.* **(Refer section 7.3)** **[3]**

b) *Write advantages and disadvantages of centrifugal compressor.* **(Refer section 8.3)** **[4]**

c) *The following details were noted in a test on a 4-cylinder, 4-stroke engine, diameter = 100 mm; stroke = 120 mm; speed of the engine = 1600 rpm; fuel consumption = 0.2 kg/min; calorific value of fuel is 44000 kJ/kg; difference in tension on either side of the brake pulley = 40 kg; brake circumference is 300 cm. If the mechanical efficiency is 80%, Calculate :*
(i) Brake thermal efficiency
(ii)Indicated thermal efficiency
(iii) Indicated mean effective pressure
(iv) Brake specific fuel consumption. **(Refer example 5.13)** **[7]**

Q.5 **a)** *Draw theoretical and actual (p-V) diagram for 4-storke petrol engine.* **(Refer section 4.16)** **[3]**

b) *Explain Van Derwalls equation of state.* **(Refer section 1.13)** **[4]**

c) *Calculate the velocity and Mach number of a supersonic aircraft flying at an altitude of 1000 m where temperature is 280 °K. Sound of the air craft is heard 2.15 seconds after the passage of air craft on the head of an observer. Take* $\gamma = 1.41$ *and* $R = 287\ J/kgK$. **(Refer example 6.6)** **[7]**

OR

Q.5 **a)** *Explain continuity equation* $\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$ **(Refer section 6.4.1)** **[3]**

b) *Explain CFD.* **(Refer section 6.12)** **[4]**

c) *The free air delivered by a centrifugal compressor is 20 kg/min. The suction condition is 1 bar and 20°C. The velocity of air at the inlet is 60 m/sec. The isentropic efficiency of the compressor is 75 %. If the total head pressure ratio of the compressor is 3, find (a) The total head temperature of air at the exit of the compressor, and (b) Indicated power required to run the compressor assuming mechanical efficiency of 90 %.*
Pressure and temperature of air at inlet are static. For air, take $C_p = 1.005\ kJ/kg\ K$, $\gamma = 1.4$. **(Refer example 8.3)** **[7]**

❑❑❑

Notes

Appendix

Refrigerant Properties Tables and Charts

[in SI Units]

Table 1 Ammonia NH_3 (Refrigerant 717) Properties of liquid and saturated vapour.

Temp. °C	Pressure Bar	Enthalpy kJ/kg		Entropy kJ/kg K		Specific volume m^3/kg	
		h_f	h_g	S_f	S_g	$v_f \times 10^{-3}$	v_g
– 60	0.2199	– 69.5330	1373.19	– 0.10909	6.6592	1.4010	4.68508
– 55	0.3029	– 47.5062	1382.01	– 0.00717	6.5454	1.4126	3.47422
– 50	0.4103	– 25.4342	1390.64	0.09264	6.4382	1.4245	2.61651
– 45	0.5474	– 3.3020	1399.07	0.19049	6.3369	1.4367	1.99891
– 40	0.7201	18.9024	1407.26	0.28651	6.2410	1.4493	1.54736
– 35	0.9349	41.1883	1415.20	0.38082	6.1501	1.4623	1.21249
– 30	1.1990	63.5629	1422.86	0.47351	6.0636	1.4757	0.960867
– 28	1.3202	72.5387	1425.84	0.51015	6.0302	1.4811	0.878100
– 26	1.4511	81.5300	1428.76	0.54655	5.9974	1.4867	0.803761
– 25	1.5216	86.0335	1430.20	0.56463	5.9813	1.4895	0.770315
– 24	1.5922	90.5370	1431.64	0.58272	5.9652	1.4923	0.736868
– 22	1.7441	99.5600	1434.46	0.61865	5.9336	1.4980	0.676570
– 20	1.9074	108.599	1437.23	0.65436	5.9025	1.5037	0.622122
– 18	2.0826	117.656	1439.94	0.68984	5.8720	1.5096	0.572875
– 16	2.2704	126.729	1442.60	0.72511	5.8420	1.5185	0.528257
– 15	2.3709	131.275	1443.90	0.74264	5.8223	1.5155	0.508013
– 14	2.4714	135.820	1445.20	0.76016	5.8125	1.5215	0.487769
– 12	2.6863	144.929	1447.74	0.79501	5.7835	1.5276	0.450971
– 10	2.9157	154.056	1450.22	0.82965	5.7550	1.5338	0.417477
– 9	3.0360	158.628	1451.44	0.84690	5.7409	1.5369	0.401860
– 8	3.1602	163.204	1452.64	0.86410	5.7269	1.5400	0.386944
– 7	3.2884	167.785	1453.83	0.88125	5.7131	1.5432	0.372692
– 6	3.4207	172.371	1455.00	0.89835	5.6993	1.5464	0.359071
– 5	3.5571	176.962	1456.15	0.91541	5.6856	1.5496	0.346046
– 4	3.6977	181.559	1457.29	0.93242	5.6721	1.5528	0.333589
– 3	3.8426	186.161	1458.42	0.94938	5.6586	1.5561	0.321670
– 2	3.9920	190.768	1459.53	0.96630	5.6453	1.5594	0.310263
– 1	4.1458	195.381	1460.62	0.98317	5.6320	1.5627	0.299340
0	4.3043	200.000	1461.70	1.00000	5.6189	1.5660	0.288880
1	4.4674	204.625	1462.76	1.01679	5.6058	1.5694	0.278858
2	4.6353	209.255	1463.80	1.03354	5.5929	1.5727	0.269253
3	4.8081	213.892	1464.83	1.05024	5.5800	1.5762	0.260046
4	4.9859	218.535	1465.84	1.06691	5.5672	1.5796	0.251216
5	5.1687	223.185	1466.84	1.08353	5.5545	1.5831	0.242745
6	5.3567	227.841	1467.82	1.10012	5.5419	1.5866	0.234618
7	5.5500	232.503	1468.78	1.11667	5.5294	1.5901	0.226817
8	5.7487	237.172	1469.72	1.13317	5.5170	1.5936	0.219326
9	5.9528	241.848	1470.65	1.14964	5.5046	1.5972	0.212132

10	6.1625	246.531	1471.57	1.16607	5.4924	1.6008	0.205221
11	6.3778	251.221	1472.46	1.18246	5.4802	1.6045	0.198580
12	6.5989	255.918	1473.34	1.19882	5.4681	1.6081	0.192196
13	6.8259	260.622	1474.20	1.21515	5.4561	1.6118	0.186058
14	7.0588	265.334	1475.05	1.23144	5.4441	1.6156	0.180154
15	7.2979	270.053	1475.88	1.24769	5.4322	1.6193	0.174475
16	7.5431	274.779	1476.69	1.26391	5.4204	1.6231	0.169009
17	7.7946	279.513	1477.48	1.28010	5.4087	1.6269	0.163748
18	8.0525	284.255	1478.25	1.29626	5.3971	1.6308	0.158683
19	8.3169	289.005	1479.01	1.31238	5.3855	1.6347	0.153804
20	8.5879	293.782	1479.75	1.32847	5.3740	1.6386	0.149106
21	8.8657	298.527	1480.48	1.34452	5.3626	1.6426	0.144578
22	9.1503	303.300	1481.18	1.36055	5.3512	1.6466	0.140214
23	9.4418	308.081	1481.87	1.37654	5.3399	1.6507	0.136006
24	9.7403	312.870	1482.53	1.39250	5.3286	1.6547	0.131950
25	10.046	317.687	1483.18	1.40843	5.3175	1.6588	0.128037
26	10.359	322.471	1483.81	1.42433	5.3063	1.6630	0.124261
27	10.680	327.284	1484.42	1.44020	5.2953	1.6672	0.120619
28	11.007	332.104	1485.01	1.45604	5.2843	1.6714	0.117103
29	11.343	336.933	1485.59	1.47185	5.2733	1.6757	0.113708
30	11.686	341.769	1486.14	1.48762	5.2624	1.6800	0.110430
31	12.037	346.614	1486.67	1.50337	5.2516	1.6844	0.107263
32	12.396	351.466	1487.18	1.51908	5.2408	1.6888	0.104205
33	12.763	356.326	1487.66	1.53477	5.2300	1.6932	0.101248
34	13.139	361.195	1488.13	1.55042	5.2193	1.6977	0.098391
35	13.522	366.072	1488.57	1.56605	5.2086	1.7023	0.095629
36	13.915	370.957	1488.99	1.58165	5.1980	1.7069	0.092957
37	14.315	375.851	1489.39	1.59722	5.1874	1.7115	0.090374
38	14.724	380.754	1489.76	1.61276	5.1768	1.7162	0.087874
39	15.143	385.666	1490.10	1.62828	5.1663	1.7209	0.085456
40	15.570	390.587	1490.42	1.64377	5.1558	1.7257	0.0831150
41	16.006	395.519	1790.71	1.65924	5.1453	1.7305	0.0808484
42	16.451	400.462	1490.98	1.67470	5.1349	1.7354	0.0786536
43	16.906	405.416	1491.21	1.69013	5.1244	1.7404	0.0765276
44	17.370	410.382	1491.41	1.70554	5.1140	1.7454	0.0744678
45	17.843	415.362	1491.58	1.72095	5.1036	1.7504	0.0724716
46	18.326	420.358	1491.72	1.73635	5.0932	1.7555	0.0705365
47	18.819	425.369	1491.82	1.75174	5.0827	1.7607	0.0686602
48	19.322	430.399	1491.88	1.76714	5.0723	1.7659	0.0668403
49	19.835	435.450	1491.91	1.78255	5.0618	1.7712	0.0650746
50	20.359	440.523	1491.89	1.79798	5.0514	1.7766	0.0633608
51	20.892	445.623	1491.83	1.81343	5.0409	1.7820	0.0616971
52	21.436	450.751	1491.73	1.82891	5.0303	1.7875	0.0600813
53	21.991	455.913	1491.58	1.84445	5.0198	1.7931	0.0585114
54	22.556	461.112	1491.38	1.86004	5.0092	1.7987	0.0569855
55	23.132	466.353	1491.12	1.87571	4.9985	1.8044	0.0555019

Table 2 Ammonia NH_3 (Refrigerant 717) Specific heat, C_p, of superheated vapour

Sat. Temp. °C	Specific heat, C_p kJ/kg K					
	Degree of superheat					
	= 10 °C	= 50 °C	= 60 °C	= 80 °C	= 100 °C	= 120 °C
– 60	2.091	2.082	2.083	2.087	2.094	2.103
– 55	2.117	2.102	2.102	2.105	2.151	2.117
– 50	2.147	2.124	2.123	2.124	2.128	2.135
– 45	2.177	2.148	2.146	2.145	2.148	2.153
– 40	2.213	2.175	2.171	2.168	2.169	2.173
– 35	2.251	2.204	2.198	2.193	2.192	2.195
– 30	2.292	2.235	2.228	2.220	2.217	2.218
– 25	2.337	2.269	2.261	2.249	2.244	2.243
– 20	2.385	2.306	2.295	2.280	2.272	2.270
– 15	2.437	2.345	2.332	2.314	2.303	2.298
– 10	2.492	2.387	2.372	2.350	2.336	2.329
– 5	2.551	2.432	2.414	2.388	2.372	2.362
0	2.614	2.480	2.459	2.429	2.409	2.396
5	2.681	2.530	2.507	2.472	2.449	2.402
10	2.752	2.584	2.558	2.518	2.491	2.472
15	2.828	2.641	2.612	2.567	2.535	2.513
20	2.908	2.701	2.668	2.618	2.582	2.556
25	2.994	2.765	2.729	2.672	2.631	2.602
30	3.084	2.832	2.792	2.729	2.684	2.655
35	3.182	2.903	2.859	2.790	2.739	2.701
40	3.286	2.979	2.931	2.854	2.797	2.755
45	3.398	3.060	3.006	2.921	2.859	2.812
50	3.518	3.145	3.087	2.993	2.924	2.872
55	3.649	3.237	3.172	3.070	2.993	2.935
60	3.791	3.335	3.264	3.151	3.066	3.002
65	3.948	3.441	3.363	3.238	3.145	3.074
70	4.122	3.555	3.469	3.332	3.229	3.151
75	4.317	3.680	3.585	3.433	3.320	3.235
80	4.537	3.817	3.711	3.556	3.418	3.322
85	4.789	3.969	3.850	3.664	3.525	3.419
90	5.083	4.138	4.005	3.798	3.642	3.524
95	5.433	4.330	4.179	3.945	3.773	3.641
100	5.858	4.550	4.378	4.118	3.919	3.772
105	6.392	4.809	4.610	4.307	4.087	3.921
110	7.085	5.119	4.886	4.536	4.283	4.093
115	8.036	5.505	5.227	4.814	4.519	4.300
120	9.439	6.011	5.669	5.169	4.819	4.560
125	11.789	6.745	6.303	5.671	4.736	4.917

Table 3 Dichlorodifluoromethane CCl_2F_2 (Refrigerant 12) Properties of liquid and saturated vapour.

Temp. °C	Pressure Bar	Enthalpy kJ/kg		Entropy kJ/kg K		Specific volume m^3/kg	
		h_f	h_g	s_f	s_g	$v_f \times 10^{-3}$	v_g
– 60	0.2262	146.463	324.236	0.77977	1.61373	0.63689	0.63791
– 55	0.2998	150.808	326.567	0.79990	1.60552	0.64226	0.49100
– 50	0.3915	155.169	328.897	0.81964	1.59810	0.64782	0.38310
– 45	0.5044	159.549	331.223	0.83901	1.59142	0.65335	0.30268
– 40	0.6417	163.948	333.541	0.85805	1.58539	0.65949	0.24191
– 35	0.8071	168.369	335.849	0.86776	1.57996	0.66563	0.19539
– 30	1.0041	172.810	338.143	0.89516	1.57507	0.67200	0.15937
– 28	1.0927	174.593	339.057	0.90244	1.57326	0.67461	0.14727
– 26	1.1872	176.380	339.968	0.90967	1.57152	0.67726	0.13628
– 25	1.2368	177.275	340.422	0.91327	1.57068	0.67860	0.13116
– 24	1.2880	178.171	340.876	0.91686	1.56985	0.67996	0.12628
– 22	1.3953	179.965	341.780	0.92400	1.56825	0.68269	0.11716
– 20	1.5093	181.764	342.682	0.93110	1.56672	0.68547	0.10884
– 18	1.6304	183.567	343.580	0.93816	1.56526	0.68829	0.10124
– 16	1.7589	185.374	344.474	0.94518	1.56385	0.69115	0.094278
– 15	1.8260	186.279	344.920	0.94868	1.56317	0.69261	0.091016
– 14	1.8950	187.185	345.365	0.95216	1.56250	0.69407	0.087895
– 12	2.0390	189.001	346.252	0.95910	1.56121	0.69703	0.082034
– 10	2.1912	190.822	347.134	0.96601	1.55997	0.70004	0.076646
– 9	2.2704	191.734	347.574	0.96945	1.55938	0.70157	0.074115
– 8	2.3519	192.647	348.012	0.97287	1.55897	0.70310	0.071686
– 7	2.4355	193.562	348.450	0.97629	1.55822	0.70465	0.069354
– 6	2.5214	194.477	348.886	0.97971	1.55765	0.70622	0.067114
– 5	2.6096	195.395	349.321	0.98311	1.55710	0.70780	0.064962
– 4	2.7001	196.313	349.755	0.98650	1.55657	0.70939	0.062895
– 3	2.7930	197.233	350.187	0.98989	1.55604	0.71099	0.060907
– 2	2.8882	198.154	350.619	0.99327	1.55552	0.71261	0.058996
– 1	2.9859	199.076	351.049	0.99664	1.55502	0.71425	0.057157
0	3.0861	200.000	351.477	1.00000	1.55452	0.71590	0.55389
1	3.1888	200.925	351.905	1.00335	1.55404	0.71756	0.053686
2	3.2940	201.852	352.331	1.00670	1.55356	0.71924	0.052048
3	3.4019	202.780	352.755	1.01004	1.55310	0.72094	0.050470
4	3.5124	203.710	353.179	1.01337	1.55264	0.72265	0.0489499
5	3.6255	204.642	353.600	1.01670	1.55220	0.72438	0.0474853
6	3.7414	205.575	354.020	1.02001	1.55176	0.72612	0.0460737
7	3.8601	206.509	354.439	1.02333	1.55133	0.72788	0.0447129
8	3.9815	207.445	354.856	1.02663	1.55091	0.72966	0.0434006
9	4.1058	208.383	355.272	1.02993	1.55050	0.73146	0.0421349
10	4.2330	209.323	355.686	1.03322	1.55010	0.73326	0.0409137

11	4.3631	210.264	356.398	1.03650	1.54970	0.73510	0.0397352
12	4.4962	221.207	356.509	1.03978	1.54931	0.73695	0.0385975
13	4.6323	212.152	356.918	1.04305	1.54893	0.73882	0.0374991
14	4.7714	213.099	357.325	1.04632	1.54856	0.74071	0.0364382
15	4.9137	214.048	357.730	1.04958	1.54819	0.74262	0.0354133
16	5.0591	214.998	358.134	1.05284	1.54783	0.74455	0.0344230
17	5.2076	215.951	358.535	1.05609	1.54748	0.74649	0.0334658
18	5.3594	216.906	358.935	1.05933	1.54713	0.74846	0.0325405
19	5.5145	217.863	359.333	1.06258	1.54679	0.75045	0.0316457
20	5.6729	218.821	359.729	1.06581	1.54645	0.75246	0.0307802
21	5.8347	219.783	360.122	1.06904	1.54612	0.75449	0.0299429
22	5.9998	220.746	360.514	1.07227	1.54579	0.75655	0.0291327
23	6.1684	221.712	360.904	1.07549	1.54547	0.75863	0.0283485
24	6.3405	222.680	361.291	1.07871	1.54515	0.76073	0.0275894
25	6.5162	223.650	361.676	1.08193	1.54484	0.76286	0.0268542
26	6.6954	224.623	362.059	1.08514	1.54453	0.76501	0.0261422
27	6.8782	225.598	362.439	1.08835	1.54423	0.76718	0.0254524
28	7.0647	226.576	362.817	1.09155	1.54393	0.76938	0.0247840
29	7.2550	227.557	363.193	1.09475	1.54363	0.77161	0.0241362
30	7.4490	228.540	363.566	1.09795	1.54334	0.77386	0.0235082
31	7.6468	229.520	363.937	1.10115	1.54305	0.77614	0.0228993
32	7.8485	230.515	364.305	1.10434	1.54276	0.77845	0.0223088
33	8.0541	231.506	364.670	1.10753	1.54247	0.78079	0.0217359
34	8.2636	232.501	365.033	1.11072	1.54219	0.78316	0.0211802
35	8.4772	233.498	365.392	1.11391	1.54191	0.78556	0.0206408
36	8.6948	234.499	365.749	1.11710	1.54163	0.78799	0.020117
37	8.9164	235.503	366.103	1.12028	1.54135	0.79045	0.019609
38	9.1423	236.510	366.454	1.12347	1.54107	0.79294	0.019115
39	9.3723	237.521	366.802	1.12665	1.54079	0.79546	0.018636
40	9.6065	238.535	367.146	1.12984	1.54051	0.79802	0.018170
41	9.8451	239.552	367.487	1.13302	1.54024	0.80062	0.017718
42	10.088	240.574	367.825	1.13620	1.53996	0.80325	0.017278
43	10.335	241.598	368.160	1.13938	1.53968	0.80592	0.016851
44	10.587	242.627	368.491	1.14257	1.53941	0.80863	0.016435
45	10.843	243.659	368.818	1.14575	1.53913	0.81137	0.016031
46	11.104	244.696	369.141	1.14894	1.53885	0.81416	0.015638
47	11.369	245.736	369.461	1.15213	1.53856	0.81698	0.015256
48	11.639	246.781	369.777	1.15532	1.53828	0.81985	0.014884
49	11.914	247.830	370.088	1.15851	1.53799	0.82277	0.014522
50	12.193	248.884	370.396	1.16170	1.53770	0.82573	0.014170
52	12.766	251.004	370.997	1.16810	1.53712	0.83179	0.013493
54	13.359	253.144	371.581	1.17451	1.53651	0.83804	0.012850
56	13.972	255.304	372.145	1.18093	1.53589	0.84451	0.012241
58	14.605	257.486	372.683	1.18738	1.53524	0.85121	0.011662
60	15.259	259.690	373.213	1.19384	1.53457	0.85814	0.011111
62	15.935	261.918	373.707	1.20034	1.53387	0.86534	0.010587

64	16.632	264.172	374.180	1.20686	1.53313	0.87282	0.010088
66	17.351	266.452	374.625	1.21342	1.53235	0.88059	0.009612
68	18.093	268.762	375.042	1.22001	1.53153	0.88870	0.009158
70	18.858	271.102	375.427	1.22665	1.53066	0.89716	0.008725
75	20.875	277.100	376.234	1.24347	1.52821	0.92009	0.000722
80	23.046	283.341	376.777	1.26069	1.52526	0.94612	0.006821
85	25.380	289.879	376.985	1.27845	1.52164	0.97621	0.006004
90	27.885	296.788	376.748	1.29691	1.51708	1.01190	0.005257
95	30.569	304.181	375.887	1.31637	1.51113	1.05581	0.004563
100	33.441	312.261	374.070	1.33732	1.50296	1.11311	0.003902

Table 4 Dichlorodifluoromethane CCl_2F_2 (Refrigerant 12) Properties of superheated vapour

- 20 °C (1.5093 bar) Temp.				-15 °C (1.8260 bar)			
Temp. °C	v, m^3/kg	h, kJ/kg	s,kJ/kg K	Temp. °C	v, m^3/kg	h, kJ/kg	s,kJ/kg
– 15	0.1 1477	345.674	1.5784	– 10	0.0932332	347.971	1.5749
– 10	0.1 4078	348.678	1.5900	– 5	0.0954209	351.031	1.5864
– 5	0.1 6652	351.696	1.6013	0	0.0975841	354.101	1.5977
0	0.1 9204	354.727	1.6125	5	0.0997255	357.182	1.6089
5	0.121735	357.774	1.6236	10	0.101847	360.277	1.6199
10	0.124247	360.836	1.6345	15	0.103951	363.384	1.6308
15	0.126741	363.913	1.6452	20	0.106039	366.506	1.6416
20	0.129221	367.008	1.6559	25	0.108113	369.643	1.6522
25	0.131686	370.119	1.6664	30	0.110173	372.794	1.6626
30	0.134139	373.247	1.6768	35	0.112222	375.961	1.6730
Temp. °C	- 14 °C (1.8950 bar)			Temp. °C	- 12 °C (2.0390 bar)		
– 10	0.0896116	347.814	1.5719	– 10	0.0828408	347.486	1.5659
– 5	0.0917325	350.884	1.5834	– 5	0.0848373	350.575	1.5775
0	0.0938286	353.962	1.5948	0	0.0868085	353.672	1.5890
5	0.0959027	357.052	1.6060	5	0.0887572	356.779	1.6003
10	0.0979570	360.154	1.6171	10	0.0906856	359.895	1.6114
15	0.0999934	363.268	1.6280	15	0.0925959	363.024	1.6223
20	0.102014	366.396	1.6387	20	0.0944898	366.165	1.6331
25	0.104019	369.538	1.6494	25	0.0963689	369.319	1.6438
30	0.106012	372.695	1.6599	30	0.0982345	372.486	1.6543
35	0.107992	375.867	1.6702	35	0.100088	375.669	1.6647
Temp. °C	- 10 °C (2.1912 bar)			Temp. °C	- 9 °C (2.2704 bar)		
– 5	0.0785299	350.245	1.5717	– 5	0.0755780	350.072	1.5688
0	0.0803875	353.363	1.5832	0	0.0773827	353.200	1.5803
5	0.0822220	356.487	1.5945	5	0.0791639	356.334	1.5917
10	0.0840358	359.620	1.6057	10	0.0809243	359.476	1.6029
15	0.08583111	362.764	1.6167	15	0.0826659	362.628	1.6139

20	0.0876097	365.919	1.6276	20	0.0843907	365.790	1.6248
25	0.0893732	369.085	1.6383	25	0.0861008	368.963	1.6355
30	0.0911231	372.265	1.6488	30	0.0877961	372.149	1.6461
35	0.0928607	375.458	1.6593	35	0.0894795	375.348	1.6566
40	0.0945870	378.665	1.6696	40	0.0911516	378.560	1.6669
- 8 °C (2.3519 bar)				- 7 °C (2.4355 bar)			
Temp. °C	v, m^3/kg	h,kJ/kg	s, kJ/kg K	Temp. °C	v, m^3/kg	h, kJ/kg	s,kJ/kg K
– 5	0.0727515	349.894	1.5658	– 5	0.0700441	349.709	1.5629
0	0.0745057	353.032	1.5774	0	0.0717501	352.859	1.5746
5	0.0762361	356.176	1.5888	5	0.0734320	356.013	1.5860
10	0.0779454	359.328	1.6001	10	0.0750926	359.174	1.5973
15	0.0796358	362.487	1.6111	15	0.0767339	362.343	1.6084
20	0.0813091	365.657	1.6220	20	0.0783581	365.520	1.6193
25	0.0829672	368.838	1.6328	25	0.0799668	368.708	1.6301
30	0.0846113	372.030	1.6434	30	0.0815616	371.907	1.6407
35	0.0862429	375.234	1.6539	35	0.0831437	375.117	1.6512
40	0.878632	378.452	1.6643	40	0.0847143	378.341	1.6616
Temp. °C	- 6 °C (2.5214 bar)			Temp. °C	- 5 °C (2.6096 bar)		
-5	0.0674498	349.518	1.5600	0	0.0665791	352.495	1.5688
0	0.0691098	352.680	1.5717	5	0.0681705	355.672	1.5804
5	0.0707454	355.845	1.5832	10	0.0697399	358.852	1.5917
10	0.0723594	359.016	1.5945	15	0.0712898	362.039	1.6028
15	0.0739540	362.193	1.6056	20	0.0728220	365.233	1.6138
20	0.0755312	365.379	1.6166	25	0.0743385	368.436	1.6247
25	0.0770927	368.574	1.6274	30	0.0758407	371.649	1.6354
30	0.0786402	371.780	1.6380	35	0.0773301	374.873	1.6459
35	0.0801749	374.997	1.6486	40	0.0788078	378.108	1.6563
40	0.0816980	378.226	1.6589	45	0.0802749	381.355	1.6666
Temp. °C	- 4 °C (2.7001 bar)			Temp. °C	- 3 °C (2.7930 bar)		
0	0.0641525	352.305	1.5660	0	0.0618250	352.108	1.5631
5	0.0657017	355.493	1.5775	5	0.0633338	355.308	1.5747
10	0.0672286	358.684	1.5889	10	0.0648201	358.510	1.5861
15	0.0687357	361.880	1.6001	15	0.0662863	361.717	1.5974
20	0.0702550	365.083	1.6111	20	0.0677345	364.929	1.6084
25	0.0716983	368.294	1.6220	25	0.0691665	368.148	1.6193
30	0.0731573	371.515	1.6327	30	0.0705841	371.376	1.6300
35	0.0746032	374.745	1.6433	35	0.0719885	374.613	1.6406
40	0.0760374	377.986	1.6537	40	0.0733810	377.861	1.6511
45	0.0774610	381.240	1.6640	45	0.0747628	381.121	1.6614

- 2 °C (2.8882 bar)				- 1 °C (2.9859 bar)			
Temp. °C	v, m^3/kg	h kJ/kg	s,kJ/kg K	Temp. °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
0	0.0595916	351.904	1.5602	0	0.0574478	351.694	1.5574
5	0.0610620	355.117	1.5719	5	0.0588814	354.921	1.5691
10	0.0625095	358.331	1.5834	10	0.0602918	358.116	1.5806
15	0.0639365	361.548	1.5946	15	0.0616815	361.374	1.5919
20	0.0653454	364.769	1.6057	20	0.0630528	364.605	1.6030
25	0.0667379	367.997	1.6166	25	0.0644075	367.842	1.6139
30	0.0681158	371.233	1.6274	30	0.0657474	371.086	1.6247
35	0.694804	374.478	1.6380	35	0.0670739	374.339	1.6354
40	0.0708331	377.733	1.6485	40	0.0683884	377.622	1.6459
45	0.0721749	380.998	1.6588	45	0.0696919	380.872	1.6562
Temp. °C	0 °C (3.0861 bar)			Temp. °C	2 °C (3.2940 bar)		
5	0.0567878	354.717	1.5663	5	0.0528450	354.292	1.5607
10	0.0581628	357.956	1.5778	10	0.0541538	357.556	1.5723
15	0.0595168	358.194	1.5892	15	0.0554409	360.819	1.5837
20	0.0608520	364.436	1.6003	20	0.0567088	364.082	1.5949
25	0.0621706	367.682	1.6113	25	0.0579595	367.348	1.6060
30	0.0634741	370.935	1.6221	30	0.0591949	370.619	1.6169
35	0.0647641	374.195	1.6328	35	0.0604164	373.895	1.6276
40	0.0660420	377.464	1.6433	40	0.0616255	377.179	1.6381
45	0.0673088	380.742	1.6537	45	0.0628234	380.471	1.6486
50	0.0685655	384.031	1.6639	50	0.0640110	383.773	1.6589
Temp. °C	4 °C (3.5124 bar)			Temp. °C	5 °C (3.6255 bar)		
5	0.0492026	353.838	1.5550	10	0.0487056	356.909	1.5640
10	0.0504510	357.131	1.5668	15	0.0499032	360.211	1.5755
15	0.0516771	360.420	1.5783	20	0.0510805	363.510	1.5869
20	0.0528833	363.706	1.5896	25	0.522400	366.809	1.5981
25	0.0540719	366.994	1.6007	30	0.0533835	370.109	1.6090
30	0.0552447	370.284	1.6116	35	0.0545127	373.412	1.6198
35	0.0564033	373.578	1.6224	40	0.0556290	376.721	1.6305
40	0.0575493	376.878	1.6330	45	0.0567337	380.036	1.6410
45	0.0586838	380.185	1.6435	50	0.0578279	383.358	1.6514
50	0.0598079	383.500	1.6538	55	0.0589126	386.689	1.6616

Temp. °C	28 °C (7.0647 bar)			Temp. °C	29 °C (7.2550 bar)		
30	0.0250741	364.291	1.5488	30	0.0242789	363.934	1.5461
35	0.0257838	367.947	1.5609	35	0.0249790	367.616	1.5581
40	0.0264739	371.573	1.5724	40	0.0256589	371.263	1.5699
45	0.0271470	375.174	1.5838	45	0.0263214	374.884	1.5813
50	0.0278052	378.756	1.5950	50	0.0269686	378.483	1.5926
55	0.0284503	382.323	1.6060	55	0.0276024	382.066	1.6036
60	0.0290837	385.880	1.6167	60	0.0282243	385.636	1.6144
65	0.0297067	389.428	1.6273	65	0.0288356	389.197	1.6250
70	0.0303204	392.971	1.6377	70	0.0294374	392.751	1.6354
75	0.0309257	396.510	1.6479	75	0.0300308	396.301	1.6457
Temp. °C	30 °C (7.4490 bar)			Temp. °C	31 °C (7.6468 bar)		
35	0.0241993	367.274	1.5555	35	0.0234435	366.291	1.5528
40	0.0248695	370.944	1.5673	40	0.0241047	370.616	1.5647
45	0.0255219	374.585	1.5788	45	0.0247474	374.278	1.5763
50	0.0261586	378.202	1.5901	50	0.0253741	377.914	1.5876
55	0.0267816	381.801	1.6011	55	0.0259868	381.529	1.5987
60	0.0273924	385.386	1.6120	60	0.0265871	385.129	1.6096
65	0.0279925	388.960	1.6226	65	0.0271764	368.717	1.6203
70	0.0285830	392.526	1.6331	70	0.0277500	392.295	1.6308
75	0.0291649	396.087	1.6434	75	0.0283268	395.867	1.6411
80	0.0297390	399.645	1.6536	80	0.0288897	399.436	1.6513
Temp. °C	32 °C (7.8485 bar)			Temp. °C	33 °C (8.0541 bar)		
35	0.0227108	367.557	1.5501	35	0.0220000	366.182	1.5474
40	0.0233634	370.277	1.5621	40	0.0226446	369.928	1.5595
45	0.0239970	373.961	1.5737	45	0.0232697	373.635	1.5712
50	0.0246142	377.617	1.5851	50	0.0238778	377.311	1.5822
55	0.0252170	381.250	1.5963	55	0.0244712	380.962	1.593
60	0.0258072	384.865	1.6072	60	0.0250517	384.594	1.604
65	0.0263862	388.467	1.6180	65	0.0256208	388.210	1.615
70	0.0269552	392.058	1.6285	70	0.0261798	391.815	1.626
75	0.0275154	395.642	1.6389	75	0.0267297	395.411	1.636
80	0.0280675	399.221	1.6491	80	0.0272714	399.001	1.646

Temp. °C	34 °C (8.2636 bar)			Temp. °C	35 °C (8.4772 bar)		
	v, m^3/kg	h, kJ/kg	s, kJ/kg K		v, m^3/kg	h, kJ/kg	s, kJ/kg K
35	0.0213103	365.793	1.5447	40	0.0212711	369.196	1.5542
40	0.0219475	369.568	1.5568	45	0.0218804	372.953	1.5661
45	0.0225644	373.299	1.5686	50	0.0224719	376.673	1.5777
50	0.0231640	376.997	1.5802	55	0.0230478	380.363	1.5890
55	0.0237485	380.667	1.5914	60	0.0236102	384.030	1.6001
60	0.0243197	984.316	1.6025	65	0.0241607	387.677	1.6109
65	0.0248793	387.947	1.6133	70	0.0247007	391.310	1.6216
70	0.0254286	391.566	1.6239	75	0.0252313	394.931	1.6321
75	0.0259686	395.174	1.6343	80	0.0257534	398.545	1.6424
80	0.0265004	398.776	1.6446	85	0.0262681	402.152	1.6525
Temp. °C	36 °C (8.6948 bar)			Temp. °C	37 °C (8.9164 bar)		
40	0.0206145	368.812	1.5515	40	0.0199770	368.416	1.5488
45	0.0212168	372.597	1.5635	45	0.0205727	372.229	1.5608
50	0.0218006	376.340	1.5751	50	0.0211492	375.997	1.5726
55	0.0223684	380.051	1.5865	55	0.0217094	379.729	1.5841
60	0.0229224	383.736	1.5977	60	0.0222554	383.434	1.5953
65	0.0234642	387.400	1.6086	65	0.0227889	387.115	1.6062
70	0.0239952	391.047	1.6193	70	0.0233113	390.778	1.6170
75	0.0245167	394.682	1.6298	75	0.0238241	394.427	1.6275
80	0.0250296	398.308	1.6402	80	0.0243281	398.065	1.6379
85	0.0255340	401.926	1.6503	85	0.0248244	401.695	1.6481
Temp. °C	38 °C (9.1423 bar)			Temp. °C	39 °C (9.3723 bar)		
40	0.0193577	368.007	1.5460	40	0.0187554	367.584	1.5433
45	0.0199473	371.849	1.5582	45	0.0193398	371.458	1.5556
50	0.0205171	375.644	1.5700	50	0.0199033	375.279	1.5675
55	0.0210700	379.399	1.5816	55	0.0204494	379.058	1.5791
60	0.0216083	383.123	1.5928	60	0.0209805	382.804	1.5904
65	0.0221339	386.822	1.6039	65	0.0214985	386.521	1.6015
70	0.0226482	390.501	1.6147	70	0.0220050	390.217	1.6123
75	0.0231525	394.165	1.6253	75	0.0226013	393.896	1.6230
80	0.0236480	397.816	1.6357	80	0.0229885	397.561	1.6334
85	0.0241356	401.458	1.6459	85	0.0234678	401.215	1.6437

40 °C (9.6065 bar)				41 °C (9.8451 bar)			
Temp. °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K	Temp. °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
45	0.0187495	371.054	1.5529	45	0.0181757	370.636	1.5502
50	0.0193072	374.904	1.5649	50	0.0187281	374.517	1.5623
55	0.0198469	378.708	1.5766	55	0.0192618	378.347	1.5741
60	0.0203711	382.475	1.5880	60	0.0197795	382.137	1.5855
65	0.0208819	386.212	1.5991	65	0.0202834	385.895	1.5967
70	0.0213809	389.926	1.6100	70	0.0207753	389.626	1.6077
75	0.0218695	393.620	1.6207	75	0.0212565	393.337	1.6184
80	0.0223489	397.299	1.6312	80	0.0217283	397.031	1.6289
85	0.0228201	400.966	1.6415	85	0.0221918	400.711	1.6393
90	0.0232819	404.625	1.6516	90	0.0226477	404.382	1.6495
Temp. °C	42 °C (10.088 bar)			Temp. °C	43 °C (10.335 bar)		
45	0.0176177	370.205	1.5475	45	0.0170748	369.758	1.5447
50	0.0181652	374.117	1.5597	50	0.0176178	373.705	1.5570
55	0.0186933	377.975	1.5715	55	0.0181408	377.592	1.5690
60	0.0192049	381.790	1.5831	60	0.0186467	381.432	1.5806
65	0.0197023	385.569	1.5943	65	0.0191383	385.233	1.5919
70	0.0201874	389.319	1.6053	70	0.0196165	389.003	1.6030
75	0.0206615	393.046	1.6161	75	0.0200840	392.748	1.6138
80	0.0211261	396.755	1.6267	80	0.0205416	396.473	1.6244
85	0.0215821	400.450	1.6371	85	0.0209905	400.182	1.6349
90	0.0220305	404.133	1.6473	90	0.0214316	403.878	1.6451
Temp. °C	44 °C (10.587 bar)			Temp. °C	45 °C (10.843 bar)		
45	0.0165463	369.296	1.5419	50	0.0165673	372.839	1.5517
50	0.0170854	373.279	1.5544	55	0.0170813	376.790	1.5638
55	0.0176037	377.197	1.5664	60	0.0175769	380.684	1.5756
60	0.0181043	381.063	1.5781	65	0.0180569	384.533	1.5870
65	0.0185897	384.888	1.5895	70	0.0185235	388.345	1.5982
70	0.0190621	388.679	1.6006	75	0.0189784	392.128	1.6092
75	0.0195231	392.442	1.6115	80	0.0194229	395.886	1.6199
80	0.0199741	396.183	1.6222	85	0.0198584	399.625	1.6304
85	0.0204161	399.907	1.6326	90	0.0202858	403.349	1.6407
90	0.0208502	403.616	1.6429	95	0.0207060	407.060	1.6509

Table 5 Refrigerant 12 Specific heat, c_p of superheated vapour

Sat. Temp. t °C	Specific heat c_p kJ/kg K		
	Degree of superheat		
	= 0 °C (Sat. Vap.)	= 10 °C	= 30 °C
100	0.448	0.456	0.470
95	0.456	0.464	0.478
90	0.464	0.471	0.485
85	0.473	0.479	0.493
80	0.481	0.488	0.500
75	0.489	0.496	0.508
70	0.497	0.504	0.516
65	0.506	0.513	0.524
60	0.515	0.521	0.532
55	0.524	0.530	0.541
50	0.534	0.539	0.549
45	0.543	0.548	0.558
40	0.554	0.558	0.567
35	0.564	0.568	0.576
30	0.575	0.578	0.585
25	0.586	0.589	0.595
20	0.597	0.600	0.605
15	0.609	0.611	0.615
10	0.622	0.623	0.626
5	0.635	0.635	0.637
0	0.648	0.648	0.648

Sat. Temp. t °C	Specific heat c_p kJ/kg K		
	Degree of superheat		
	= 0 °C (Sat. Vap.)	= 10 °C	= 30 °C
5	0.662	0.661	0.660
10	0.677	0.675	0.673
15	0.693	0.689	0.685
20	0.709	0.704	0.699
25	0.727	0.721	0.713
30	0.746	0.737	0.727
35	0.766	0.756	0.743
40	0.788	0.776	0.759
45	0.813	0.798	0.777
50	0.839	0.820	0.796
55	0.871	0.847	0.817
60	0.906	0.876	0.840
65	0.947	0.910	0.866
70	0.997	0.950	0.895
75	1.065	0.997	0.928
80	1.145	1.056	0.968
85	1.255	1.131	1.016
90	1.420	1.231	1.075
95	1.703	1.369	1.151
100	2.265	1.579	1.256
105	3.817	1.933	1.416

Table 6 Monochlorodifluoromethane, $CHClF_2$ (Refrigerant 22) Properties of liquid and saturated vapour

Temp. t °C	Pressure p, Bar	Enthalpy kJ/kg		Entropy kJ/kg K		Specific volume m^3/kg	
		h_f	h_g	s_f	S_g	$v_f \times 10^3$	v_g
– 60	0.3748	134.763	379.114	0.73254	1.87886	0.68208	0.537152
– 55	0.4947	139.830	381.529	0.75599	1.86389	0.68856	0.414827
– 50	0.6439	144.959	383.921	0.77919	1.85000	0.69526	0.324557
– 45	0.8271	150.153	386.282	0.80216	1.83708	0.70219	0.256990
– 40	1.0495	155.414	388.609	0.82490	1.82504	0.70936	0.205745
– 35	1.3168	160.742	390.896	0.84743	1.81380	0.71680	0.166400
– 30	1.6348	166.140	393.138	0.86976	1.80329	0.72452	0.135844

– 28	1.7776	168.318	394.021	0.87864	1.79927	0.72769	0.125563
– 26	1.9299	170.507	394.896	0.88748	1.79535	0.73092	0.116214
– 25	2.0098	171.606	395.330	0.89190	1.79342	0.73256	0.111859
– 24	2.0922	172.708	395.762	0.89636	1.79152	0.73420	0.107701
– 22	2.2648	174.919	396.619	0.90509	1.78779	0.73753	0.099936
– 20	2.4483	177.142	397.467	0.91386	1.78415	0.74091	0.092843
– 18	2.6429	179.376	398.305	0.92259	1.78059	0.74436	0.86354
– 16	2.8493	181.622	399.133	0.93129	1.77711	0.74786	0.080410
– 15	2.9570	182.749	399.544	0.93564	1.77540	0.74965	0.077625
– 14	3.0678	183.878	399.951	0.93997	1.77371	0.75143	0.074957
– 12	3.2989	186.147	400.759	0.94862	1.77039	0.75506	0.069947
– 10	3.5430	188.426	401.555	0.95725	1.76713	0.75876	0.065339
– 9	3.6701	189.571	401.949	0.96155	1.76553	0.76063	0.063174
– 8	3.8006	190.718	402.341	0.96585	1.76394	0.76253	0.061095
– 7	3.9347	191.868	402.729	0.97014	1.76237	0.76444	0.059099
– 6	4.0723	193.021	403.114	0.97442	1.76082	0.76636	0.057182
– 5	4.2135	194.176	403.496	0.97870	1.75928	0.76831	0.055339
– 4	4.3584	195.335	403.876	0.98297	1.75775	0.77028	0.053568
– 3	4.5070	196.497	404.252	0.98724	1.75624	0.77226	0.05186
– 2	4.6594	197.662	404.626	0.99150	1.75475	0.77427	0.05022
– 1	4.8157	198.828	404.994	0.99575	1.75326	0.77629	0.4865
0	4.9759	200.000	405.361	1.00000	1.75279	0.77834	0.04713
1	5.1401	201.174	405.724	1.00424	1.75034	0.78041	0.04567
2	5.3083	202.351	406.084	1.00848	1.74889	0.78249	0.04427
3	5.4806	203.530	406.440	1.01271	1.74746	0.78460	0.0429166
4	5.6571	204.713	406.793	1.01694	1.74604	0.78673	0.0416124
5	5.8378	205.899	407.143	1.02116	1.74463	0.78889	0.0403556
6	6.0228	207.089	407.489	1.02537	1.74324	0.79107	0.0391441
7	6.2122	208.281	407.831	1.02958	1.74185	0.79327	0.0379759
8	6.4059	209.477	408.169	1.03379	1.74047	0.79549	0.0368493
9	6.6042	210.675	408.504	1.03799	1.73911	0.79775	0.0357624
10	6.8070	211.877	408.835	1.04218	1.73775	0.80002	0.0347136
11	7.0144	213.083	409.162	1.04637	1.73640	0.80232	0.0337013
12	7.2265	214.291	409.485	1.05056	1.73506	0.80465	0.0327239
13	7.4433	215.503	409.804	1.05474	1.73373	0.80701	0.0317801
14	7.6650	216.719	410.119	1.05892	1.73241	0.80939	0.0308683
15	7.8915	217.937	410.430	1.06309	1.73109	0.81180	0.0299874
16	8.1229	219.160	410.736	1.06726	1.72978	0.81424	0.0291361
17	8.3593	220.386	411.038	1.07142	1.72848	0.81671	0.0283131
18	8.6008	221.615	411.336	1.07559	1.72719	0.81922	0.0275173
19	8.8475	222.848	411.629	1.07974	1.72590	0.82175	0.0267477
20	9.0993	224.084	411.918	1.08390	1.72462	0.82431	0.0260032
21	9.3564	225.324	412.202	1.08805	1.72334	0.82691	0.0252829
22	9.6189	226.568	412.481	1.09220	1.72206	0.82954	0.0245357
23	9.8867	227.816	412.755	1.09634	1.72080	0.83221	0.0239107
24	10.160	229.068	413.025	1.10048	1.71953	0.83491	0.0232572

25	10.439	230.324	413.289	1.10462	1.71827	0.83765	0.0226242
26	10.723	231.583	413.548	1.10876	1.71701	0.84043	0.0220111
27	11.014	232.847	413.802	1.11290	1.71576	0.84324	0.0214169
28	11.309	234.115	414.050	1.11703	1.71450	0.84610	0.0208411
29	11.611	235.387	414.293	1.12116	1.71325	0.84899	0.0202829
30	11.919	236.664	414.530	1.12530	1.71200	0.85193	0.0197417
31	12.232	237.944	414.762	1.12943	1.71075	0.85491	0.0192168
32	12.552	239.230	414.987	1.13355	1.70950	0.85793	0.0187076
33	12.878	240.520	415.207	1.13768	1.70826	0.86101	0.0182135
34	13.210	241.814	415.420	1.14181	1.70701	0.86412	0.0177341
35	13.548	243.114	415.627	1.14594	1.70576	0.86729	0.0172686
36	13.892	244.418	415.828	1.15007	1.70450	0.87051	0.0168168
37	14.243	245.727	416.021	1.15420	1.70325	0.87378	0.0163779
38	14.601	247.041	416.208	1.15833	1.70199	0.87710	0.0159517
39	14.965	248.361	416.388	1.16246	1.70073	0.88048	0.0155375
40	15.335	249.686	416.561	1.16659	1.69946	0.88392	0.0151351
41	15.712	251.016	416.726	1.17073	1.69819	0.88741	0.0147439
42	16.096	252.352	416.883	1.17486	1.69692	0.89097	0.0143636
43	16.487	253.696	417.033	1.17900	1.69564	0.89459	0.0139938
44	16.885	255.042	417.174	1.18315	1.69435	0.89828	0.0136431
45	17.290	256.396	417.308	1.18730	1.69305	0.90203	0.0132841
46	17.702	257.756	417.432	1.19145	1.69174	0.90586	0.0129436
47	18.121	259.123	417.548	1.19560	1.69043	0.90976	0.0126122
48	18.548	260.497	417.655	1.19977	1.68911	0.91374	0.0122895
49	18.982	261.877	417.752	1.20393	1.68777	0.91779	0.0119753
50	19.423	263.264	417.838	1.20811	1.68643	0.92193	0.0116693
52	20.328	266.062	417.983	1.21648	1.68370	0.93047	0.0110806
54	21.265	268.89	418.083	1.22489	1.68091	0.93939	0.0105214
56	22.232	271.754	418.137	1.23333	1.67805	0.94872	0.00998952
58	23.232	274.654	418.141	1.24183	1.67511	0.95850	0.00948319
60	24.266	277.594	418.089	1.25038	1.67208	0.96878	0.00900062
62	25.333	280.577	417.978	1.25899	1.66895	0.97960	0.00854016
64	26.435	283.607	417.802	1.26768	1.66570	0.99104	0.00810023
66	27.573	286.690	417.553	1.27647	1.66231	1.00317	0.00767934
68	28.747	289.832	417.226	1.28535	1.65876	1.01608	0.00727605
70	29.959	293.033	416.809	1.29436	1.65504	1.02987	0.00688899
75	33.161	301.399	415.299	1.31758	1.64472	1.06916	0.00598334
80	36.623	310.424	412.898	1.34223	1.63239	1.11810	0.00514862
85	40.368	320.505	409.101	1.36936	1.61673	1.18328	0.00435815
90	44.425	332.615	402.653	1.40155	1.59440	1.28230	0.00356440
95	48.835	351.767	386.708	1.45222	1.54712	1.52064	0.00255133

Table 7 (Refrigerant 22) Properties of superheated vapour

t, °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
Saturation temperature, - 20 °C			
– 20	0.0928432	397.467	1.7841
– 15	0.0951474	400.737	1.7969
– 10	0.0974256	404.017	1.8095
– 5	0.0996808	407.307	1.8219
0	0.101915	410.610	1.8341
5	0.104130	413.926	1.8461
10	0.106328	417.258	1.8580
15	0.108510	420.606	1.8697
20	0.110678	423.970	1.8813
25	0.112832	426.353	1.8928
Saturation temperature, 10 °C			
0	0.0471354	405.361	1.7518
5	0.0483899	408.969	1.7649
10	0.0496215	412.567	1.7777
15	0.0508328	416.159	1.7903
20	0.0520259	419.649	1.8026
25	0.0532028	423.339	1.8148
Saturation temperature, 10 °C			
10	0.0347136	408.835	1.7377
15	0.0356907	412.651	1.7511
20	0.0366454	416.442	1.7642
25	0.0375804	420.215	1.7769
30	0.0384981	423.974	1.7894
35	0.0394002	427.724	1.8017
40	0.0402884	431.469	1.8137
45	0.0411642	435.211	1.8256
50	0.0420286	438.954	1.8373
Saturation temperature, 20 °C			
20	0.0260032	411.918	1.7246
25	0.0267900	415.977	1.7383
30	0.0275542	419.91	1.7517
35	0.0282989	423.970	1.7646
40	0.0290264	427.922	1.7774
45	0.0297389	431.852	1.7899
50	0.0304379	435.766	1.8021
55	0.0311250	439.668	1.8141
60	0.0318012	443.561	1.8258
65	0.0324678	447.450	1.8374

t, °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
Saturation temperature, - 10 °C			
– 10	0.0653399	401.555	1.7671
– 5	0.0670081	404.983	1.7800
0	0.0686524	408.412	1.7927
5	0.0702751	411.845	1.8052
10	0.0718785	415.283	1.8174
15	0.0734644	418.730	1.8295
20	0.0750346	422.186	1.8414
25	0.0765904	425.653	1.8531
Saturation temperature, 5 °C			
5	0.0403556	407.143	1.7446
10	0.0414580	410.851	1.7578
15	0.0425379	414.542	1.7708
20	0.0435979	418.222	1.7834
25	0.0446401	421.894	1.7958
30	0.0456665	425.562	1.8080
35	0.0466786	429.229	1.8200
40	0.0476779	432.897	1.8319
45	0.0486656	436.569	1.8435
50	0.0496427	440.247	1.8550
Saturation temperature, 15 °C			
10			
15	0.0299874	410.430	1.7311
20	0.0308606	414.362	1.7556
25	0.0317114	418.260	1.7578
30	0.0325427	422.133	1.7707
35	0.0333568	425.985	1.7833
40	0.0341556	429.823	1.7956
45	0.0349409	433.650	1.8078
50	0.0357139	437.470	1.8197
Saturation temperature, 25 °C			
25	0.0226242	413.289	1.7183
30	0.0233389	417.487	1.7322
35	0.0240306	421.627	1.7458
40	0.0247027	425.721	1.7590
45	0.0253575	429.779	1.7718
50	0.0259974	433.807	1.7844
55	0.0266239	437.813	1.7967
60	0.0272386	441.801	1.8087
65	0.0278427	445.777	1.8206

t, °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
Saturation temperature, - 30 °C			
30	0.0197417	414.530	1.7120
35	0.0203962	418.881	1.7262
40	0.0210272	423.159	1.7400
45	0.0216381	427.378	1.7534
50	0.0222316	431.549	1.7664
55	0.0228101	435.683	1.7791
60	0.0233733	439.787	1.7915
65	0.0239288	443.867	1.8036
Saturation temperature, 34 °C			
35	0.0178590	416.325	1.7099
40	0.0184675	420.792	1.7243
45	0.0190526	425.174	1.7382
50	0.0196178	429.487	1.7517
55	0.0201660	433.747	1.7647
60	0.0206994	437.963	17775
65	0.0212199	442.143	1.7899
70	0.0217289	446.294	1.8021
75	0.0222278	450.424	1.8141
80	0.0227176	454.535	1.8258
Saturation temperature, 38 °C			
40	0.0161865	418.076	1.7080
45	0.0167545	422.664	1.7225
50	0.0172991	427.155	1.7365
55	0.0178240	431.568	1.7501
60	0.0183320	435.918	1.7632
65	0.0188255	440.218	1.7760
70	0.0193063	444.477	1.7885
75	0.0197760	448.703	1.8008
80	0.0202358	452.901	1.8127
Saturation temperature, 42 °C			
45	0.0146964	419.779	1.7061
50	0.0152286	424.496	1.7208
55	0.0157373	429.101	1.7349
60	0.0162264	433.617	1.7486
65	0.0166987	438.062	1.7618
70	0.0171568	442.449	1.7747
75	0.0176024	446.788	1.7872
80	0.0180371	451.090	1.7995
85	0.0184622	455.360	1.8115

t, °C	v, m^3/kg	h, kJ/kg	s, kJ/kg K
Saturation temperature, - 32 °C			
35	0.0190907	417.648	1.7182
40	0.0197093	422.014	1.7322
45	0.0203062	426.310	1.7458
50	0.0208847	430.549	1.7591
55	0.0214471	434.743	1.7719
60	0.0219956	438.900	1.7845
65	0.0225318	443.028	1.7968
70	0.0230571	447.133	1.8089
75	0.0235726	451.219	1.8207
80	0.0240794	455.292	1.8323
Saturation temperature, 36 °C			
40	0.0172953	419.483	1.7162
45	0.0178708	423.961	1.7304
50	0.0184247	428 358	1.7442
55	0.0189603	432.690	1.7575
60	0.0194802	436.970	1.7704
65	0.0199865	441.207	1.7830
70	0.0204807	445.410	1.7954
75	0.0209643	449.586	1.8074
80	0.0214385	453.739	1.8193
Saturation temperature, 40 °C			
40	0.0151350	416.561	1.6995
45	0.0156982	421.274	1.7144
50	0.0162355	425.871	1.7287
55	0.0167514	430.374	1.7426
60	0.0172491	434.803	1.7560
65	0.0177313	439.171	1.7690
70	0.0182001	443.491	1.7817
75	0.0186571	447.771	1.7940
80	0.0191038	452.019	1.8061
85	0.0195412	456.241	1.8180
Saturation temperature, 45 °C			
45	0.0132841	417.308	1.6931
50	0.0138136	422.241	1.7084
55	0.0143154	427.025	1.7231
60	0.0147946	431.693	1.7372
65	0.0152550	436.268	1.7509
70	0.0156995	440.769	1.7641
75	0.0161303	445.209	1.7769
80	0.0165492	449.599	1.7895
85	0.0169578	453.950	1.8017
90	0.0173571	458.267	1.8137

Table 8 Specific heat, c_p of superheated vapour

Sat. Temp. t, °C	Specific heat, c_p kJ/kg K			
	Degree of superheat			
	= 0 °C	= 10 °C	= 30 °C	= 50 °C
– 100	0.494	0.501	0.513	0.525
– 95	0.502	0.507	0.518	0.532
– 90	0.508	0.515	0.527	0.539
– 85	0.516	0.522	0.534	0.546
– 80	0.523	0.529	0.541	0.553
– 75	0.531	0.537	0.549	0.561
– 70	0.540	0.545	0.557	0.568
– 65	0.548	0.553	0.565	0.576
– 60	0.558	0.563	0.574	0.584
– 55	0.567	0.572	0.582	0.593
– 50	0.577	0.582	0.592	0.602
– 45	0.588	0.592	0.602	0.611
– 40	0.600	0.603	0.612	0.621
– 35	0.612	0.615	0.623	0.632
– 30	0.625	0.628	0.634	0.642
– 25	0.639	0.641	0.647	0.654
– 20	0.653	0.655	0.660	0.666
– 15	0.669	0.670	0.673	0.678
– 10	0.686	0.686	0.688	0691
– 5	0.704	0.702	0.703	0.705
0	0.723	0.720	0.719	0.720
5	0.744	0.740	0.736	0.736
10	0.766	0.761	0.755	0.752
15	0.790	0.783	0.774	0.770
20	0.817	0.807	0.795	0.788
25	0.846	0.834	0.817	0.808
30	0.879	0.863	0.842	0.830
35	0.916	0.895	0.869	0.853
40	0.957	0.931	0.898	0.878
45	1.001	0.972	0.930	0.905
60	1.215	1.139	1.055	1.007
70	1.470	1.321	1.177	1.102
80	2.050	1.643	1.367	1.242
90	2.460	2.388	1.743	1.498

Table 9 Tetrafluoroethane CH_2 FCF_3 (Refrigerant 134 a) Properties of Saturated Liquid and Saturated Vapour

Temp °C	Absolute Pressure, bar	Density, kg/m^3 Liquid	Volume, m^3/kg Vapour	Enthalpy kJ/kg		Entropy kJ/(kg K)		Specific heat, c_p kJ/(kg K)	
				Liquid h_t	Vapour h_g	Liquid s_t	Vapour s_g	Liquid	Vapour
-103.30	0.0039	1591	35.26	71.89	335.07	0.4143	1.9638	1.147	0.585
– 100.00	0.0056	1582	25.04	75.71	337.00	0.4366	1.9456	1.168	0.592
– 90.00	0.0153	1554	9.719	87.59	342.94	0.5032	1.8975	1.201	0.614
– 80.00	0.0369	1526	4.250	99.65	349.03	0.5674	1.8585	1.211	0.637
– 70.00	0.0801	1499	2.053	111.78	355.23	0.6286	1.8269	1.215	0.660
– 60.00	0.1594	1471	1.077	123.96	361.51	0.6871	1.8016	1.220	0.685
– 50.00	0.2948	1443	0.5056	136.21	367.83	0.7432	1.7812	1.229	0.712
– 40.00	0.5122	1415	0.3609	148.57	374.16	0.7973	1.7649	1.243	0.740
– 30.00	0.8436	1386	0.2260	161.10	380.45	0.8498	1.7519	1.260	0.771
– 28.00	0.9268	1380	0.2068	163.62	381.70	0.8601	1.7497	1.264	0.778
– 26.07	1.0132	1374	0.1902	166.07	382.90	0.8701	1.7476	1.268	0.784
– 26.00	1.0164	1374	0.1896	166.16	382.94	0.8704	1.7476	1.268	0.785
– 24.00	1.1127	1368	0.1741	168.70	384.19	0.8806	1.7455	1.273	0.791
– 22.00	1.2160	1362	0.1601	171.26	385.43	0.8908	1.7436	1.277	0.798
– 20.00	1.3268	1356	0.1474	173.82	386.66	0.9009	1.7417	1.282	0.805
– 18.00	1.4454	1350	0.1360	176.39	387.89	0.9110	1.7399	1.286	0.812
– 16.00	1.5721	1344	0.1256	178.97	389.11	0.9211	1.7383	1.291	0.820
– 14.00	1.7074	1338	0.1161	181.56	390.33	0.9311	1.7367	1.296	0.827
– 12.00	1.8516	1332	0.1075	184.16	391.55	0.9410	1.7351	1.301	0.835
– 10.00	2.0052	1326	0.09963	186.78	392.75	0.9509	1.7337	1.306	0.842
– 8.00	2.1684	1319	0.09246	189.40	393.95	0.9608	1.7323	1.312	0.850
– 6.00	2.3418	1313	0.08591	192.03	395.15	0.9707	1.7310	1.317	0.858
– 4.00	2.5257	1307	0.07991	194.68	396.33	0.9805	1.7297	1.323	0.866
– 2.00	2.7206	1300	0.07440	197.33	397.51	0.9903	1.7285	1.329	0.875
0.00	2.9269	1294	0.06935	200.00	398.68	1.0000	1.7274	1.335	0.883
2.00	3.1450	1287	0.06470	202.68	399.84	1.0097	1.7263	1.341	0.892
4.00	3.3755	1281	0.06042	205.37	401.00	1.0194	1.7252	1.347	0.901
6.00	3.6186	1274	0.05648	208.08	402.14	1.0291	1.7242	1.353	0.910
8.00	3.8749	1267	0.05284	210.80	403.27	1.0387	1.7233	1.360	0.920
10.00	4.1449	1260	0.04948	213.53	404.40	1.0483	1.7224	1.367	0.930
12.00	4.4289	1253	0.04636	216.27	405.51	1.0579	1.7215	1.374	0.939
14.00	4.7276	1246	0.04348	219.03	406.61	1.0674	1.7207	1.381	0.950
16.00	5.0143	1239	0.04081	221.80	407.70	1.0770	1.7199	1.388	0.960
18.00	5.3706	1232	0.03833	224.59	408.78	1.0865	1.7191	1.396	0.971

20.00	5.7159	1225	0.03603	227.40	409.84	1.0960	1.7183	1.404	0.982
22.00	6.0777	1218	0.03388	230.21	410.89	1.1055	1.7176	1.412	0.994
24.00	6.4566	1210	0.03189	233.05	411.93	1.1149	1.7169	1.420	1.006
26.00	6.8531	1203	0.03003	235.90	412.95	1.1244	1.7162	1.429	1.018
28.00	7.2678	1195	0.02829	238.77	413.95	1.1338	1.7155	1.438	1.031
30.00	7.7008	1187	0.02667	241.65	414.94	1.1432	1.7149	1.447	1.044
32.00	8.1530	1179	0.02516	244.55	415.90	1.1527	1.7142	1.457	1.058
34.00	8.6250	1171	0.02374	247.47	416.85	1.1621	1.7135	1.467	1.073
36.00	9.1172	1163	0.02241	250.41	417.78	1.1715	1.7129	1.478	1.088
38.00	9.6301	1155	0.02116	253.37	418.69	1.1809	1.7122	1.489	1.140
40.00	10.165	1147	0.01999	256.35	419.58	1.1903	1.7115	1.500	1.120
42.00	10.721	1138	0.01890	259.35	420.44	1.1997	1.7108	1.513	1.138
44.00	11.300	1129	0.01786	262.38	421.28	1.2091	1.7101	1.525	1.156
46.00	11.901	1120	0.01689	265.42	422.09	1.2185	1.7094	1.539	1.175
48.00	12.527	1111	0.01598	268.49	422.88	1.2279	1.7086	1.553	1.196
50.00	13.177	1102	0.01511	271.59	423.63	1.2373	1.7078	1.569	1.218
52.00	13.852	1093	0.01430	274.71	424.35	1.2468	1.7070	1.585	1.241
54.00	14.553	1083	0.01353	277.86	425.03	1.2562	1.7061	1.602	1.266
56.00	15.280	1073	0.01280	281.04	425.68	1.2657	1.7051	1.621	1.293
58.00	16.033	1063	0.01212	284.25	426.29	1.2752	1.7041	1.641	1.322
60.00	16.815	1052	0.01146	287.49	426.86	1.2847	1.7031	1.663	1.354
62.00	17.625	1042	0.01085	290.77	427.37	1.2943	1.7019	1.686	1.388
64.00	18.464	1031	0.01026	294.08	427.84	1.3039	1.7007	1.712	1.426
66.00	19.334	1019	0.00970	297.44	428.25	1.3136	1.6993	1.740	1.468
68.00	20.234	1008	0.00917	300.84	428.61	1.3234	1.6979	1.772	1.515
70.00	21.165	996	0.00867	304.29	428.89	1.3332	1.6963	1.806	1.567
72.00	22.130	983	0.00818	307.79	429.10	1.3430	1.6945	1.846	1.626
74.00	23.127	970	0.00772	311.34	429.23	1.3530	1.6926	1.890	1.693
76.00	24.159	957	0.00728	314.96	429.27	1.3631	1.6905	1.941	1.770
78.00	25.227	942	0.00686	318.65	429.20	1.3733	1.6881	2.000	1.861
80.00	26.331	927	0.00646	322.41	429.02	1.3837	1.6855	2.069	1.967
85.00	29.259	886	0.00550	332.27	427.91	1.4105	1.6775	2.313	2.348
90.00	32.445	837	0.00461	343.01	425.48	1.4392	1.6663	2.766	3.064
95.00	35.916	772	0.00374	355.43	420.60	1.4720	1.6490	3.961	4.942
100.00	39.721	647	0.00265	374.02	407.08	1.5207	1.6093	-	-
101.03	40.560	513	0.00195	389.79	389.79	1.5593	1.5593	∞	∞

Table 10 Tetrafluoroethane CH_2 FCF_3 (Refrigerant 134 a) Properties of superheated vapour

Pressure = 1.01325 bar, Saturation temperature = 26.07 °C				Pressure = 2.00 bar, Saturation temperature = 10.07 °C			
Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)	Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)
Saturated Liquid	**1374.34**	**166.07**	**0.8701**	Saturated Liquid	1325.78	186.69	**0.9506**
Vapour	**5.26**	**382.90**	**1.7476**	Vapour	**10.01**	392.71	**1.7337**
- 20	5.11	387.68	1.7667	- 10	10.01	392.77	1.7339
- 10	4.89	395.65	1.7976	0	9.54	401.21	1.7654
0	4.69	403.74	1.8278	10	9.13	409.73	1.7961
10	4.50	411.97	1.8574	20	8.76	418.35	1.8260
20	4.34	420.34	1.8864	30	8.42	427.07	1.8552
30	4.18	428.85	1.9150				
Pressure = 4.00 bar, Saturation temperature = 8.94 °C				Pressure = 6.00 bar, Saturation temperature = 21.58 °C			
Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)	Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)
Saturated Liquid	**1263.84**	**212.08**	**1.0432**	Saturated Liquid	**1219.08**	**229.62**	**1.1035**
Vapour	**19.52**	**403.80**	**1.7229**	Vapour	**29.13**	**410.67**	**1.7178**
10	19.41	404.78	1.7263	30	27.79	418.97	1.7455
20	18.45	414.00	1.7583	40	26.41	428.72	1.7772
30	17.61	423.21	1.7892	50	25.21	438.44	1.8077
40	16.87	432.46	1.8192	60	24.16	448.16	1.8374
				70	23.22	457.93	1.8662
				80	22.37	467.75	1.8944
Pressure = 8.00 bar, Saturation temperature = 31.33 °C				Pressure = 10.00 bar, Saturation temperature = 39.39 °C			
Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)	Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)
Saturated Liquid	**1181.92**	**243.58**	**1.1495**	Saturated Liquid	**1149.06**	**255.44**	**1.1874**
Vapour	**38.99**	**415.58**	**1.7144**	Vapour	**49.16**	419.31	**1.7117**
40	36.98	424.61	1.7437	40	48.95	419.99	1.7139
50	35.03	434.85	1.7758	50	45.86	430.91	1.7482
60	33.36	444.98	1.8067	60	43.34	441.56	1.7807
70	31.90	455.08	1.8366	70	41.21	452.05	1.8117
80	30.62	465.17	1.8656	80	39.36	462.47	1.8416
Pressure = 12.00 bar, Saturation temperature = 46.32 °C				Pressure = 14.00 bar, Saturation temperature = 52.43 °C			

Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)	Temp, °C	Density, kg/m^3	Enthalpy, kJ/kg	Entropy, kJ/(kg K)
Saturated Liquid Vapour	**1118.89 59.73**	**265.91 422.22**	**1.2200 1.7092**	**Saturated Liquid Vapour**	**1090.50 70.76**	**275.38 424.50**	**1.2488 1.7068**
50	58.09	426.51	1.7226	60	66.61	433.38	1.7347
60	54.32	437.83	1.7571	70	62.25	445.31	1.7691
70	51.26	448.81	1.7896	80	58.74	456.56	1.8014
80	48.69	459.61	1.8206	90	55.79	467.60	1.8322
90	46.49	470.30	1.8504				
Pressure = 16.00 bar, Saturation temperature = 57.91 °C				**Pressure = 18.00 bar, Saturation temperature = 62.90 °C**			
Temp, °C	**Density, kg/m^3**	**Enthalpy, kJ/kg**	**Entropy, kJ/(kg K)**	**Temp, °C**	**Density, kg/m^3**	**Enthalpy, kJ/kg**	**Entropy, kJ/(kg K)**
Saturated Liquid Vapour	**1063.28 82.34**	**284.11 426.27**	**1.2748 1.7042**	**Saturated Liquid Vapour**	**1036.81 94.53**	**292.26 427.59**	**1.2987 1.7014**
60	80.74	428.99	1.7124	70	88.23	437.17	1.7296
70	74.43	441.47	1.7493	80	81.54	449.76	1.7657
80	69.61	453.30	1.7833	90	76.38	461.74	1.7992
90	65.71	464.76	1.8153	100	72.17	473.36	1.8308
100	62.43	476.01	1.8458				
Pressure = 20.00 bar, Saturation temperature = 67.49 °C				**Pressure = 22.00 bar, Saturation temperature = 71.74 °C**			
Temp, °C	**Density, kg/m^3**	**Enthalpy, kJ/kg**	**Entropy, kJ/(kg K)**	**Temp, °C**	**Density, kg/m^3**	**Enthalpy, kJ/kg**	**Entropy, kJ/(kg K)**
Saturated Liquid Vapour	**1010.74 107.46**	**299.96 428.52**	**1.3209 1.6983**	**Saturated Liquid Vapour**	**984.76 121.25**	**307.32 429.08**	**1.3147 1.6948**
70	104.37	432.22	1.7091	80	110.03	441.49	1.7303
80	94.85	445.86	1.7483	90	100.70	454.98	1.7680
90	87.97	458.49	1.7835	100	93.78	467.61	1.8023
100	82.58	470.57	1.8164	110	88.25	479.75	1.8344

Pressure = 24.00 bar, Saturation temperature = 75.70 °C			
Temp, °C	**Density, kg/m^3**	**Enthalpy, kJ/kg**	**Entropy, kJ/(kg K)**
Saturated Liquid Vapour	**958.58 136.07**	**314.40 429.27**	**1.3616 1.6908**
80	127.96	436.42	1.7112
90	114.90	451.12	1.7523
100	105.89	464.44	1.7885
110	99.00	477.04	1.8218

Table 11 specific heats, c_p, of liquid refrigerants

Refrigerant	Specific heat, c_p, kJ/kg K				
	– 34.4 °C	– 17.8 °C	– 1.1 °C	15.6 °C	32.2 °C
R11	0.816	0.829	0.842	0.854	0.871
R12	0.888	0.904	0.921	0.950	0.993
R13	1.005	1.089	1.235		
R21	0.98	0.996	1.013	1.043	1.076
R22	1.097	1.130	1.197	1.298	1.407
R113	0.816	0.846	0.862	0.883	0.909
R114	0.871	0.907	0.950	0.988	1.030
R500	1.08	1.122	1.164	1.214	1.271
R600		2.010	2.135	2.172	2.219
R717	4.396	4.605	4.605	4.601	4.815
R744	1.884	2.010	2.596	3.140	

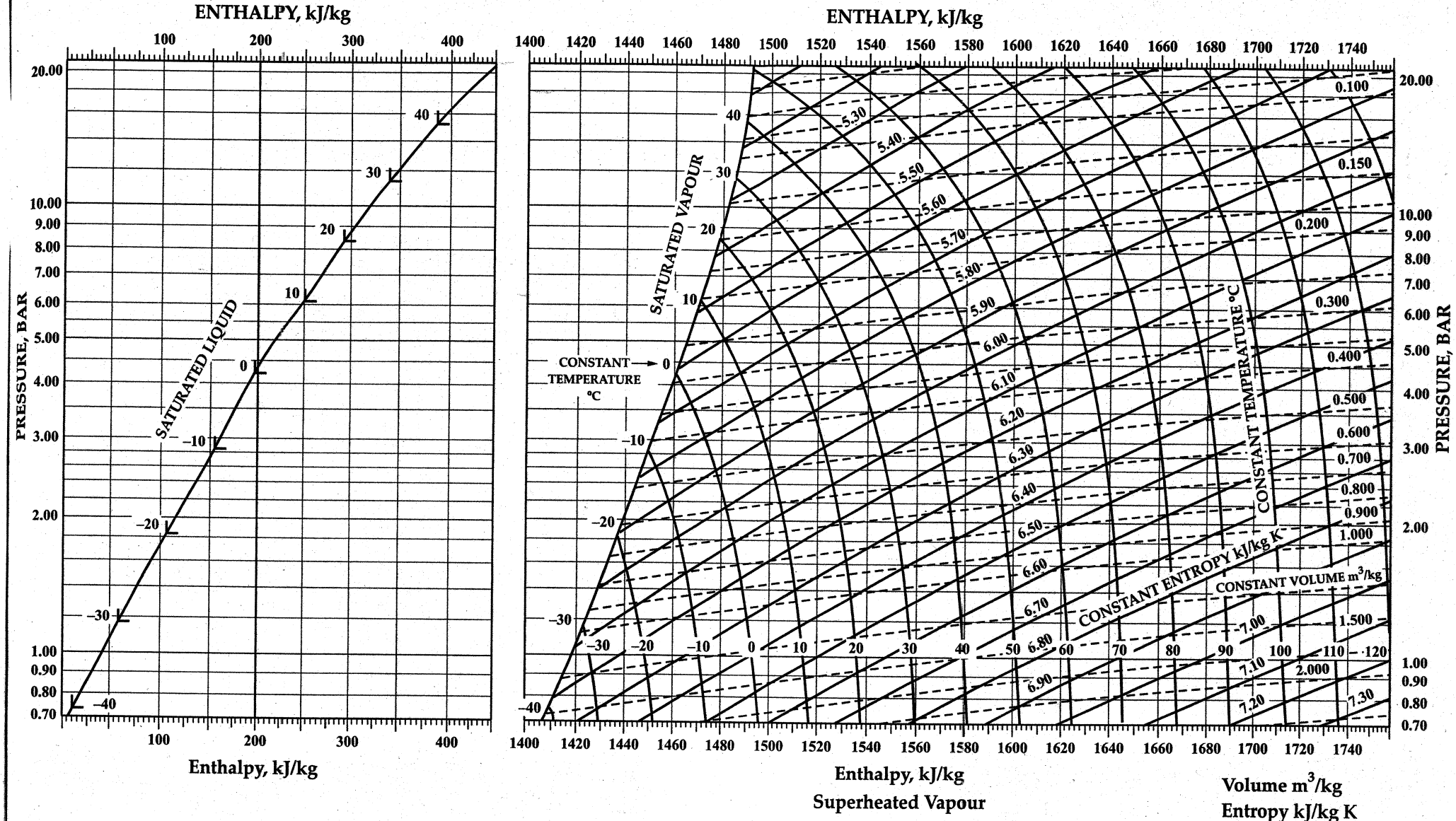
Table 12

Properties of air at atm pressure

t, °C	Vapour pressure bar	Humidity ratio, kg/kg dry air	Specific volume, m^3/kg dry air	Enthalpy, kJ/kg dry air
20	0.023373	0.01475	0.8498	57.544
21	0.024861	0.01572	0.8540	61.021
22	0.026431	0.01674	0.8583	64.646
23	0.028086	0.01781	0.8626	68.425
24	0.029832	0.01896	0.8671	72.366
25	0.031671	0.02016	0.8716	76.481
26	0.033609	0.02144	0.8763	80.777
27	0.035649	0.02279	0.8811	85.263
28	0.037797	0.02422	0.8860	89.952
29	0.040055	0.02572	0.8910	94.851
30	0.042431	0.02732	0.8961	99.977
31	0.044928	0.02900	0.9014	105.337
32	0.047552	0.03078	0.9068	110.946
33	0.050308	0.03266	0.9124	116.819
34	0.053201	0.03464	0.9182	122.968
35	0.056237	0.03674	0.9241	129.411
36	0.059423	0.03895	0.9302	136.161
37	0.062764	0.04129	0.9365	143.239
38	0.066265	0.04376	0.9430	150.660
39	0.069935	0.04636	0.9497	158.445
40	0.073778	0.04911	0.9567	166.615
41	0.077803	0.05202	0.9639	175.192
42	0.082016	0.05509	0.9713	184.206
43	0.086424	0.05833	0.9790	193.662
44	0.091036	0.06176	0.9871	203.610
45	0.095856	0.06537	0.9954	214.067
46	0.100896	0.06920	1.0040	225.068
47	0.106161	0.07324	1.0130	236.643
48	0.111659	0.07751	1.0224	248.828
49	0.117402	0.08202	1.0322	261.667
50	0.123397	0.08680	1.0424	275.198
52	0.136176	0.09720	1.0641	304.512
54	0.150072	0.10887	1.0879	337.182
56	0.165163	0.12198	1.1141	373.679
58	0.181531	0.13674	1.1429	414.572

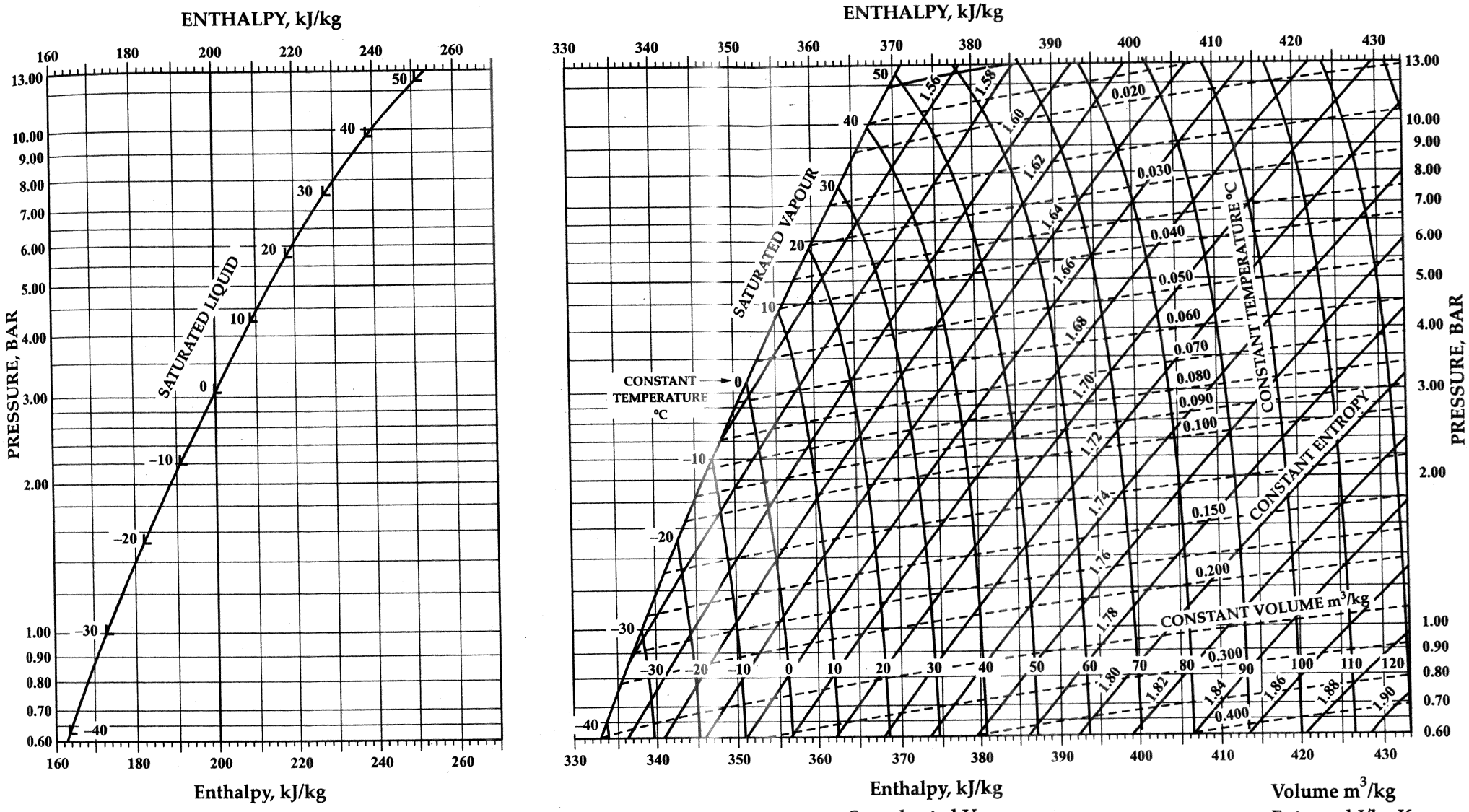
Appendix ends...

PRESSURE, BAR



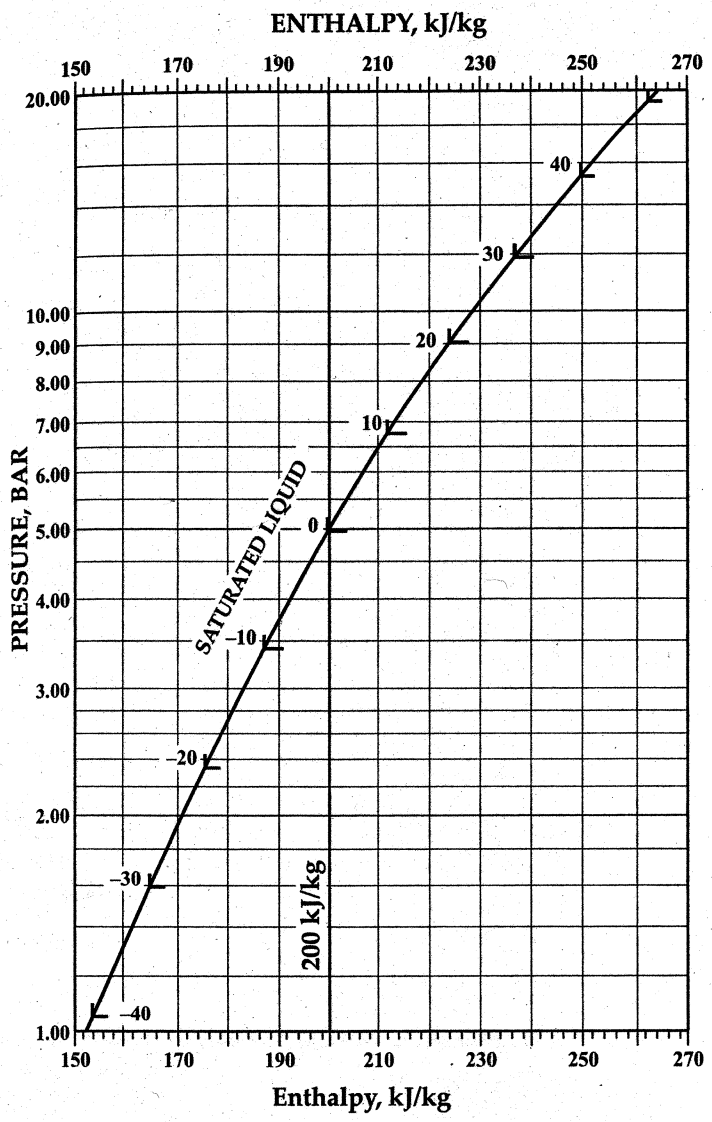
Pressure - Enthalpy Diagram of R - 717 (Ammonia)

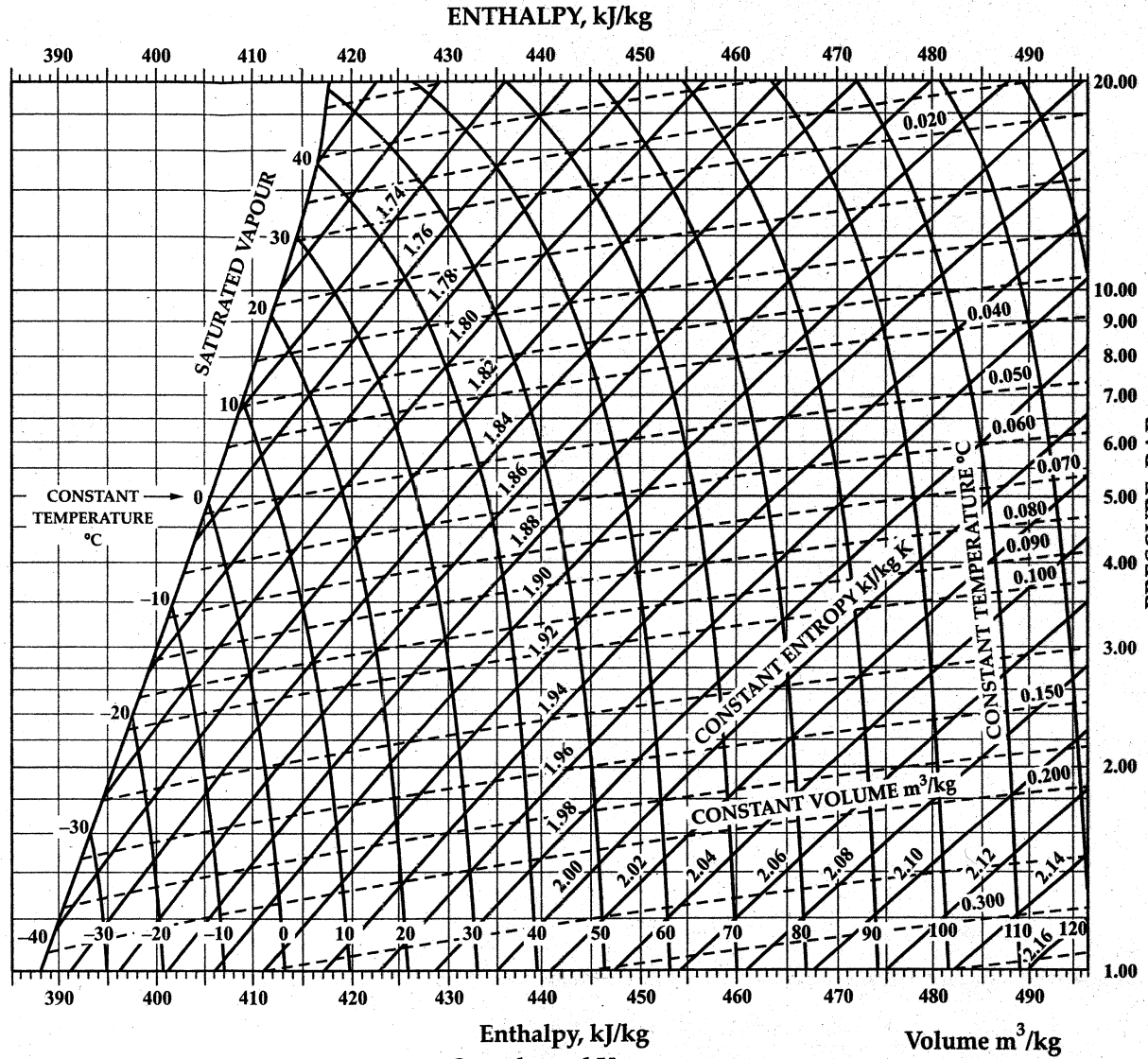
Chart - 1



Pressure - Enthalpy Diagram of R - 12

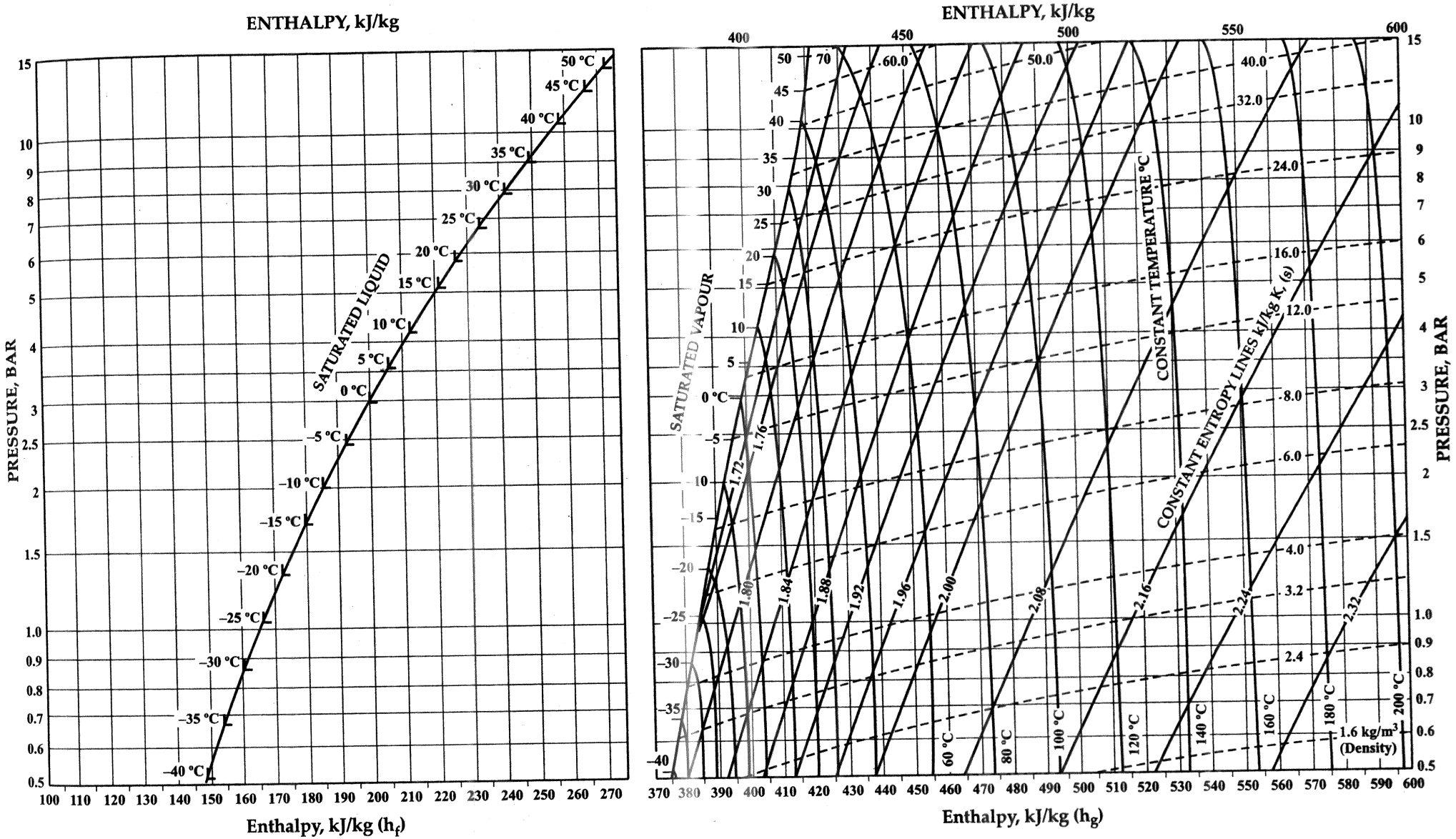
Chart - 2





Pressure - Enthalpy Diagram of R - 22

Chart - 3

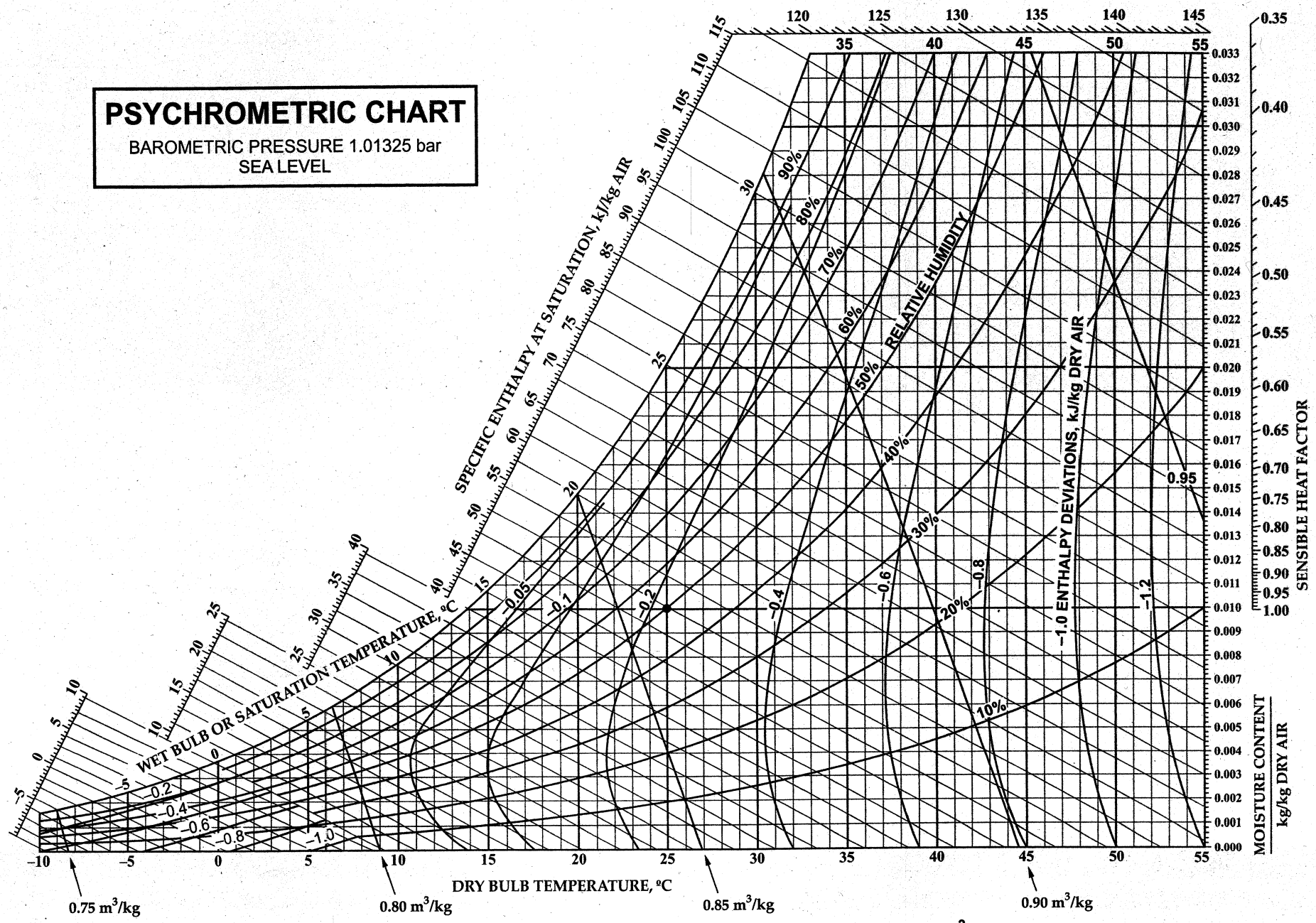


Pressure - Enthalpy Diagram of R - 134a

Chart - 4

PSYCHROMETRIC CHART

BAROMETRIC PRESSURE 1.01325 bar
SEA LEVEL



BELOW 0 °C PROPERTIES AND ENTHALPY DEVIATION LINES ARE FOR ICE

Chart - 5